FRANK'S PERCEPTUAL SUBITIZING ACTIVITY RELATIVE TO NUMBER UNDERSTANDING AND ORIENTATION: A TEACHING EXPERIMENT

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This proposal explores how the activity of subitizing – quickly apprehending the numerosity of a small set of items – changes with the development of number concepts. We describe how varying the orientations of items in teaching experiment sessions promoted one pre-schooler, Frank, to attend to subgroups of items and change his thinking about conjoining two numbers. The results illustrate how game-play oriented subitizing activities may support growth in number understandings.

Keywords: Number Concepts and Operations; Pre-School Education; Learning Trajectories

Introduction

Subitizing is a quick apprehension of the numerosity of a small set of items (Sarama& Clements, 2009). Sarama and Clements (2009) suggested that subitizing processes transition from a reliance upon orientations to a reliance upon number understandings. MacDonald (2013) conducted four concurrent teaching experiments to investigate how subitizing activity changed in relation to understanding of number and perceived space between items. In this study, we focus on how one student's (Frank) understanding of number changed over time to rely on more conceptual processes.

Literature Review

Subitizing

Sarama and Clements's (2009) argue that children rely on either *perceptual subitizing* or *conceptual subitizing* when subitizing. Perceptual subitizing, an innate ability to discriminate different quantities, emerges in infants as young as three to five months of age and is limited to five items. Conceptual subitizing is grounded in a child's number understanding due to a child's ability to subitize groups and then compose the total number of items (Sarama & Clements, 2009). When children in kindergarten through grade two engaged in *Building Blocks,* a computer learning environment, their subitizing activity became more sophisticated and included conceptual processes (Clements & Sarama, 2007).

Number Understanding

Number understandings will be grounded in theories stemming from number conservation (Piaget, 1941/1965). This is characterized as a child's simultaneous coordination of their *serial* (number follows a sequential order) and *algebraic* (number is composed by smaller subgroups) thinking structures (Piaget, 1968/1970). Number understanding in this study will be centered on the following four areas: (a) counting (Steffe, Cobb, & von Glasersfeld, 1988), (b) composition and decomposition of number (Fuson et al., 1997), (c) links between quantity and number words (Krajewski & Schneider, 2009), and (d) perceived dimensionality (Piaget & Inhelder, 1948/1967).

Counting. Counting is described as a child relying primarily on serial thinking structures, as items are empirically pointed to and coordinated with a sequence of words. (Steffe et al., 1988). Multiple sets of research findings emphasize the importance counting has in children's mathematical development (Chan, Au, & Tang, 2014; Jones et al, 1994; Steffe et al., 1988). Steffe et al.'s (1988) research findings essentially describe how counting promotes children's number development through a coordination of units. Van Nes & van Eerde (2010) found that relationships between spatial reasoning and counting exist, as children's counting changed in relation to block arrangements. Thus, spatial orientations of

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objects promote children to rely on different types of counting and construct sophisticated number understandings.

Composition and decomposition of number. Fuson et al. (1997) found that children able to construct multi-digit number understandings had more sophisticated grouping techniques. Essentially when children compose and decompose number they progress through six stages of development. Jones, Thornton, and Putts' (1997) also suggest that many aspects of number understanding, including composition and decomposition of number, are foundational for students' development of number. Thus, these findings imply that young children build multi-digit number understandings through effective composition and decomposition of numbers less than ten.

Link between quantity and number words. As number is understood in a more abstract manner, number words are said to link to quantities (quantity-number competencies [QNC]) (Krajewski & Schneider, 2009). Krajewski and Schneider found that kindergarten children's QNC explained about 25% of these children's achievement scores four years later. Implications from this study suggested that future research consider how young children's empirical activity with concrete material promotes children's QNC ability prior to entering kindergarten.

Dimensionality. Topological thinking structures involve a child's attention towards the perceived topology of objects and sets of objects (Piaget, 1968/1970). One aspect of topological thinking structures, *dimensionality,* is described by Piaget and Inhelder (1948/1967) as directly promoting the flexible thinking necessary for children's later conceptualizations of formal Euclidean Geometry. Four areas of development characterize dimensionality: (a) proximity (nearbyness), (b) separation (betweeness), (c) continuity (connecting objects in spatial fields), and (d) enclosure of shape (surrounding) (Piaget & Inhelder, 1948/1967).

Purpose

Parallels between children's construction of number and subitizing activity have been suggested in the research literature (e.g. Freeman, 1912; Sarama & Clements, 2009), but a fine-grained analysis of this relationship is absent from the literature. To understand number, children need to engage with empirical items to group, partition, compose, decompose, and count. Linking number to number words would allow for number to be abstracted. Subitizing activity would essentially promote several of these empirical and mental activities to provide a child a vehicle in which to construct number understandings. The purpose for this study was to investigate how one child's understanding of number changed as he engaged in subitizing activity.

Methodology

Teaching Experiment Methodology

This study uses teaching experiment methodology (Steffe & Ulrich, 2014) and is grounded in the *radical constructivist paradigm* suggesting that mathematics understanding is actively constructed (von Glasersfeld, 1995). A teaching experiment includes a teacher-researcher, a witness for each teaching episode, at least one student, and a way to record student actions and words in each teaching session (Steffe & Ulrich, 2014). In this study, the first author was the teacher-researcher, and the second and third authors alternated as the witness. The teacher-researcher and witnesses initially met prior to the start of the experiment to establish a similar theoretical grounding and establish the functional aspects of each of our roles in this study.

Participants

Fifteen students between the ages of three years, 11 months, and five years, five months were initially recruited to participate in a larger study. The 15 students were enrolled in a preschool located near a university campus located in the southeastern portion of the United States. Four

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students spoke a second language at home. Eleven students were male and four students were female. Following an initial screening, six of these preschool students were selected to participate in a teaching experiment. The selection was based on their ability or lack of ability to conserve number, count, and subitize two to five items. An in-depth analysis of one student, Frank, is the focus of this study.

Frank. Frank is a male student whose family is from China. He was four years and five months old at the onset of this study. He spoke English, and in his home spoke Mandarin. Frank was interviewed two separate times on June 5th to determine if he was able to conserve number and to determine his counting and subitizing abilities. Throughout the interviews, Frank wanted to have the "correct" answer. This disposition promoted Frank to reflect more often on his activity. Frank engaged in 22 teaching experiment sessions.

Procedures

Interviews. Frank's first two interviews were used to determine (a) whether he conserved number, (b) whether he could keep track of items when counting, and (c) whether he perceptually or conceptually subitized. Frank was found to not be able to conserve number, and used perceptual subitizing. When counting, it seemed as if Frank was able to initially "count on" from 12 items. Knowing that Frank was able to count on and not conserve number seemed atypical, as Steffe et al.'s (1988) findings indicated that "counting on" required a more comprehensive understanding of number.

Teaching experiment session tasks. The teaching experiment was comprised of 22 sessions occurring two days per week no more than 20 minutes each. Tasks were designed to either assess or provoke change in Frank's thinking. Item orientation, reliance upon empirical material, and QNC were considered in the formation of the tasks throughout analysis. Tasks required him to subitize a set of items, draw or use counters to show what he remembered, and use words or actions to justify his response. They were refined prior to the teaching experiment sessions, allowing for orientation, quantity, or color of items to change to test and expand the limits of Frank's thinking. The five following tasks were used: (a) Draw what you saw, (b) Camera game, (c) Concentration, (d) Board games, and (e) Hidden Pictures. Below, we focus on the first two.

Draw what you saw. The teacher asked Frank to subitize a set of dots or counters and then to draw or use counters to show what he "saw" or "remembered." This activity was also followed up with, "How do you know you saw _______?" Frank was given material to draw what he remembered or use counters to represent what he remembered.

Camera game. The camera game was adapted from Clements and Sarama's (2007) activities. Clements and Sarama's (2007) camera game used a computer program, but in this study the activity had a series of camera pictures on a three-ring notebook. Frank was shown quickly an image of the viewfinder of a camera with dots arranged. He was asked how many dots were seen, and then he drew what the picture would look like when it came out of the camera.

Analysis

Two forms of analysis, conceptual and retrospective, were used to model and describe Frank's thinking (Steffe & Ulrich, 2014). *Conceptual analysis* regards students' responses between tasks and sessions, and *retrospective analysis* regards changes over a longer period of time. Each session was videotaped with two video cameras. Each session's video footage was reviewed after each session (conceptual analysis) and sections of video footage from the entire study were reviewed six times throughout the study (retrospective analysis).

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Results

Conceptual Analysis

Subitizing activity relative to the perceived symmetry of items. Initially, Frank subconsciously relied on symmetry when subitizing, as it seemed the symmetrical orientations of the items afforded Frank the opportunity to build towards four. In Frank's first teaching experiment session he was shown four dots arranged in a square-like orientation. Frank stated that he saw "T…four," but when asked about almost stating two, he responded that he did not remember seeing two. When asked to draw what he remembered, he drew two dots and wrote the numeral two beside them. After seeing the orientation a second time, he stated that he saw four and drew the four dots in the same square-like orientation and wrote the numeral four beside them. His response suggests a subconscious attention to the two by which to build towards four.

Subitizing activity relative to the perceived space between items. In the middle of his fourth teaching experiment session Frank was playing the "Camera Game." He was shown an image with five dots (see Figure 1). The space between the square and the one dot seemed to disrupt Frank's ability to subitize the total group. This is evident when Frank draws four dots and one dot, and then writes the corresponding numerals beside his drawings (see Figure 1). After Frank describes seeing "four and one," he is asked "how many is there altogether?" This question elicits his response of "fourteen." This happened in subsequent sessions when Frank used counters, and it suggests that Frank's QNC was grounded in a procedural understanding for two-digit numbers (Krajewski & Schneider, 2009).

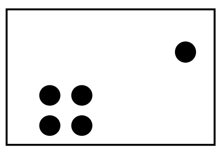
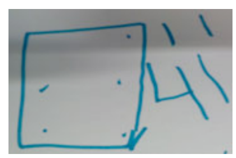

Figure 1: The orientation shown to Frank and the drawing Frank made respectively.

In Frank's earlier sessions, it seemed as if Frank's QNC was primarily procedural. This was evident when he used two-digit numbers to describe what he saw (i.e. "four and one…that makes fourteen"). Space between clustered items played a critical role in Frank's subitizing activity, as it seemed to promote him to attend towards subgroups, but he lacked the ability to compose these two groups of number. Thus, Frank would rely on a procedural QNC by stating a two-digit number. Symmetrical orientations with a regular amount of space between items prompted Frank to build towards the total number of items with one subgroup. This was evident when Frank said, "T…four." This activity does not require Frank to compose groups but to count up after subitizing two, eliciting a serial thinking structure. Thus, it seemed as if he was having difficulty coordinating his serial and algebraic thinking structures.

Changes in Frank's subitizing activity relative to changes in Frank's understanding of number. Throughout the first three sessions, Frank was capable of subitizing four items, but when shown five items, he needed space between clustered groups of two or three to subitize the subgroups of two or three. Frank did not have a conceptual understanding of five at this point because he could not compose the subgroups to name the total group of five. When Frank was shown five items without a space between the clustered items, he either named this as "six," "seven," or rearranged the items to look like the "X" orientation shown on the face of a typical die before describing this total set of items as "five."

Frank continually described two-digit numbers (i.e. fourteen, twenty-three) when attempting to conjoin the two subgroups he subitized, which indicates that his number understanding remained procedural. He may have been taught to name and identify two-digit numbers before understanding

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single digit numbers. To perturb this notion, we asked Frank to count items and name the total number, or covered up portions of an orientation, incrementally building (+1) from a group of three to the total number of items. This task design utilized Frank's counting and subitizing ability, to perturb what he understood number to entail.

In the early portion of Frank's seventh teaching experiment, Frank is describing "two" and "three" items as "twenty-three." To perturb his thinking, the teacher asked him to count the counters he placed on his mat. He counts out three counters and then counts out two counters. After two more attempts, Frank's counting responses remain the same, so the teacher counted the counters in front of her and had him "parrot" her counting. Immediately following this task, Frank is shown items clustered to represent "two," "one," and "two," (see Figure 2) and Frank describes seeing "two plus one plus two makes five." This is the first time Frank begins to construct five by composing groups of numbers, suggesting a change in how Frank understands five.

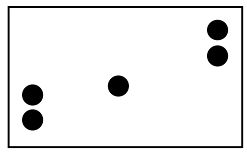

Figure 2: This orientation was shown to Frank in the middle of his seventh session.

Throughout the subsequent sessions, Frank's responses reveal more conceptual understandings of number (i.e. "three and then one is four," "two and two is four," "three and one is what tells me four"). These responses seem to reflect how Frank is perceiving the space between clustered groups of items, as he simply subitizes the total groups of items when no space is evident. Also, it is important to note that each time Frank explains his thinking he needs to rely upon his description of the subgroups to describe the total group. This seems less abstract than if Frank were to state the total group and then reverse his thinking to then describe the subgroups. Thus, it seems as if his number understanding is still developing.

Frank's number understanding in session 17. Prior to session 17, Frank was capable of subitizing four with subgroups "three and one," "two and two," and "one and three," and his QNC was more conceptually grounded with regard to four. However, Frank was not able to describe the total group and then reverse his thinking to describe the subgroups of four. For Frank to describe the total group and then the subgroups would suggest a cognitive reorganization of what Frank knew about four because he would have to compose four and then decompose four. Composing and decomposing a number would require Frank to reflect on his actions and be cognizant of the subgroups he used to compose four.

Additionally, Frank's understanding of five was still limited, and after his seventh session, he was not able to carry his description of five as "two plus one plus two" into subsequent tasks. Often when shown an orientation that promoted "two, two, and one" he would describe seeing four because he saw two and two or state that he saw, "four…five." These responses suggested that Frank was still solidifying what he knew "four" to be and was not able to coordinate the composition of two subgroups to build towards four and then coordinate a third subgroup to build towards five. Thus, we planned to use symmetrical orientations to promote Frank's subitizing of five in session 17.

In the Data Excerpt below, Frank was in his 17th teaching experiment session and his teacher showed him the circular counters in an orientation (see Figure 3), and asked him how many he saw. Frank brought his stuffed mouse to this teaching experiment session, and at times he pretends the mouse is the one responding to the tasks. This task was near the end of the session, and just before this task Frank was shown five counters which he determined were four and one, which then made fourteen. Once Frank counted these five counters, he determined the group was five. So again, Frank was having difficulty with composing five. As Frank described the subgroups which made up the composite groups, it seems evident that symmetry supported this activity, as only one two is mentioned.

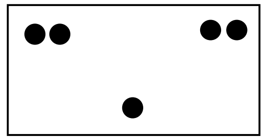

Figure 3: This orientation was shown to Frank near the end of his 17th session.

Data Excerpt.

Teacher: Okay, set mousey aside. Okay, one, two, three. [Teacher-researcher lifts the top piece of cardstock revealing counters arranged so that two counters are on the left-hand portion of the mat, and two counters are on the right-hand portion of the mat. In the middle there is one counter (see Figure 3).]
Frank: Five. [Frank talks in a squeaky "mouse-like" voice.]
Teacher: Five? How did you know that so fast?
Frank: Mousey says that.
Witness: How did mousey know it so fast?
Teacher: Yeah, how did mousey know it that quickly? Do you agree with mousey?
Frank: Yeah.
Teacher: Yeah? Why?
Frank: Because mousey said five [says five again in a squeaky "mouse-like" voice.] But mousey wins.
Teacher: He did win, but why did mousey know it was five? I don't know why that's five.
Frank: But you put it…you put two and down [motions with both his hands to show two on his left hand and right-hand portion on the bottom portion of his mat] and one and up [motions in the middle top portion of his mat.]

This data excerpt illustrates three cognitive changes in how Frank is understanding five. First, the symmetrical aspects of the orientation seemed to scaffold a change in Frank's understanding of five, as he described "two and down and one and up." Second, Frank composed subitized groups to quickly state that he saw five, but then "unpacked" or decomposed five to describe the actions and the groups he saw when subitizing five. This activity is more sophisticated than building up to five, and the symmetrical nature of the orientation seemed to promote this activity. Third, Frank no longer needed to "make" the orientation, but pointed to the imagined areas where the counters are located. This step away from the perceptual material was an important one; it seems as if Frank relied on more abstract actions when subitizing.

Retrospective Analysis

Frank's subitizing activity initially relied on common images or patterns to describe four, and at times, five. Also, Frank's initial QNC was grounded in procedural knowledge related to what he "knew" two-digit numbers "to be" which did not support a conceptual conjoining of number. To press Frank to attend to subgroups, Frank was shown orientations with large amounts of space between small clustered items that Frank was capable of subitizing. To connect what Frank knew about five to his subitizing activity, Frank reflected upon his counting activity and the relationship between subgroups when items were covered up. Frank developed more sophisticated number understandings as a result of

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subitizing symmetrical orientations. Frank was replacing the visual patterns with actions, as he was able to explain his thinking without having the perceptual material in front of him as evidenced in the Data Excerpt. Frank used prepositional phrases to explain what he saw (i.e. down, up) which suggests he may also be describing his eye movements. Thus, it seemed that Frank developed strategies that promoted changes in his number understanding (i.e. composition, decomposition, QNC). These strategies seemed to result from counting, subitizing, and the orientations (i.e. symmetry and space between items).

Conclusions

The purpose of this teaching experiment was to investigate how a child's understanding of number changed as he engaged in subitizing activity. Item orientation, reliance upon empirical material, and QNC were considered throughout the analysis. The two main findings of the study were related to (a) how Frank composed number and (b) the nuances in his perceptual subitizing activity.

Number Composition and Decomposition

With such a societal push to promote early childhood instruction in mathematics, it is important to understand what is (in) appropriate to teach young children. Entering the teaching experiment, Frank's understanding of conjoining two groups of items was a procedure resulting in a two-digit number word. This understanding of composition changed for situations in which the procedure was linked to a conceptual conjoining of groups to explain five. Though it seems that his procedure for naming two-digit numbers distracted from his construction of five, perhaps developing the procedure helped him attend to subgroups. Future research that focuses on helping children make appropriate connections between procedures and conceptual number understandings would be important for developing early childhood curricula. Findings from this study brought new ideas to light about how subitizing activity can press students to engage in meaningful activities when beginning to understand number.

Nuances in Perceptual Subitizing Activity

Throughout the teaching experiment, Frank's subitizing activity was described as perceptual instead of conceptual because he relied heavily upon perceptual material when discussing subgroups and total groups of items. The identification of sub-stages of perceptual subitizing are useful for further understanding topological, serial, and algebraic thinking prior to number conservation (Piaget, 1941/1965). First, shapes and patterns were initially described when explaining how a number was understood. This reliance upon patterns seemed to provide Frank a template to work from when recreating the orientations. Second, attention to subgroups before describing the total group of items indicated a "building up" of number. Third, near the end of the study, it seemed that Frank could compose items quickly when subitizing and then decompose these groups to explain his thinking. Children making connections between early perceptual activities and conceptual processes gives purpose for particular curriculum choices. Thus, early childhood educators' utilization of findings from this study could inform their pedagogical choices when designing subitizing tasks embedded in game play.

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