WHEN "HALF AN HOUR" IS NOT "THIRTY MINUTES": ELEMENTARY STUDENTS SOLVING ELAPSED TIME PROBLEMS

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This paper presents assessment study results addressing the question: Do students treat elapsed time problems differently if phrased as "half an hour" versus "thirty minutes"? A paper-and-pencil assessment was given to second (n=292) and fourth (n=205) grade students in six New England elementary schools. I compare responses on tasks presented in hour units and minute units. Results indicate that children respond differently to elapsed time questions as a function of the units provided in the question (half hour or thirty minutes) depending on the provided starting time (e.g., on the half hour versus on the second half of the clock).

Keywords: Measurement; Elementary School Education; Problem Solving

Objective

The teaching and learning of STEM topics has been identified as a key to our country's innovation as well as a gateway to individuals' job opportunities. Uniting the various STEM domains, time underlies many scientific explorations of *how things work*, from determinations of speed and impact in physical phenomena to the Earth's orbit and rotation to graphs of functions over time. At the same time, our understanding of how children interpret time in standard units is minimal. While a staple of early elementary mathematics instruction (National Governors Association Center for Best Practices [NGA Center] & Council of Chief State School Officers [CCSSO], 2010), time has been repeatedly referred to as one of the *least* studied mathematical symbol systems (Blume, Galindo, & Walcotte, 2007; Burny, Valcke & Desoete, 2009; Kamii & Russell, 2012), with little known about how children problem solve using conventional notation and units for time. This is of particular concern given the prevalent role of time underlying the mathematics of change in middle and high school (see Yerushalmy & Shternberg, 2000). Although our culture's pervasive use of digital clocks may imply a collective mastery of time, such tools mask the rich and complex mathematics that underlies the unitizing of time and in determining elapsed time.

Theoretical Framework

My theoretical framework coordinates two areas: time is an area of measurement situated within children's developing *theory of measure* (Lehrer, Jaslow, & Curtis, 2003); and cultural tools and representations mediate our thought and communication (Cole, 1996; Sfard, 2007, 2008).

First, children develop a theory of measure through everyday examinations of the attributes of objects or events. Ideally, instruction provides opportunities for children to coordinate sensorimotor actions and everyday experiences with principles of measure (such as unit iteration, the need for equal units, or tiling; see Lehrer, 2003). Such principles cohere across different measures, including length, weight, volume, and time. Like length measure, time is a measureable quantity for which humans have developed standard units. While developmentally we know how students develop an understanding of length measure and also when notions of sequencing and duration develop (Piaget, 1969), we have little understanding about how children draw upon notation for time as related to their theory of measure.

To support the development of children's theory of measure, research has emphasized the role of units (see Lehrer et al., 2003; Stephans & Clements, 2003). Unitizing is a core concept of measuring, with many measurement principles focusing on the role of unit or resulting entailments, such as the

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need for a zero-point. With standard units of time measure, a long social history has led to our current system of notation in which units of time are in groups of 12 or 24 (hours) and 60 (minutes). The groupings of these hierarchical units greatly contrast with base-10 system of numeration underlying much of the mathematics content in elementary school, suggesting that drawing upon the proportional relation between hour and minute units may pose challenges. However, little research exists that explores how children may apply and coordinate standard units for time in their problem solving. The present study seeks examines how children solve elapsed time problems in order to document whether such challenges involving unit exist and, if so, how to characterize them.

Second, I consider thinking and learning to be inextricably linked to culture (Cole, 1996; Earnest, 2015; Sfard, 2007), with tools (i.e., a digital or analog clock) and conventional notation serving mediating roles in thought and communication. This may include cultural and mathematical referents such as “half an hour” and “thirty minutes.” Analog and digital clocks represent time and its properties in different ways, with the analog clock’s intervals of time translating duration into spatial distance (Lakoff & Nuñez, 2000; Williams, 2012). Digital time provides a precise time to the minute without reflecting hour-to-minute relations. The digital time 5:10, for example, provides a quick and precise numeric realization of time. Contrasting with this, the analog clock indicates time through a length based representational context. If we consider the hour hand for 5:10, for example, one may interpret its position as not just showing the “5” as with digital notation, but its displacement from 5:00 to 5:10 as well as the length corresponding to the 50 minutes remaining in the hour from 5:10 to 6:00. Tapping into children’s developing theory of measure, the analog clock is a cultural tool that builds on principles of measure reflected on a number line. A premise of this work is that, in children’s problem solving involving time, prior experiences with notation and tools serve mediating roles, even when solving problems involving digital notation alone.

Mathematically, elapsed times are equivalent whether provided in hour-units (e.g., half an hour) or minute-units (e.g., 30 minutes). However, children may interpret such units in different ways, though prior research has not considered this. Various articles have provided important examples about how children may treat time notation in terms of base ten (e.g., Breyfogle & Williams, 2008; Kamii & Russell, 2012), such as adding 30 minutes to the time 4:40 to reach 4:70. An underlying complexity underlying standard time notation is that hierarchical units of hours and minutes are grouped by 12 (hours) and 60 (minutes and seconds), a stark contrast to the base-10 groupings underlying place value and standard algorithms in elementary mathematics. The present study seeks to contribute systematic research to support whether such issues are pervasive in children’s problem solving.

Research Questions

The present study investigates the research questions: Do children perform differently on elapsed time tasks as a function of the units of elapsed time? Do such differences depend on the starting time provided in the task? Based on these questions, I investigate patterns of responses across the focal tasks.

Methods

Participants included 292 Grade 2 students and 205 Grade 4 students drawn from six elementary schools in urban, rural, and suburban contexts in New England. Grade 2 students were selected because standards indicate children in this grade have already mastered time to the hour and half hour and are currently working on time at the 5 minutes (NGA Center & CCSSO, 2010). Grade 4 students were selected because, according to standards, time concepts including elapsed time have been mastered in prior grades, and their performances therefore illuminate any persisting differences in performance on problems involving time.

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The assessment featured 31 items, the design of which was informed by classroom observations and informal interviews with students in second and fourth grades over 1.5 years. This paper focuses on a subset of six assessment items: three items featured minute-units for elapsed time ("30 minutes") and three analogous items involving hour-units ("half an hour") (see Figure 1). For each problem type, one problem had a starting time at x:30, another corresponding to the first half of the clock (x:10), and the final to the second half of the clock (x:40).

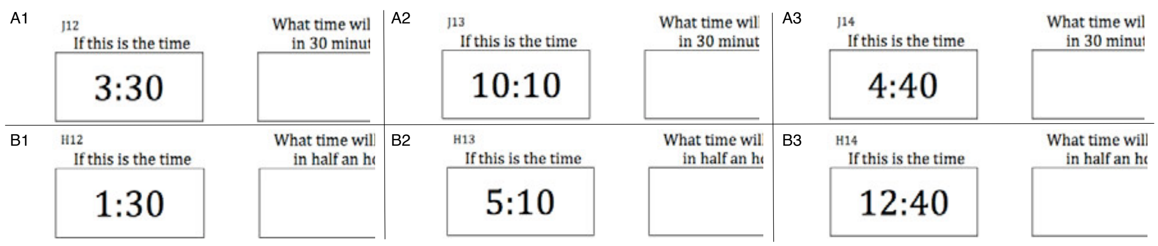

Figure 1: Assessment tasks by Problem Type (rows A and B) and Starting Time (columns 1, 2, or 3).

Teachers administered the assessments in December 2014 and January 2015 (dates were staggered due to weather-related school cancellations). Students had 25 minutes to complete the assessment; students that did not finish the assessment were not included in this analysis. The researcher was present to oversee each administration. All assessments were collected and scanned. Though not reported in the current paper, a subset of grade 2 (n = 72) and grade 4 (n = 72) students participated in problem solving interviews with analog or digital clocks.

Analysis and Results

The analysis is presented in two parts. I first present quantitative results that reveal students perform differently on "30 minutes" versus "half an hour" questions depending on the starting time. Following this, I present an analysis of responses on tasks in order to demonstrate that particular responses arise for one unit type but not for the other.

Performance by Problem Type and Starting Time

Six problems on the assessment addressed elapsed time in digital notation (see Figure 1). Each student was given one point for each correct response; incorrect responses were assigned 0 points. Means and standard deviations are represented in Figure 2. To determine whether there was a difference in performance, I conducted a Two (Problem Type) x Three (Starting Time) x Two (Grade) repeated measures analysis of variance (ANOVA) for performance.

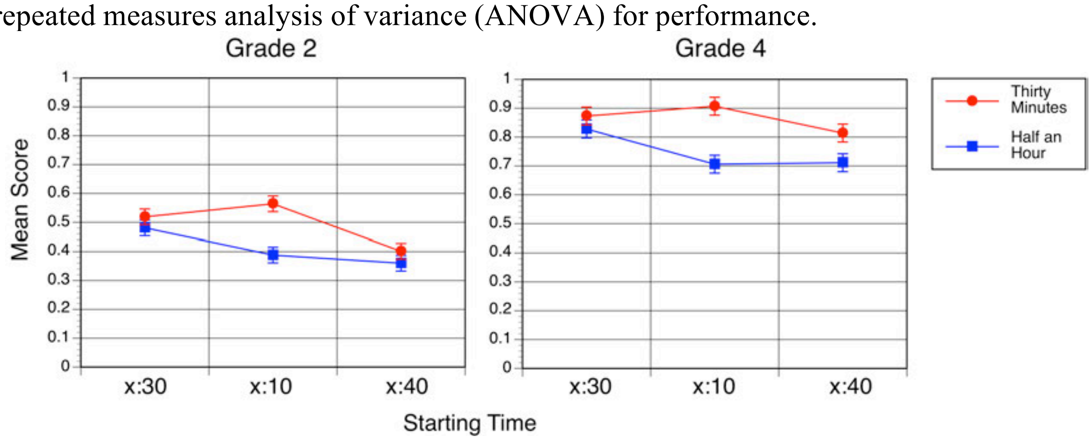

Figure 2: Mean scores with standard deviations on focal tasks.

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I first report three-way and two-way interactions in the data. While there was not a statistically significant three-way interaction between Grade, Problem Type, and Starting Time, $p = .558$, there was a statistically significant two-way interaction between Problem Type and Starting Time, $F(2, 982) = 17.798$, $p < .0001$. There were no significant interactions between Problem Type and Grade ($p = .204$) or Starting Time and Grade ($p = .204$), suggesting a similar performance profile across problems independent of grade. Given the significant two-way interaction between Problem Type and Starting Time, I also compare performance for each Problem Type for each of the three starting times for the sample (corresponding to data points in the columns of Figure 1). There were statistically different performances on "half an hour" and "thirty minute" tasks for each of the three starting times: x:30 ($p = .020$), x:10 ($p < .0001$), and x:40 ($p < .0001$).

Results thus far confirm that students interpret questions phrased as "half an hour" versus "thirty minutes" differently, with students performing better on tasks phrased as "thirty minutes." These results lead to the following question: How do students' responses vary on tasks based on the units of elapsed time? In order to consider these questions, I turn to an analysis of students' provided responses for across the six tasks.

Examining Patterns in Student Responses

I now consider patterns in incorrect responses for the six tasks presented in Figure 1 identify incorrect responses according to how the final response reflects a particular displacement from the starting time. Five types of displacements emerged across problems (see Figure 3): (1) Displacement by an hour (e.g., 4:40 with 30 minutes elapsed is 5:40); (2) Displacement by 1.5 hours; (3) Displacement to a x:00 or x:30 point (e.g., 4:40 with 30 minutes elapsed is 5:30); (4) Displacement to a multiple of 5 between 5 and 55 minutes after the starting time (e.g., 4:40 with 30 minutes elapsed is 4:50); (5) Accurate displacement with inaccurate notation (e.g., 4:40 with 30 minutes elapsed is 4:70). In addition, Figure 3 reflects students that provided as a response: (6) the starting time or one minute after the starting time (e.g., 4:40 with 30 minutes elapsed is 4:40 or 4:41); (7) Idiosyncratic responses with frequencies of 1 or 2 in the sample; and (8) "I don't know" or no response. I draw attention to parts of the graphs in Figure 3 in which particular incorrect response codes were provided for one Problem Type and not the other. Note that since Figure 3 presents the proportion based only on incorrect responses, the *n*s for each problem do not include correct responses and, therefore vary across problems. When possible for each code, I reflect on how responses relate to children's theory of measure and/or the role of tools and notation.

1. Displacement by 1 hour. The first incorrect response code reflected displacement by one hour. Students provided this response across problems regardless whether phrased as "half an hour" or "thirty minutes." The proportion was greatest for the starting time of x:30; this may be related to children treating the "o'clock" time (e.g., 4:00) as a zero-point rather than the provided starting time. In this case, solving problem A1 (3:30 plus 30 minutes) may treat the next hour mark as the zero point, thereby adding 30 minutes to 4:00 to solve the problem. While this code was applied most for when the starting time was x:30 and when the problem was phrased as "half an hour," this does not well explain how this particular response occurs for starting times of x:10 or x:40, leading to further questions regarding children's problem solving.

2. Displacement by 1.5 hours. The second strategy code reflected a 1.5-hour displacement from the starting time. Students drew upon the use of mathematical words in "half an hour" to quantify both *half* and *hour*, adding them together for a total displacement of 1.5 hours. Across the sample, only once did this response occur when the problem was phrased as "thirty minutes." In this particular case, the two units—hours and minutes—differently mediate children's problem solving.

3. Displacement to x:00 or x:30. The third code reflected a translation of either "thirty minutes" or "half an hour" into a final time with notation ending with x:00 or x:30. This category arose for

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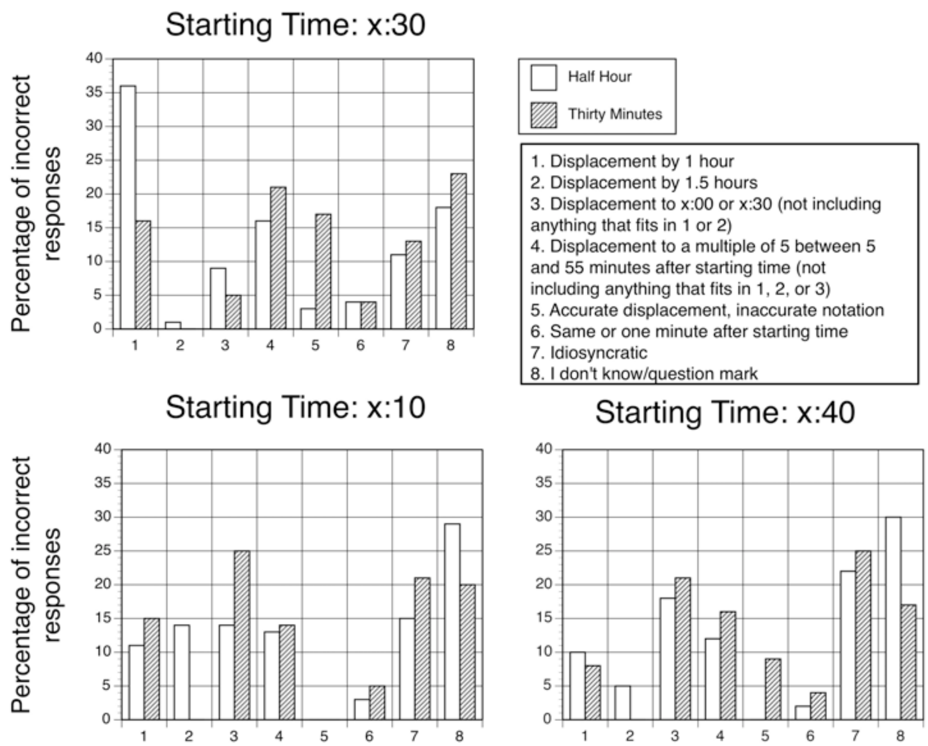

Figure 3: Proportions of incorrect response codes comparing tasks for "half an hour" or "thirty minutes" by starting time.

problem phrased both as "half an hour" and "thirty minutes." Students provided a variety of responses that resulted in this code. For example, for problem B3, students responding with "2:00" or "11:00" each received this code, though the different response choices do not reveal children's microgenetic constructions leading to these solutions. Assessment data alone do not well reveal the logic underlying the variety of responses receiving this code, though a conjecture underlying the frequency of x:00 or x:30 responses pertains both to the elapsed duration of 30 minutes/half an hour as well as the social relevance of times to the "o'clock" or "thirty."

4. Displacement to a multiple of 5. A common incorrect response for problems phrased both as "half an hour" and "thirty minutes" involved providing a new time at a multiple of 5. This particular category includes many different responses that do not fit into categories provided above. At the same time, the pattern of responses at multiples of 5 was striking. A conjecture is that the analog clock as a tool highlights time at five- minute intervals. Despite the fact that focal problems did not feature analog clocks, students' prior experiences with time, particularly the instructional and cultural emphases on identifying discrete positions in time to the five minutes (NGA Center & CCSSO, 2010), likely informed their problem solving leading to responses at five minute intervals.

5. Accurate displacement, inaccurate notation. The fifth strategy code reflected an accurate displacement with inaccurate notation, specifically adding minutes without regrouping such that the final minutes were greater than 60. This code was employed only for those tasks that reached or crossed the hour, and almost exclusively for those problems presented as "thirty minutes." This may be related to students' developing understanding of addition across elementary grades in which minutes to the right of the colon in digital notation are treated in terms of base-10 when asked to add thirty minutes. Conversely, "half an hour" tasks did not lead to this particular code. This may be because the unit "hour" cues for students spatial representations of time related to the analog clock, such that adding half an hour (as opposed to 30 minutes) to 12:40 involves does not cue the same

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base-10 reasoning. This suggests that issues with composition and decomposition in groups of 60 may result from problems phrased as “thirty minutes” in more prevalent ways than when phrased as “half an hour.”

Concluding Remarks

Results indicate that students treat problems phrased as “half an hour” and “thirty minutes” differently. While such displacements are mathematically equivalent, they pose differential challenges to children solving such problems. Based on assessment data, this paper makes the following claims and conjectures, though all must be further substantiated through interviews of children’s in-the-moment problem solving:

- Students treat “half an hour” and “thirty minute” displacements differently in their problem solving, despite the fact that such durations are equivalent;
- Students in each grade may translate an elapsed time of either “half an hour” or “30 minutes” into an hour of displacement;
- Students translate “half an hour” into 1.5 hours, but do not do this with “thirty minutes;”
- Students may translate displacements of “half an hour” and “thirty minutes” in coordination with treating the “o’clock” or “-thirty” time as a zero-point, leading to solutions ending in x:00 or x:30;
- Students draw upon the cultural emphasis of time to the five minutes when solving problems involving time;
- Accurate duration with inaccurate notation is observed when the resulting time either reaches or crosses the next hour, and when tasks are presented in minute- units.

While these findings provide information about children’s problem solving with time notation, the study leads to new questions that are yet unanswered in the current analysis. This paper reports on trends in data from an assessment given to elementary students, yet does not answer how or why students respond in such ways. For example, might students’ success on problems with a starting time of x:10 be related to whole number understandings, such that 30 is added to 10 without regard to time notation? Does “half an hour” cue for students part-whole relations and their understandings of fractions in a way that “30 minutes” does not?

Given the limited research in this area, the present paper begins to address questions about how elementary children respond to questions about elapsed time. Rather than answer such questions completely, results of this study lead to further questions. While clocks are certainly pervasive in culture, results of this study underscore that children’s conceptions of time in standard notation are quite varied. A concern of this project is in supporting all students in coordinating their developing notions of duration with standard time units, as such understandings are generative both in children’s developing theory of measure as well as engaging in any scientific investigation requires relying on time.

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