

MAINTAINING CONVENTIONS AND CONSTRAINING ABSTRACTION

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Conventions play an important communicative role in mathematics. Likely due to the complex relationship between conventions and school mathematics, few education researchers have questioned or investigated the consequences of instruction and curricula that primarily, if not unquestionably, maintain conventions. Drawing on Piagetian notions of abstraction and our work with students and teachers, we argue that students' repeated experiences with instruction and curricula that maintain conventions likely constrain students' learning opportunities. We hypothesize that by 'breaking' conventions, educators could better support students in differentiating those aspects of their activity essential to a concept from those that are unessential. We characterize student work on two tasks to illustrate potential relationships between the nature of students' abstractions and what we perceive to be conventions.

Keywords: Abstraction; Radical constructivism; Student cognition; Multiple representations

...la mathématique est l'art de donner le même nom à des choses différentes. (Poincaré, 1908, p. 29)

Mathematicians and mathematics educators widely hold the study of mathematics as dependent upon abstraction. Said simply, abstraction is the process of coming to understand some sense of invariance among seemingly different activities, situations, or objects so that this understanding is not tied to particular features of any one activity, situation, or object; translating the Poincaré quote above, “mathematics is the art of giving the same name to different things.” Despite the central role of abstraction in the study of mathematics, students' abstraction processes and how educators can support students in constructing productive (in the short- and long-term) meanings through abstraction remain pressing areas of research (Oehrtman, 2008; Simon et al., 2010). We agree with Thompson (2013), who argued that making fundamental improvements to U.S. mathematics education requires that educators at all levels take the meanings that students abstract more seriously.

We draw on theoretical accounts of abstraction and our work with students (and teachers) to clarify students' abstracted meanings. Namely, we argue that students' actions and opportunities to abstract productive meanings for mathematical concepts are likely unintentionally constrained by educators who maintain conventions common to school mathematics (e.g., using the Cartesian horizontal axis to represent a function's input). After discussing our motivation and background, we provide a theoretical framing of abstraction and examples of student activity that clarify how educators maintaining particular conventions might constrain students' experiences and hence abstracted meanings. We also illustrate students operating productively in non-canonical situations so that we can clarify how experiences with such situations might influence the meanings that students abstract. We close with connections to related areas of research and ideas about lines of inquiry that can contribute insights into students' abstraction processes.

Background and Motivation

The present work emerged from a collection of studies (e.g., Moore, 2014a, 2014c; Moore, Paoletti, & Musgrave, 2013; Thompson & Silverman, 2007). Our goal in each study was to understand students' and teachers' *quantitative and covariational reasoning* (Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Saldanha & Thompson, 1998; Thompson, 1993)—how individuals conceive of a situation as composed of measureable attributes and relationships between these attributes—

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including how teachers support said reasoning. Our research settings have included teaching experiments (Steffe & Thompson, 2000) and task-based clinical interviews (Goldin, 2000), with these settings sometimes occurring in the larger context of professional development projects. Our data analyses efforts have followed a combination of conceptual analysis techniques (Thompson, 2008) and open/axial methods (Strauss & Corbin, 1998). Such efforts entail fine-grained, iterative analyses for the purpose of constructing viable models of individuals' thinking. We direct the reader to our prior studies for more detailed explanations of the work that informed this theoretically oriented paper.

Motivating our present focus, it was in trying to support students' and teachers' quantitative reasoning that we noticed their difficulty assimilating non-canonical situations in quantitative ways (Moore, Silverman, Paoletti, & LaForest, 2014; Moore, Silverman, et al., 2013). We found this outcome noteworthy for two reasons. First, the students' difficulties assimilating situations that we had designed to be non-canonical often led to their (consciously or subconsciously) imposing conventions on situations. For instance, in cases that we had designed tasks to explicitly define—in spoken or written text—a Cartesian axis oriented vertically as representing a function's input values (see the following section for task examples), students persisted in conceiving the axis oriented horizontally as a function's input (Moore, Silverman, et al., 2013). Second, the students and teachers often exhibited actions within canonical situations that they were unable to relate to and re-present in non-canonical situations. As an example, students would describe a graph of a function in terms of how two quantities varied in tandem, but then be unable to describe a non-canonical graph of that same (to us) function in an equivalent way (Moore et al., 2014). Due to the frequency of these outcomes, we inferred from the students' actions that what we perceived to be conventions were instead inherent aspects of students' meanings (i.e., not conventions) (Moore et al., 2014). This inference has led us to question the relationships between students' and teachers' abstracted meanings and what we, as educators and researchers, perceive to be conventions. In addition to discussing our theoretical framing, we use student work on two tasks to clarify important differences with respect to these relationships.

Abstraction and Student Illustrations

A detailed comparison of theories of abstraction is beyond the scope of this work given space constraints, and thus we discuss those constructs most relevant to our focus: Piaget's constructs of (pseudo-)empirical and reflective abstractions. The former type of abstraction, whether empirical or pseudo-empirical, concerns itself with the results of activity, whereas reflective abstraction concerns itself with internalized coordinations (Chapman, 1988; Piaget, 1980; von Glasersfeld, 1991). Empirical abstractions involve characteristics of experiential objects (e.g., color) including the results of sensorimotor activity on those objects. Empirical abstractions support repeated actions and motor patterns based in sensorimotor experience. Pseudo-empirical abstractions are similar to empirical abstractions in that they foreground abstraction from objects and results of activity, but they differ from empirical abstractions due to the individual introducing properties of these results into objects at the level of mental actions (Dubinsky, 1991). For instance, after working several problems graphing linear functions, a student might abstract rate of change or slope as an indicator of direction (e.g., all lines with a positive rate of change means slope upward left-to-right). Such an abstraction is not constrained solely to sensorimotor operations and observables (e.g., empirical abstractions), but the generalization does stem from patterns tied to the product of an activity (e.g., graphing linear functions) and conditions for this activity (e.g., having a rate of change and graph).

Reflective abstractions involve re-presentation, symbolization, and the coordination of (mental) actions so that the locus of abstraction is on activity itself, as opposed to activity results (Chapman, 1988; Piaget, 2001; von Glasersfeld, 1991). Relative to graphing linear functions, a student might

conceive rate of change as entailing the imagery of coordinating the relative change in two quantities. The student might also coordinate this process with graphs in several orientations or coordinate systems; the linear function and rate of change become a coordinated system so that each is a “pointer” to coordinating relative changes, but the process itself need not be carried out. In the case of pseudo-empirical abstraction, the rate of change value or slope signals executing an activity and obtaining a particular graph-as-picture; the linear function or rate of change symbolizes a structure of mental actions and associated representations all at once. In what follows, we provide student responses to two non-canonical tasks and then synthesize their activities with respect to these notions of abstraction.

Student Illustrations 1

We presented the graph in Figure 1 to undergraduate (secondary) mathematics education students (*Student Quotes for Figure 1*) with the claim that a hypothetical student deemed it a graph of the inverse sine function. We also provided a statement by the hypothetical student: “Well, because we are graphing the inverse of the sine function, we just think about x as the output and y as the input.” Among other considerations, we designed the task to incorporate a non-canonical representation of the inverse sine function and to capture fundamental aspects of the inverse relationship, namely the understanding that if $y = \sin(x)$, then $x = \sin^{-1}(y)$ with appropriate restrictions on x .

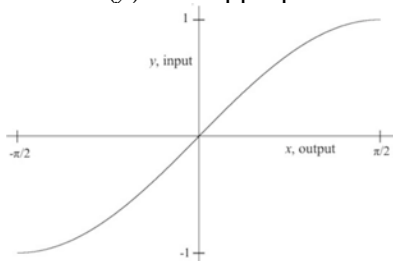


Figure 1: A Non-canonical Graph of the Inverse Sine Function

Student Quotes for Figure 1

Molly: I feel like he’s missing the whole concept of a graph...I know you can call whatever axis you know if you are doing time and weight or volume or whatever. You can flip-flop those and be OK. But not necessarily with the sine graph. Like a sine graph’s like a, it’s a graph like everyone knows about, you know.

Rowena: I’m thinking this just kind of looks like the sine graph, like the plain sine graph [laughs]. Which is going to be different...I don’t know if, or like an inverse function, like the graph of an inverse function, like, can’t be the same as the original graph.

Ariana: You could just like disregard the y and x for a minute, and just look at, like, angle measures. So it’s like here [referring to graph of $\sin^{-1}(x) = y$, see Figure 2, left], with equal changes of angle measures [denoting equal changes along the vertical axis] my vertical distance is increasing at a decreasing rate [tracing graph]. And then show them here [referring to graph of $\sin^{-1}(y) = x$, see Figure 2, right] it’s doing the exact same thing. With equal changes of angle measures [denoting equal changes along the horizontal axis] my vertical distance is increasing at a decreasing rate [tracing graph]. So even though the curves, like, this one looks like it’s concave up [referring to graph of $\sin^{-1}(x) = y$ from $0 < x < 1$] and this one concave down [referring to graph of $\sin^{-1}(y) = x$ from $0 < x < \pi/2$], it’s still showing the same thing [denotes equivalent changes on graphs, see Figure 2].

We contend that each student *understood* the given graph, and we interpret their actions to suggest differences in their meanings. Molly and Rowena understood the sine function to be uniquely

associated or in a one-to-one relationship with the given graph. An implication of this understanding is that “the sine function” or “inverse sine function” were, in the moment of their assimilating the graph, as much about associating a function name to a unique shape as about associating a function name and graph to a particular relationship. In contrast, Ariana understood “the sine function,” “inverse sine function,” and graphs in terms of a relationship between covarying quantities. Moreover, she was not constrained to reasoning about this relationship in a particular coordinate orientation. Hence, when we presented Ariana an alternative graph, she understood differences in shape did not imply a difference in the represented relationship; to Ariana, both graphs represented an invariant relationship associated with “the sine function,” written as $y = \sin(x)$, and “the arcsine function,” written as $x = \sin^{-1}(y)$.

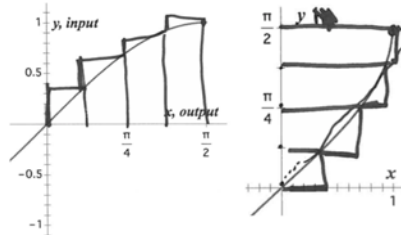


Figure 2: Two Graphs, One Relationship

Student Illustrations 2

We presented a graph (Figure 3) to undergraduate (secondary) mathematics education students (*Student Quotes for Figure 3*) with the claim that a hypothetical student produced the graph to represent $y = 3x$. We designed the task to explore rate of change (or slope as rate of change) in non-canonical axes orientations. The following student responses are in reply to our asking them to comment on the student’s solution including their interpretation of its correctness.

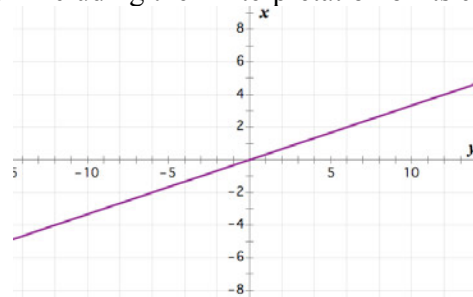


Figure 3: A Non-canonical Graph of $y = 3x$

Student Quotes for Figure 3

Rowena: Because if you turn it this way [referring to Figure 3 rotated 90-degrees counterclockwise] then this [traces left to right along the x-axis which is now oriented horizontally] and this [traces top to bottom along the y-axis] and it would be still not right though...this [laying the marker on the line which is sloping downward left-to-right] is negative slope. So I would...show them like the difference between positive and negative slopes also. Because that's something that, like, when I was in middle school we, like, learned kind of like a trick to remember positive, negative, no slope, and zero [making hand motions to indicate a direction of line for each]. Like where the slopes were...it’s important to know which direction they’re going...

Rubeus: They messed up the placement of x and y...They are looking at it like this [rotating graph 90-degrees counterclockwise]...If you are looking at it this way, it’s a negative slope

[tracing graph] and it should be a positive slope [tracing imagined graph upward left-to-right]...slope is wrong.

Amelia: I think it demonstrates understanding of the relationship...I don't care what axis you put it on...[interview rotates graph and says a student claims it has negative slope] [The way we teach slope] is just a very visual thing and no understanding of like what slope means or where it came from...your negatives are over here and your positives are over here [referring to horizontal axis value orientations]...So if we look for like a change of x of one [identifies change with a segment], zero to one, we see that y changes by positive three [identifies change with a segment]...positive slope because you are looking at change...If you're so obsessed with convention and the way things are supposed to be, you're going to take more work to get it to the way that you are comfortable with...than to just interpret the graph.

Rowena and Rubeus drew on meanings for slope rooted in visual queues including the direction of a line (e.g., “where the slopes are”). When they rotated the graph, which we interpret to be for the purpose of maintaining conventional x - y axes orientations, the students understood the “slope” to be different and hence they understood the line to represent a different relationship once rotated. In contrast, Amelia assimilated the graph in terms of reasoning about how two quantities vary in tandem while remaining attentive to how these quantities were represented with respect to the axes. No matter the rotation, Amelia understood the graph and slope in terms of a relationship such that the rate of change of y with respect to x is 3 (e.g., $y = 3x$). Amelia also distinguished between someone constrained to a conventional understanding of slope (e.g., “visual” understandings) and someone who is focused on interpreting the graph as a relationship.

Looking Across the Illustrations

Despite differences in the tasks and student responses, we interpret an underlying commonality to Molly, Rowena, and Rubeus' actions and meanings. Likewise, we interpret there to be an underlying commonality to Ariana and Amelia's actions and meanings. In the former case, the students primarily focused on observables or perceptual features of the given graphs in relation to particular topics (e.g., function name or slope). In the latter case, the students operated beyond the level of observables and perceptual features. They understood the graphs in terms of interiorized covariation schemes, and they drew on these schemes to make sense of the non-canonical representations and conceive invariance among perceptually different graphs.

Returning to theories of abstraction, we interpret Molly, Rowena, and Rubeus' actions to be compatible with meanings stemming from pseudo-empirical abstractions due to their focus on observables and the products or results of activity (e.g., produced shapes and associations with these shapes). One explanation for this result is that the students had experiences constrained to associating a function name with *one particular* graph or associating slope values with lines in *one particular* coordinate system orientation. Through repeatedly and habitually assimilating experiences to these meanings, the students abstracted function names and slope as essentially facts of perceptual shape. We interpret Ariana and Amelia's actions to be compatible with meanings stemming from reflective abstractions. That is, their understandings of “the sine function” and slope entailed internalized coordinated actions that they re-presented to make sense of the non-canonical graphs. One explanation for this understanding, which we expand upon in the next section as a future line of inquiry, is that the students had sustained opportunities to make sense of functions, function properties, and function graphs (including those that are non-canonical) in terms of covarying quantities. Hence, the students came to understand slope (or rate of change) and function names in ways not restricted to visual shape, but instead as properties of internalized covariation schemes.

Connections and Moving Forward

Mamolo and Zazkis (Mamolo & Zazkis, 2012; Zazkis, 2008) hypothesized that individuals face difficulties constructing sophisticated understandings of mathematical ideas when they only experience instruction that maintains particular conventions. Zazkis (2008) explained, “Of course, conventions are to be respected...but there is a need to become aware of them” (p. 138). Zazkis (2008) proposed that by becoming aware of conventions and structure specific to a representational system, individuals have an opportunity to develop “richer or more abstract schema” (p. 154). We agree with Mamolo and Zazkis, and we contend that students have said opportunities due to the nature of abstractions that can occur when students have experiences with both canonical and non-canonical representations. These experiences give students the opportunity to abstract meanings that differentiate what is essential to a mathematical idea from what is a convention or structure of the representational system. Students who only have experiences with conventional representations (e.g., *one* graph in a particular axes orientation) do not have the same collection of experiences or activity to reflect upon and coordinate in order to differentiate aspects of their activity that are critical to an idea from those that are not.

One area of research that the relationship between conventions and abstraction is most relevant to is that of multiple representations. Educators often frame a focus on multiple representations in terms of graphical, tabular, analytic, etc. representational contexts. We argue that it is productive to extend the ‘multiple representations’ perspective to incorporate the use of canonical and non-canonical representations within these representational contexts. Without offering students an opportunity to work with both canonical and non-canonical representations, we likely unintentionally enable students’ construction of meanings that take what we perceive to be conventions of a representational context to be essential or unquestionable facts of mathematical ideas. On the other hand, offering students experiences with both canonical and non-canonical representations in a multitude of representational contexts enable them to reflect on and coordinate their activity both *within* and *among* representational contexts. In doing so, students have a collection of experiences to reflect upon in order to come to understand an idea in terms of what is invariant among their activity within and among representational contexts; we envision that these experiences might address Thompson’s (1994) call for a focus on “representations of something that, from the students’ perspective, is *representable*” (p. 40).

We also conceive relationships between abstraction and conventions to be relevant to researchers who explore students’ quantitative and covariational reasoning. Specifically, we have found it necessary to be more careful and nuanced in our claims about students’ covariational reasoning, especially in distinguishing between a student who is able to carry out a particular activity in one coordinate orientation versus a student who has internalized and coordinated their actions in a way that is not constrained to one coordinate orientation. For instance, despite students exhibiting actions in one coordinate orientation compatible with that of Ariana (e.g., drawing and comparing segments within a canonical representation of a function in ways that suggest covariation mental actions outlined by Carlson et al. (2002)), we have found that students encounter much difficulty extending their activity to other coordinate orientations and systems. We interpret such difficulties to highlight the importance of researchers identifying the extent that students’ meanings are tied to their activity and its results (e.g., pseudo-empirical abstractions versus reflected abstractions). With respect to characterizing students’ quantitative reasoning, researchers might explain students’ learning through constructing an *increasingly abstract quantitative structure*. By increasingly abstract quantitative structure, we mean a student constructing and reconstructing a quantitative structure that becomes so internalized and operational that the student is increasingly able to assimilate novel representational contexts and situations to that structure, as opposed to being dependent on activity tied to particular representational contexts and conventions.

We close by noting that we do not intend the reader to interpret that pseudo-empirical abstractions are not desirable, nor are we arguing for the dismissal of conventions. Pseudo-empirical abstractions are a critical part of the learning process and are the source material for reflective abstractions (Dubinsky, 1991). For instance, in the context of graphing, students abstracting shape-based associations is a natural outcome (Weber, 2012). An issue arises when a student repeatedly assimilates their experiences to these shape-based associations so that the shape associations become nearly the entirety of her or his operating meanings. This outcome stands in contrast to a student coming to know graphical representations so well that they can operate at the level of shape associations while anticipating these associations in terms of internalized processes that can be carried out and adjusted if necessary. Despite the critical role of pseudo-empirical abstractions to the learning process, as educators we must look to support students in pushing beyond pseudo-empirical abstractions, lest the results of those abstractions become the entirety of their meanings. Simon et al. (2010) argued that gaining insights into nuances of students' abstraction processes with respect to their activity is a promising area of research. In accomplishing this goal in the context of using canonical and non-canonical representations, we might come to better understand how to support students in experiencing mathematics as "l'art de donner le même nom à des choses différentes" (Poincaré, 1908, p. 29).

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