CURRICULAR TREATMENT OF FRACTIONS IN JAPAN, KOREA, TAIWAN, AND THE UNITED STATES

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This paper investigates how the selected three East Asian countries—Japan, Korea, and Taiwan—introduce and develop ideas related to fractions and fraction addition and subtraction compared to the Common Core State Standards of Mathematics and EngageNY. Looking at curricular approaches used across countries can provide a better picture of what is of importance in instruction aimed at developing students' mathematical proficiency. Understanding how the aforementioned three Eastern Asian materials treat fractions will offer both mathematics teachers and teacher educators some concrete images of the visions of the Common Core State Standards of Mathematics and specific ideas on teaching and learning of fractions.

Keywords**:** Curriculum Analysis; Elementary School Education; Number Concepts and Operations

Introduction

Teaching and learning of fractions in middle grades remain a major challenge for many teachers and students. Developing deep understanding of fractions, which has been identified as a foundation for algebra (Math commission), is a major focus of the Common Core State Standards: Mathematics (CCSSM, Common Core State Standards Initiatives, 2010). The authors of the CCSSM examined the mathematics standards from the high-achieving countries including Asian countries because one finding from the previous cross-national studies is that, in general, United States students do not perform as well as the Asian students in mathematics especially Hong Kong, Japan, Korea, Singapore, and Chinese Taipei (Taiwan) (Mullis, et al., 2008).

The observed performance differences among students in different countries might be attributed to variations in mathematical curricula (Reys, Reys, & Chavez, 2004). A growing body of research has begun to investigate the content of mathematics textbooks as a possible factor for the achievement gaps as reflected in the large international assessments such as Trends in International Mathematics Science Study (TIMSS) and the Programme for International Student Assessment (PISA). As Kilpatrick, Swafford and Findell (2001) pointed out, "what is actually taught in classrooms is strongly influenced by the available textbooks" (p. 36).

The purpose of this study is to examine how curriculum materials from the selected three East Asian countries—Japan, Korea, and Taiwan--introduce and develop ideas related to fractions and fraction addition and subtraction compared to the recommendations from the CCSSM. In this study, we seeks to understand intended students' learning opportunities in three Asian countries by analyzing how textbooks from these three countries develop a mathematics topic known to be challenging to school children. In particular, we compare and contrast the treatment of fraction concepts and fraction addition and subtractions in the three Asian textbooks compared to that in *EngageNY*. *EngageNY* is curriculum modules and resources in preK-12 developed by New York State Education Department to support teachers implement key aspects of the CCSSM (https://www.engageny.org/). The research questions that guide this study are: 1) What are the similarities and differences of the intended learning progressions of fraction concepts development among the three Asian curricula and those recommended by CCSSM? and 2) What are the similarities and differences in the development of fraction addition and subtraction fluency among the three Asian curricula and those presented in *EngageNY*?

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Cross-national comparative studies in the teaching and learning of mathematics provide unique opportunities to understand the current state of students' learning and to explore how students' learning can be improved (Stigler & Hiebert, 1999; Son & Senk, 2010).

Theoretical Background

Teaching and learning fractions has traditionally been problematic. Prior research identified one of the predominant factors contributing to the complexities of teaching and learning fractions lies in the fact that fractions comprise a multifaceted construct (Lamon, 2007). Kieran(1976)articulated that fractions consist of five subconstructs — part-whole, measure, quotient, operator, and ratio. Behr, Lesh, Post and Silver (1983) further developed Kieren's ideas and proposed a theoretical model linking the different interpretations of fractions to the basic operations of fractions as shown in Figure 1.

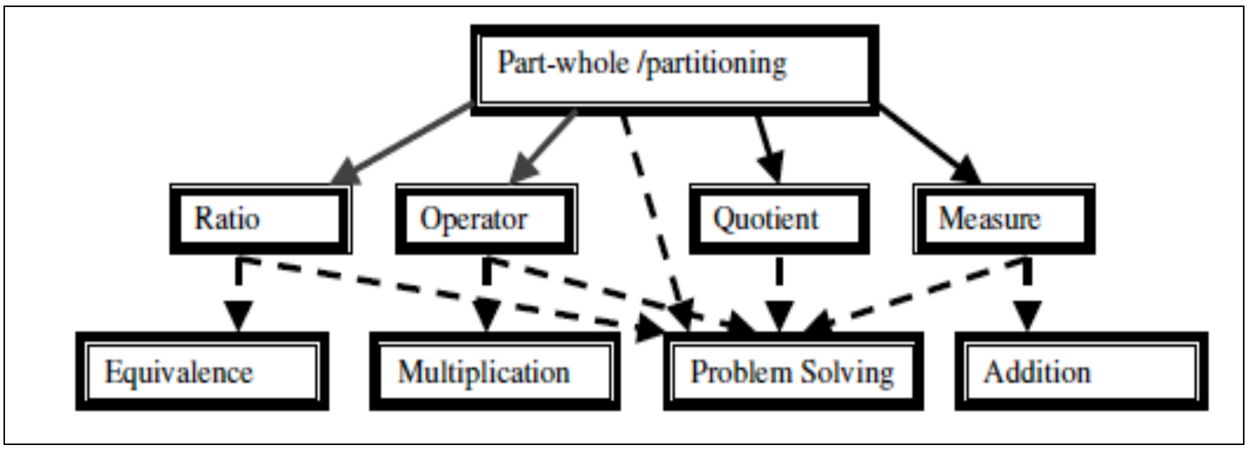

Figure 1. Five subconstructs of fractions and their relationships (Behr, et al., 1983)

According to Behr, et al., the part-whole subconstruct of rational numbers is fundamental for developing understanding of the four subordinate constructs of fractions. Moreover, the operator and measure subconstructs are helpful for developing understanding of the multiplication and addition of fractions, respectively. Although there is a consensus that fraction instruction that focuses solely on the part-whole subconstruct is limiting, there are many unanswered questions about how to incorporate these subconstructs in a mathematics curriculum (Lamon, 2007).

Research on Fractions and Fraction Addition and Subtraction

As mentioned before, over the last three decades researchers and scholars have identified several factors contributing to students' difficulties in learning fractions. The NCTM (2000) provides the following instructional guidelines in developing deep understanding of fractions: (1) begin with a simple contextual task and (2) have students explore each of the operations using a variety of representations and models. We took account of them in our analytical framework. In particular, as word problems serve as a way to contextualize mathematical operations (Carpenter et al., 1999), this study investigated how the meanings of fraction addition and subtraction are addressed and developed through the word problems in each curriculum. The Cognitively Guided Instruction framework (CGI) and CCSSM's problem types, which categorizes addition and subtraction word problems based on the semantic structure of problems shown in Table 1, was utilized for the analysis.

Table 1: Four problem types for addition and subtraction (from Carpenter, et al., 1992)

Problem Type	Unknown Factors		
Join (Add to)	**(Result Unknown)** Connie had 5 marbles. Juan gave her 8 more marbles. How many marbles does Connie have altogether?	**(Change Unknown)** Connie had 5 marbles. How many marbles does she need to have 13 marbles altogether?	**(Start Unknown)** Connie had some marbles. Juan gave her 5 more. Now she has 13 marbles. How many marbles did Connie have to start with?
Separate (Take from)	**(Result Unknown)** Connie had 13 marbles. She gave 5 to Juan. How many marbles does Connie have left?	**(Change Unknown)** Connie had 13 marbles. She gave some to Juan. Now she has 5 marbles left. How many marbles did Connie give to Juan?	**(Start Unknown)** Connie had some marbles. She gave 5 to Juan. Now she has 8 marbles left. How many marbles did Connie have to start with?
Part-Part-Whole (puttogether/take apart)	**(Whole Unknown)** Connie has 5 red marbles and 8 blue marbles. How many marbles does she have altogether?		**(Part Unknown)** Connie has 13 marbles: 5 are red, and the rest are blue. How many blue marbles does Connie have?
Compare	**(Difference Unknown)** Connie has 13 marbles. Juan has 5 marbles. How many more marbles does Connie have than Juan?	**(Bigger Unknown)** Juan has 5 marbles. Connie has 8 more than Juan. How many marbles does Connie have?	**(Small Unknown)** Connie has 13 marbles. She has 5 more marbles than Juan. How many marbles does Juan have?

Method

The primary data source of this study includes the national curriculum guidelines and selected textbook from the three Asian countries. CCSSM and fraction modules from *EngageNY* were analyzed. This study applied the content analysis method to analyze the problems presented in the mathematics textbooks (Confrey & Stohl, 2004). The data analysis of this study went through several iterations with respect to the following aspects:

- Examine the overall curricular flow on fractions – what topics are introduced in which grade
- Analyze in details how textbooks develop the concept of fractions, paying specific attention on fraction subconstructs
- Analyze in details how textbooks develop addition/subtraction of fractions:
- Word problem types, e.g., add to, take from, put together/take apart, compare.
- Types of representations

Note that the textbook series from each country were analyzed in their respective languages by the three authors who are native speakers of the respective languages. Because problem contexts and fraction subconstructs are not always visually verifiable, we needed to calibrate our coding of these two factors. We used the English translation of the Japanese series (Fujii & Iitaka, 2012) and analyzed them independently. We then compared our analysis, and whenever there was a discrepancy in our analyses, we discussed the particular instance until a consensus was reached.

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Summary of Findings

Overall Curricular Flow on Fractions

Table 2 shows the overall curricular flow on fractions across the three Asian countries as well as the CCSSM. All Asian curricula shared similar overall flows with some embedded variations. Japanese curriculum introduces the initial concept of fractions in grade two with a brief introduction of the concepts of 1/2 and 1/4. In third grade, the focus is on the continued development of fraction concepts along with the introduction of the addition and subtraction of fractions with the same denominator. Fraction concepts are further extended in grade four to the improper fractions and mixed fractions. The quotient meaning of fractions is introduced in Grade 4 in the Korean and the Taiwanese curriculum, while the idea is introduced in Grade 5 in the Japanese curriculum.

In the Japanese and the Korean curricula, simple cases of equivalent fractions are discussed in Grade 4 even though the formula for creating equivalent fractions is not discussed until Grade 5, while the Taiwanese curriculum addresses this topic in Grade 4. All three curricula took time develop fraction addition and subtraction over three grades: first with proper fractions with the same denominators, then with improper fractions or mixed numbers with the same denominators. Fraction addition and subtraction with unlike denominators follows the discussion of equivalent fractions in the fifth grade curricula. In all three curricula, multiplication of fractions are also discussed in multiple grades. Both the Japanese and the Korean curricula formally discuss multiplication of fractions by whole numbers in Grade 5 and multiplication by fractions in Grade 6. The Taiwanese curriculum follows the same sequence but one grade level earlier. The Japanese and the Korean curricula follow the similar sequence with division while the Taiwanese curriculum discuss division of fractions only in Grade 6.

Table 2:Curricula Flow on Fractions in curriculum in three countries

	Japan	Korea	Taiwan	CCSSM
Fractions as equal shares	2	3	3	1/2/3
Fraction as number	3/4	¾	3/4	3/4
Comparison	3/4/5	3/4/5	3/4	3/4
Equivalent fractions	4/5	4/5	4	3/4
Fractions as quotients	5	4	4	5
Addition/subtraction	3/4/5	3/4/5	3/4/5	4/5
Multiplication	5/6	3/5	4/5	4/5
Division	5/6	5/6	6	5/6

The overall flow in the three Asian curricula is fairly similar to the overall flow in the CCSSM. In the CCSSM, simple fraction ideas are introduced in Grades 1 and 2 through equal partitioning of geometric shape like circles and rectangles. This approach is similar to the Japanese curriculum in Grade 2. The formal introduction of fraction as numbers occurs at Grade 3 for all curricula and CCSSM. However, while the three Asian curricula discuss addition and subtraction soon after they start the formal instruction of fractions in Grade 3, the CCSSM delays the discussion of addition and subtraction until Grade 4. Although the timing of addition/subtraction instruction is different, the three Asian curricula and the CCSSM all rely heavily on the measure subconstruct of fractions, that is, fractions are collections of unit fractions. While multiplication of fractions in the CCSSM follows the same sequence as the three Asian curricula, that is, multiplication of fractions by whole numbers first, then multiplication by fractions, the way division of fractions is developed in the CCSSM is different. Unlike the Asian curricula which first discusses the division of fractions by whole numbers then division by fractions, the CCSSM discusses division of unit fractions by whole numbers and

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whole numbers by unit fractions in Grade 5 before discussing division of fractions in general in Grade 6.

The Development of Fraction Subconstructs and Contexts/Models Used

Table 3 summarizes which of the five subconstructs offractions are present in different grade levels of themathematics curricula from the three Asian countries and from *EngageNY*.It is clear from this table that there are variations in the ways different fraction subconstructs are used in the three Asian curricula. However, one commonality is that the foundational role the part-whole subconstruct appears to play in the three curricula. Moreover, the part-whole and the measure subconstructs are the two primary subconstructs undergirding the initial instruction of fractions, including addition and subtraction of fractions, in the three curricula. This approach is similar to the way the *EngageNY* (and the CCSSM) introduces fractions, starting with the partitioning of wholes in Grades 1 and 2, and then in Grade 3, developing the understanding of non-unit fractions as collections of unit fractions. Students in *EngageNY* are then expected to use that knowledge to think about fraction equivalence, ordering, and addition and subtraction of fractions in Grades 4 and 5.

Table 3: Fraction subconstructs that appeared in the Asian curricula and in EngageNY

Grades	Japanese	Korean	Taiwanese	EngageNY (US)
2	**Part-whole**			**Part-whole**
3	Part-whole **Measure**	**Part-whole** **Measure** **Operator**	**Part-whole** **Measure**	Part-whole **Measure**
4	Part-Whole Measure	Part-whole Measure **Quotient**	Part-whole Measure **Quotient**	Part-Whole Measure **Operator**
5	Part-Whole **Quotient** Measure	Part-whole Measure Quotient Operator	Part-whole Measure Quotient **Operator**	Part-Whole Measure **Quotient** Operator
6	Part-Whole **Operator** **Ratio**	Part-whole Measure Operator **Ratio**	Part-Whole Measure Quotient **Ratio**	Part-Whole Measure Operator **Ratio**

Note: A sub-construct that is newly addressed in each grade is bolded in Table 3.

While all five subconstructs of fractions are present in each curriculum, there exist different emphases on the operator and ratiosubconstructsamong the four curricula. The Japanese curriculum emphasizes a unitary view of fractions thus putting more emphasis on the measure subcontstruct than the othersubconstructs. The primary representation is linear (either tape or number line) (see example below).

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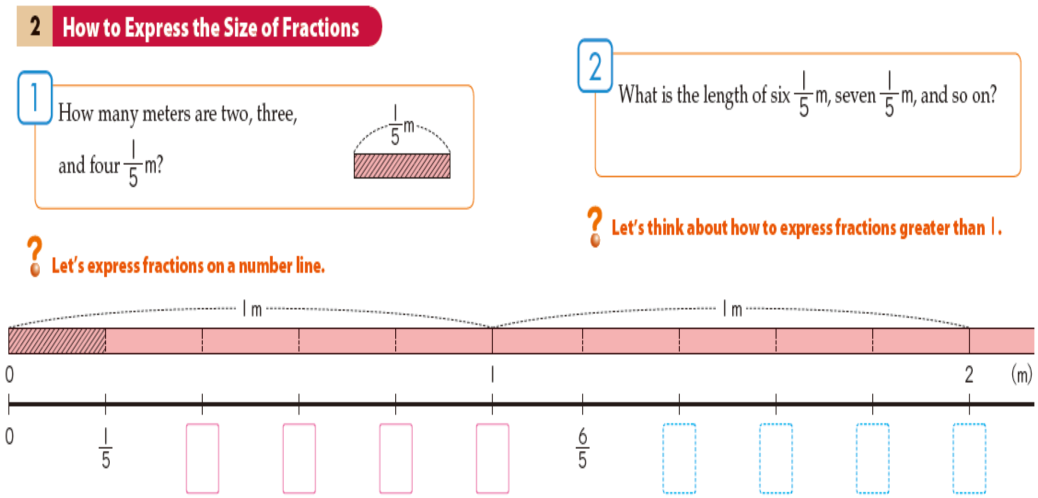

Figure 2. Typical representations used in the Japanese textbooks

One unique feature of Korean curriculum is to introduce operator construct much earlier than Japanese and Korean curricula. Below is an example from the 3rd Grade Korean textbook asking students to find what is ¾ of 8. A fraction as operator typically implies partitioning followed by iterating: for example, 3/4 as operator (3/4 of 8) implies first partitioning the object into four equal parts and then making three copies of (iterating) one of those parts as shown below. In addition, the Korean curriculum addresses the meaning of fractions as operator first in grade 3 and then in grade 5 and 6 as multiplication of fractions is introduced and developed. Furthermore, the ratio subconstruct is used to promote the concept of equivalence and, subsequently, the process of finding equivalent fractions. Thus the Korean curriculum appears to intentionally introduce students to the variety of subconstructs sooner than the other two curricula.

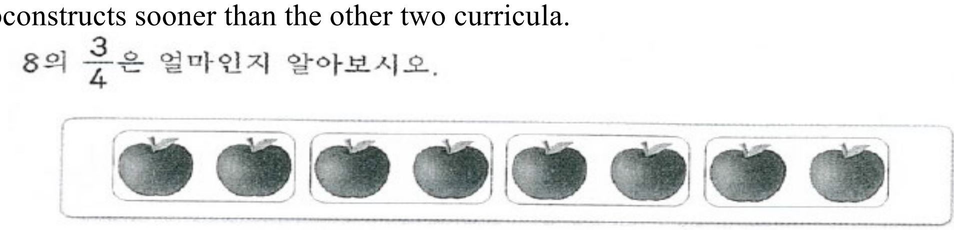

Figure 3. An example problem introducing operator construct in the Korean curriculum

How are addition and subtraction of fractions introduced and developed?

Table 4 describes the types of word problems presented in the mathematics curricula from the four countries. We found that all three Asian curricula included relatively small number of word problems as they discussed addition and subtraction of fractions. Moreover, the problem types found are generally simpler types such as Join/Separate-Result-Unknown and Part-Part-Whole-Whole-Unknown. However, *EngageNY* and Taiwanese textbooks include other types of word problems such as Separate-Initial\Change-Unknown and Part-Part-Whole Part Unknown, which emphasize the relationship between fraction addition and subtraction. Note that the Compare-Smaller-Unknown type was only found in the *EngageNY*

In addition, we found that a significant percentage of pure computation exercises of fraction additions and subtractions with different denominators are included in each curriculum with the following number and percentage: Korean 76 (76%), Japan 29 (74%), Taiwan 24 (75%), and EngageNY (66%).

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Table 4:Types of addition and subtraction word problems in four curricula

Join Result Unknown *JKTUS*	Join Change Unknown *TUS*	Join Initial Unknown
Separate Result Unknown *JKTUS*	Separate Change Unknown *TUS*	Separate Initial Unknown *TUS*
Part-Part-Whole Whole Unknown *JKTUS*	Part-Part-Whole Part Unknown *TUS*	
Compare Difference Unknown *JKT US*	Compare Smaller Unknown US	Compare Larger Unknown *T*

Note: J stands for Japanese textbooks with K for the Korean texts, T for the Taiwanese texts, and US for *EngageNY*.

While discussing addition and subtraction, all three curricula incorporated various models including linear, area, and discrete. Below is an example from the 3rd Grade Taiwanese textbook using linear model. The question asked how much would left if Wei-Ting had a 7/10 meter rope and used 3/10 meters for her art project. The students are encouraged to think "7/10 meter is 7 units of 1/10 meter, used 3 units of 1/10 meter. So left will 4 units of 1/10 meter, which is…."

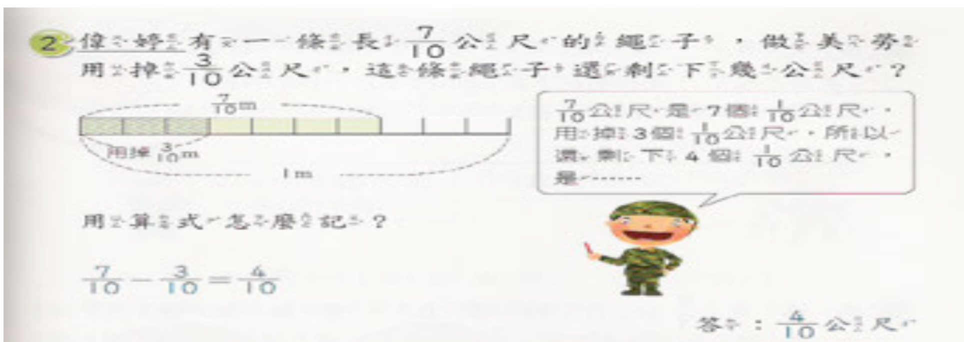
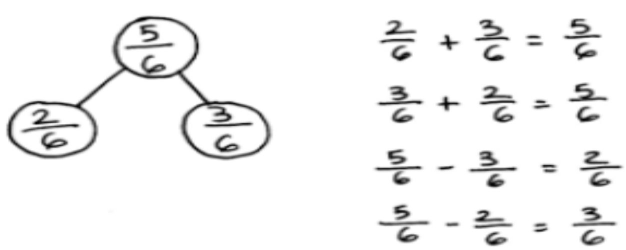


Figure 4. Typical representations used in the Taiwanese textbooks

Similar to the three Asian textbooks, modules from EngageNY incorporated various models. The most frequently model is a tape diagram, followed by a number line and area model. One unique model (or an approach) used in EngageNY is using a number bond to find the sum or the difference as shown below.



Discussion

The findings of this study showed more similarities than differences among the three Eastern Asian curricula in terms of overall flow, approach, and grade level expectations. Moreover, as far as the early discussion of basic fraction concepts, there is a significant alignment between the three Asian curricula and the CCSSM. However, some of the differences may have significant implications. For example, we noted that how the Korean curriculum incorporates all five fraction subconstructs from early grades while the Japanese curriculum takes a much more deliberate pace to introduce the subconstructs beyond part-whole and measure. Further research in the way students in these countries understand fractions may tell us how their learning may be impacted by the curricular decision. Research has long reported that many students and even teachers have difficulty understanding fractions (Ball, 1990; Behr et al. 1992; Ma, 1999).An increased understanding how fractions and fraction addition and subtraction are introduced and developed in other countries provides us with a tool to critically reflect on our current practices and that can help us to improve the quality of both curriculum materials and fraction instruction.

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