STUDENTS' STRATEGIES FOR ASSESSING MATHEMATICAL DISJUNCTIONS

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This paper presents results from three teaching experiments intended to guide students to reinvent truth-functional interpretations for mathematical disjunctions. The initial teaching experiments revealed that students' emergent strategies for assessing disjunctions did not entail or facilitate the development of a relevant partitioning of example space (comparable to Venn diagrams). Students were unable to form generalizable strategies for finding relevant exemplars to evaluate quantified disjunctions. The latter teaching experiment, in contrast, successfully prompted students' to attend to reference and partitioning of the referent space through an alternative instructional sequence. I set forth the methodology and findings of this study to demonstrate how conventions of mathematical logic can emerge within students' mathematical activity toward the end of their apprenticeship into proof-oriented mathematics.

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In proof-oriented mathematics, mathematicians embed mathematical meaning in mathematical language (definitions, theorems, proofs, etc.). This requires a high level of clarity and precision in mathematical language, which is why mathematicians were the first to invent formal languages (Azzouni, 2009). Formalizing language requires attending to the relationship between linguistic form and meaning. For mathematicians, this involved 1) creating equivalences – *but* and *and* are mathematically equivalent connectives, 2) disambiguation – distinguishing *or* and *either…or* as capturing the inclusive and exclusive everyday meanings of *or*, and 3) creating truth-functions relating truth-values of component and compound predicates. These aspects of formal language and its acquisition stand in contrast to natural language, which is learned through many more implicit or preconscious processes and which entails looser relations between form and meaning (see Stenning, 2002).

How then can mathematics students in proof-oriented mathematics courses learn formal, mathematical language, specifically as it pertains to the relation between form and meaning? Using the guided reinvention heuristic of Realistic Mathematics Education (Gravemeijer, 1994), I sought to engage students in conscious and effortful systematization of their use of mathematical language. I guided students to reinvent truth-functional definitions for mathematical disjunctions and conditionals in a series of short teaching experiments (Steffe & Thompson, 2000). In this paper I report on the major patterns of student interpretation of mathematical disjunctions, how failure to partition example spaces inhibited their ability to reinvent normative interpretations of quantified disjunctions, and an alternative instructional sequence that supported the emergence of normative interpretations of quantification.

Studies of students' interpretations of linguistic form

Because there is ample evidence that students' untrained interpretations of mathematical language differ significantly from that of mathematicians (e.g. Durand-Guerrier, 2003; Epp, 2003), many Introduction to Proof courses include a unit on logic. However, my method departs from many of the common approaches to teaching these topics in mathematical logic because 1) I want students to impose logical form on meaningful mathematical statements rather than abstract or nonsensical ones and 2) I problematize students' *reasoning toward an interpretation* (Stenning, 2002; Stenning & van Lambalgen, 2004) of language. The majority of psychological and mathematics education studies of student's interpretations of linguistic form tend to elicit students' preconscious interpretive processes (Evans, 2005; Inglis & Simpson, 2008), but assess those interpretations against a single, formalized linguistic meaning. This assumes some logical structure is embedded in language or semantic content and that people are irrational for reasoning alternatively (e.g., Stanovich, 1999). As Stenning (2002) eloquently argues, everyday linguistic interpretation is far too complex and varied for this approach. One may distinguish three views of *logic*'s relation to language use that clarify my stance. Logic can be thought of as a description of language use (common in later 20th century logic, Stenning, 2002), a prescription for proper language use (as many psychologists deem it), or the constructed product of a learning process of systematizing language (my proposal). I call this third view students *reasoning about logic* to denote its conscious and reflective nature. I adopt this lens because many studies suggest formal logic is a poor model of most students' reasoning, but proof-oriented mathematics requires that students conform their reasoning to mathematical norms. Stenning's (ibid.) findings support this study's use of meaningful mathematical statements, as he states, "formal teaching can be effective as long as it concentrates on the *relation* between formalisms and what it formalizes" (p. 187) and "logic teaching has to be aimed at teaching how to find form in content" (p. 190). So, I operationalize logic not in terms of students learning formalisms (e.g. truth-tables, Venn diagrams), but as their progressive systematization of their interpretations of mathematical statements till they impose a consistent, generalizable, and normative form.

Methods

I recruited three pairs of Calculus 3 students from a medium-sized Midwestern university to take part in short teaching experiments. I chose this course to find students who were mathematically proficient, could benefit from learning formal mathematical language, and who had not taken proof-oriented mathematics courses. I identified their background in learning logic via an online survey. Students met in pairs with the author for six one-hour sessions, and were compensated monetarily for their participation. The first three sessions focused on mathematical disjunctions and the latter three sessions on mathematical conditionals.

The guided reinvention approach helped to identify the interpretations students imposed upon the statements, how those interpretations shifted upon reflection, and which tasks elicited reasoning that approximated normative interpretations. I asked students to determine whether provided mathematical statements were true or false, then to systematize and describe their method, before asking them to negate the statements. It was initially anticipated that disjunctions would be easier for students to formalize and provide a foundation for interpreting conditionals (known to be a problematic linguistic form, Evans, 2005). Instead, disjunctions were quite challenging for mathematically important reasons. Thus, I only report on data from the first three days of each teaching experiment. The first two pairs met simultaneously and employed the same instructional activities. The third met several months later using modified activities. As such, data is presented as two experiments distinguished by their anticipated learning trajectories and instructional tasks. Table 1 presents a selection of the statements used in the study, with "D1:3" denoting the third statement on Day 1. An apostrophe denotes an item from Experiment 2.

Consistent with the teaching experiment methodology (Steffe & Thompson, 2000), during Experiment 1 the author served as teacher/researcher and another researcher served as outside observer. The observer kept field notes during each session and the researchers debriefed after each teaching session. Video recordings were also reviewed each day to form and test hypotheses about student learning to inform the teaching activities for the following session. Full retrospective analysis commenced after the experiment ended. The author coded data in the open and axial method of grounded theory (Strauss & Corbin, 1998). Codes related to 1) truth-value assessment strategies (e.g., one condition false makes the disjunction false), 2) paraphrases of provided statements (e.g., introducing "either…or" language), 3) modes of reasoning about logic (e.g., attending to the meaning of the *or* connective), 4) clarification of semantic information (e.g., identifying relevant warrants such as "all squares are rectangles"), and 5) negating actions (e.g. negating [A or B] with [not A or not B]). I report on common trends in students' interpretive behavior and emergent links among their strategies, interpretations, and particular disjunctions.

Table 1: Sample disjunctions provided to study participants

D1:1 "Given an integer number x, x is even or x is odd" D1:2:"The integer 15 is even or 15 is odd. D1:7 "The real number 0 has a reciprocal $\frac{1}{0}$ such that $0 \cdot \frac{1}{0} = 1$ or 0=0." D1:9 "Given any even number z, z is divisible by 2 or z is divisible by 3" D2:6 "Given any triangle, it is equilateral or it is not acute." D2:7 "Given any triangle, it is acute, or it is not equilateral."	D3:3 "10 is an even number or 20 is an even number" D3:4 "13 is an even number or 6 is an even number." D2':6 "For which integer numbers *z* is it true that '*z* is divisible by 4 or *z* is divisible by 3'" D2':10 "For which real numbers y is it true that "y<3 or y>5" D2':12 "For which triangles is it true that 'it is equilateral or it is not acute.'"

Experiment 1

Of the four participants in this experiment, one pair had no training in logic and the other pair had completed a philosophy course in logic. Their patterns of reasoning were nearly identical. When initially assessing the truth-values of non-quantified disjunctions, both groups declared a disjunction with a false component false (e.g. D1:2 and D1:7). Only on D1:9 did either group begin explicitly attending to the connective *or* and its role in the statements' meaning. Both groups at this point also began distinguishing the truth-values of the two components from that of the disjunction. For instance, Ron said, "This just got interesting cause when you say "or" only one of 'em has to work, not both of 'em." When asked to revisit their initial decisions, both groups reinterpreted D1:2 as true because of the *or* connective. Students were more reluctant to affirm D1:7 as it seemed more mathematically absurd, but later decided that the 0=0 condition also made it true. By the end of the first session, both groups were consistently interpreting non-quantified disjunctions in a manner consistent with the normative truth-functional definition. As Ovid said, "Cause "or" for me means either it could be one or the other or both."

Students generally had more trouble with quantified disjunctions where the truth-function was not sufficient to assess the truth-value of the statement. Some statements afforded semantic affirmation without testing particular cases as with D1:1 where the categories are exhaustive. In other cases, though, students used a *sentential testing strategy* in which they picked examples and reread the statement to evaluate whether it "covered" the given case (often reading left to right). This strategy reduced quantified disjunctions to a sequence of non-quantified disjunctions, but it also necessitated a strategy for picking cases and organizing the example space.

Partitioning the example space

Experiment 1 participants did not spontaneously develop an intentional way to partition the spaces of examples because each space was pre-organized according to familiar mathematical categories (e.g. even, acute, rectangle). The normative logical partitioning of examples (as portrayed in Venn diagrams) distinguishes cases that satisfy each component condition of the disjunction such that the examples fall into four categories (TT, TF, FT, and FF). Because students' reasoning stayed focused on the statements themselves, they failed to attend to how the statement provided a novel partitioning of example spaces. This was not problematic for cases that could be easily and exhaustively seriated such as the integers or even integers. Students tested cases sequentially (2, 4, 6, 8…), usually assigning a truth-value after 3-5 examples. However, both pairs of students struggled to

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assess geometric statements such as D2:6 and D2:7. This is because they reasoned about triangles in terms of familiar semantic categories – equilateral, acute, right, obtuse – rather than treating right and obtuse as equivalent relative to the given statements – non-equilateral, non-acute. Without a simple way of exhausting the example space, they were never sure if a statement was true of all triangles.

Students developed some other normative and non-normative strategies that helped them resolve quantification issues. First, recognizing that all equilateral triangles are acute, some students incorrectly concluded that not equilateral meant not acute, implying that statements like D2:6 were true. Students struggled in other ways with how to interpret negative predicates, often substituting a positive predicate that was non-complementary, such as replacing “<” with “>” or “not acute” with “is obtuse.” In general, students did not seem aware that a negative predicate (“not acute”) could be thought of as denoting the complement of the set of cases satisfying the positive predicate (“is acute”). As a result, students were disinclined to reason about negative predicates without paraphrasing (“can’t be acute”) or substituting positive conditions.

However, students’ sentential testing strategy led to some other strategies that more closely approximated normative interpretations and led students to make appropriate determinations of truth-values. Two participants began anticipating that anything satisfying the first condition made the statement true. So, they began ignoring such cases, as when Ron interpreted D2:6 as, “if it's not equilateral, it must be obtuse.” I call this strategy an *“if not…then” paraphrase*. While not identical to the Venn diagram partitioning of examples, this strategy allowed students to reduce the set of cases they had to attend to by excluding cases satisfying the first predicate. Their reasoning also implicitly approximated the negation of the disjunction – negating both predicates – because it led students to question whether anything failing the first condition must necessarily satisfy the second. For instance, Ron rejected that any non-equilateral triangle must be obtuse, which led him to find the non-equilateral, acute counterexample[1]. Though they used it repeatedly, students in Experiment 1 did not consciously identify or abstract their “if not…then…” strategy. Furthermore, without specific guidance students did not reinvent the Venn diagram partitioning of examples and consistently used sentential testing of a few cases.

Negating disjunctions

One of the challenges in reinventing logic is to find experientially real activities (Gravemeijer, 1994) that foster language systematization as entailed in *reasoning about logic*. Assessing truth-values successfully prompted students’ reinvention of truth functions. Negating statements appeared a natural next activity, but it was unclear how to describe logical negation to participants unfamiliar with the notion. In Experiment 1, I asked students to find a systematic way to produce an opposite statement that always had the opposite truth-value. This description of the negation of a statement proved to be underspecified for reinvention.

Experiment 1 participants commonly engaged in syntactic manipulations of the statements to produce a negation such as 1) negating both conditions with the same connective ($\neg(A \vee B) \leftrightarrow \neg A \vee \neg B$) or 2) negating with a non-complementary property ($\neg(x < y) \leftrightarrow x > y$). Even when provided with various statements intended to dissuade such strategies, students were unperturbed to negate different statements in different ways. Ron and Drew negated “10 is even” with “10 is odd,” but more appropriately negated “π is even” with “π is not even.” Anticipating that the *and* connective in the negation would not be obvious, I asked participants to negate D3:3 and D3:4. Students in both pairs negated D3:3 as “10 is an odd number or 20 is an odd number,” which yielded the opposite truth-value as desired. They then recognized a problem when the same transformation of D3:4 yielded another true statement. Drew responded by negating D3:4 with “13 is an even number or 6 is an odd number.” This statement is false, as required, but Drew did not show evidence of anticipating whether this method would work for any other statements. Ron and Drew proposed and tested various transformations of the statement before introducing an *and* connective,

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seemingly by trial and error. Upon testing, they recognized that this method properly negated the given statements, but could not semantically justify why. Contrary to the researcher's intentions, study participants did not perceive any connection between why a counterexample falsified a disjunction (it satisfied neither condition) and a systematic method of negating a disjunction (not A and not B). There was also no evidence that they understood why the negation of a universally quantified disjunction should be an existentially quantified conjunction, though the counterexample heuristic might suggest it. It appears from these two teaching experiments that the negation criterion of merely having the opposite truth-value was too underspecified to foster students' recognition of a generalizable method. They relied on trial and error syntactic transformations of the statement in lieu of any intentional, semantic strategy.

An alternative criterion for mathematical negation

One episode from the third interview with Ovid (his partner Eric was absent) suggested an alternative approach to reinventing negation. Ovid was already comfortable with abstracting from each non-quantified disjunction the two component truth-values and applying their truth function. Ovid recognized that negating a disjunction with a disjunction would not work because each of the two components would reverse truth-values. He said, "If this is false-true, then the opposite would be a true-false statement." Like Drew, Ovid initially wanted to change the way he negated the components rather than changing the connective. I invited him to explore all such component patterns and the desired outcomes for the negation, which produced the table in Figure 1. Analyzing this representation, Ovid said, "So we would have to do, probably would be an *and* statement. Because then it would have to fit both criteria rather than either, or, or both." He went on to check that "13 is not an even number and 6 is not an even number" properly negated D3:4 (i.e. produced the opposite truth-value as desired).

Parts	St	Parts	Neg
TT	T	FF	F
FT	T	TF	F
TF	T	FT	F
FF	F	TT	T

Figure 1: Reproduction of Ovid's truth table for negating disjunctions.

Ovid's discovery that the negation of a disjunction must be a conjunction was significant for two reasons. First, this was one of the clearest cases where the formalization and abstraction of the truth-value structure of the mathematical statements led a student to reinvent a normative logical theorem. As Stenning (2002) discussed, Ovid learned from the relation between the formalization and what it formalized. Ovid translated the semantic statements into a logical representation system (a truth table), deduced the appropriate pattern from that representation, and then translated it back to the semantic system of mathematical statements.

The second reason I highlight Ovid's discovery is that it suggested an alternative way of characterizing negations. Each of the statements Ovid reasoned about in this episode could be viewed as a case of the condition "x is even or y is even." As in Ovid's truth table, the negation of the condition must yield the opposite truth-value for each pair of numbers. Thus the negation of a quantified disjunction must be a case-wise negation (yielding opposite truth values for each example) in addition to a global negation (having the opposite truth-value overall). This insight, combined with the need to guide students to attend to partitioning the example space suggested the revised teaching activities employed in Experiment 2.

Experiment 2

The teaching activities on the first day of this teaching experiment were nearly identical to those in the former, except that some uninformative items were removed and the geometry items from the second day were added. The participants in Experiment 2, Cid and Macy, attended to the *or* connective much earlier (on D1:2), but still rejected D1:7 as false due to its apparent absurdity. Unlike the participants in Experiment 1, Macy had been taught logic in a mathematical context. Despite this, she consistently imposed a non-normative "exclusive or" interpretation, though it took her some time to recognize when it applied to quantified disjunctions. Like the first group, Macy and Cid distinguished and coordinated the three truth-values in a non-quantified disjunction according to truth-functions, though they disagreed about the output when both predicates were true. Like the previous pairs, neither Cid nor Macy developed a generalizable strategy for finding example cases, especially for the geometric items. Cid himself explained that he was merely "stabbing at examples in [his] head." He implicitly used a *sentential testing strategy* and *"if not…then"* paraphrases, but did not recognize or abstract these approaches. Macy attempted semantic substitution, inappropriately paraphrasing D2:6 as "is acute or is not acute" because "equilateral triangles are acute."

Alternative activities intended to emphasize quantification of predicates

On day 2, many of the same conditions were presented to the students, but the activity was reframed from assigning truth-values to quantified disjunctions to finding the set of cases that satisfied a disjunctive predicate (see the D2' items in Table 1). This sequence of tasks was intended to guide students to associate classes of examples with each condition rather than single examples, leading to the normative interpretation that mathematical predicates entail sets (rather than simply describing cases). The close association between "divisibility by 2" and $\{x \in \mathbb{Z} : 2|x\}$ (the set of even integers) is commonplace in proof-oriented mathematics, but analysis of Experiment 1 suggested that it was not a natural association for study participants.

I anticipated that visually representing some of the sets described by these disjunctive predicates might lead students to an interpretation approximating the Venn diagram for *or*, the principle of which is that a disjunctive predicate entails the union of the sets of cases entailed by the two component predicates. For this reason, I included items such as D2':10 that would easily lend themselves to visual representation.

The third reason to shift from declaring quantified disjunctions true or false to identifying the set of cases that satisfied a disjunctive predicate was to provide a natural segue to case-wise negation. Rather than negating statements by other statements that have opposite truth-values, the negation of a disjunctive predicate "X or Y" is the predicate that entails the complement of the cases. Thus I anticipated modifying the task to, "For which integers is the condition false?"

Results of the alternative instructional activities

Cid and Macy approached the second day's activities initially using verbal strategies as they had done the day before. In some cases, they could describe the set easily as "all real numbers" or "all even integers." They ran into difficulty when they tried to use "and" to denote the union of two sets, which confused the intended meaning of the connectives. Beginning with the geometric items, they began instead describing the cases that do not satisfy the condition. Regarding the correlate task to D2:6, Cid said, "An acute triangle doesn't satisfy it. I think." Macy clarified, "An acute triangle that's not equilateral." The interviewer invited them to extend this strategy and specify the set of cases making each condition false. By the next item, Macy generalized the strategy, saying "I am trying to think if there's any counterexamples where you can make both of those statements false. Cause then you can exclude some of the triangles." In this way, Macy recognized that the statement was false for cases that failed both component predicates. Both Cid and Macy later noted that it was much easier to describe the set of counterexamples to disjunctive conditions than describing the cases

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that satisfied them. They could not articulate why this was easier, though. In addition, they implicitly recognized the complement relation between the satisfying and falsifying cases.

I intended for a visual representation to suggest the relationship that the cases satisfying a disjunctive condition consist of the union of the cases satisfying each condition (as the Venn diagram suggests). Cid and Macy drew number lines for D2’:10, but they described the resulting set in spatial terms (“It’s false when y is between 3 and 5.”) rather than in inequality language. This dissociated the set from the negations of the two component predicates and led to no generalizable strategy. The interviewer then revisited D2’:6 and invited the students to create two number lines that demonstrated which numbers satisfied and falsified the given condition. The students did so (Figure 2) including the truth-values of the two components for each number. This led them to rediscover Ovid’s observation that the component truth pattern of the negation will invert that of the original and that the connective *and* will ensure the proper pattern of truth-values for the disjunction and negations overall. While this alternative visual representation did not emphasize the union property, it clearly fostered truth functional analysis leading Cid and Macy to reinvent the case-wise negation of a disjunction.

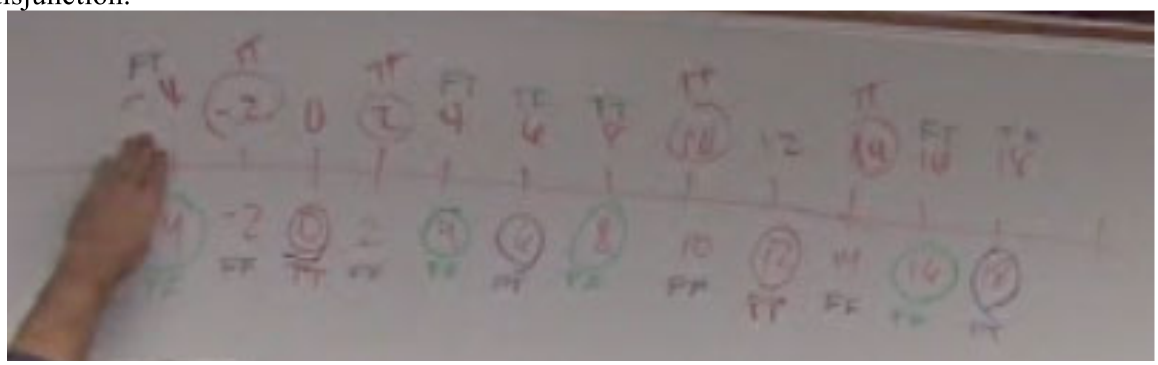

Figure 2: Representing the sets entailed by a disjunctive predicate and its negation.

Summary and discussion

From the diversity of strategies employed, and the frequency with which study participants paraphrased the given statements in various ways, it was clear that study participants had not systematized the meaning of *or* in mathematical sentences prior to the teaching experiments. Rather, students *reasoned toward an interpretation* (Stenning & van Lambalgen, 2004) of each sentence trying to find some way to have the language or content suggest a means of assessing each statement. Several key strategies emerged repeatedly and independently such as sentential testing and “if not…then,” but students did not apply such strategies consistently. Due to the methodological choice to work in meaningful mathematical contexts, participants had to impose logical form in their mathematical interpretations. Participants only slowly developed meta-language for describing and abstracting patterns and strategies across various contexts.

In each study, students reinvented the standard truth-function for inclusive or exclusive *or* in one session. The activity of assessing truth-values did not lead study participants to develop strategies for quantified disjunctions that approximated the normative Venn diagram partition of examples. Students selected examples according to the semantic structure of the content of each sentence (numbers, triangles, etc.) rather than according to the predicates in the disjunction. This suggests that instruction in proof-oriented classes must help students begin associating any property or predicate with the set of examples satisfying that predicate and its complement. Properties describe single cases, but they also organize or partition sets of examples. None of the study participants approached the given tasks in this quantified way without being guided to do so. Motivated by the need to attend to quantification of predicates and to develop a case-wise meaning of negation, I developed the instructional sequence used in Experiment 2. This approach successfully led Cid and Macy to

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formulate the normative negation of a disjunctive condition and to identify that a condition and its negation entailed complementary sets.

A theoretical goal of this project is to recast mathematical logic within students' activity. These results provide several instances of students *reasoning about logic* such as problematizing linguistic interpretation, reinventing the standard definition of *or*, comparing interpretations across statements, developing meta-language to abstract patterns, and truth-table analysis leading to new discoveries. These data support the hypothesis that students can reinvent the structures of logic when engaged in the activity of logic: systematizing language. However, further studies are needed to better understand reinventing logic's instructional affordances and implications.

Endnote

[1]While Ron's paraphrase also falsely suggests right triangles are counterexamples, his line of reasoning led him to the correct counterexample.

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