NUMBER LINE ESTIMATION WITH NEGATIVES

Laura Bofferding
Purdue University
lbofferd@purdue.edu

Andrew Hoffman
Purdue University
hoffma45@purdue.edu

Elizabeth Suazo
Purdue University
esuazo@purdue.edu

Nicole Lisy
Purdue University
nlisy@purdue.edu

When to introduce negative integers to children is an important issue in school mathematics; delaying their introduction can lead to lasting misconceptions such as one cannot subtract a larger whole number from a smaller. Yet understanding negatives involves a complex extension of whole-number knowledge. It is not known whether this extension is only possible after whole-number concepts are learned or whether simultaneous acquisition of positive and negative integer concepts is possible. This study used an established whole-number intervention (playing linear board games), extended to include negatives, with kindergartners and first graders. Performance placing integers on empty number lines provided evidence of students' understanding of integer concepts.

Keywords: Number Concepts and Operations; Cognition; Elementary School Education

Purpose of Study

One of the enduring challenges students face when learning number concepts is determining how to revise and build on their whole number understanding to include new numbers. In particular, incorporating negative integers into their number system is challenging, and many students will continue to assert that you cannot subtract a larger number from a smaller one, even if they can solve other problems with negative integers (Murray, 1985). At a basic level, when learning about negative integers, students must extend their backward counting sequence to below zero, using the positive number names with the word "negative" before them. Likewise, they must reinterpret the meaning of the minus sign to mean "negative" when attached to a numeral and referring to the numbers less than zero (Vlassis, 2004).

When given the opportunity to explore negative numbers, even first graders were able to talk about their values (Bofferding, 2014) and use them in arithmetic problems (Behrend & Mohs, 2005/2006). Other researchers have identified kindergarteners who were able reason about negative numbers (e.g., Bishop et al., 2010). However, questions still remain about the extent of knowledge possible for young students, whether typical kindergarteners can learn about negative numbers, and what types of activities might support their understanding. We explore these issues in this paper.

Theoretical Framework

According to Case's (1996) theory of Central Conceptual Structure for Number, before the age of four, children have two cognitive structures for number. The first allows them to count a set of objects, and the second allows them to make visual comparisons of sets of objects. However, they cannot use counting to help determine which set has more or less; these cognitive structures remain separate. Around the beginning of kindergarten, children begin to coordinate the two structures and can reason that adding one object to a set corresponds to moving up one number in the counting sequence. They also learn to map the numerals to quantities and number words. By first grade, these structures are often fully integrated if students have had supportive numerical experiences (Griffin, Case, & Capodilupo, 1995). This integration is referred to as a mental number line. As mentioned previously, to extend their mental number line to include negative integers, students must accept that there are numbers less than zero and learn the new notation (i.e., the importance of the negative sign), number names, how they are ordered, and their values.

One experience that helps students develop their whole-number mental number line is playing linear board games (Ramani & Siegler, 2008). On a simple board game labeled with squares from 1 to 10, preschoolers counted on as they moved toward the finish. The experience of seeing and saying the number sequence helped the children progress in their ability to identify the numerals and determine which number is bigger. Further, they outperformed a control group on a series of number line estimation tasks, which were the main measures of interest. When given a number line marked with 0 and 10 and asked to mark where numbers 1-9 go, the students who played the game were more likely to space the numbers evenly. Therefore, a linear model more completely explained their plots (based on R^2 values) and slope values of their lines were nearer to one. The results indicated that playing the board game helped the children develop a mental number line for whole numbers (Ramani & Siegler, 2008).

Unlike with whole numbers, where students have experiences both counting and working with sets of objects, children cannot work with negative sets of objects (unless we artificially impose a negative value onto objects). Therefore, playing a similar linear board game that includes negative integers may be a helpful way to give children experiences with the order and values of negative numbers. On the one hand, this extension might only make sense to children after they have developed a whole-number mental number line (in first grade). On the other hand, they may be able to learn about negatives while they are simultaneously developing the whole-number mental number line (in kindergarten). Based on these possibilities, we explore the following research question: To what extent can playing a linear board game including negative integers help kindergarteners and first graders develop a linear representation of the integers?

Methods

This study took place over two years. In the first year, we worked with first graders, and in the second year, we replicated the study with kindergarteners.

Setting and Participants

The participants came from an elementary school located in a low-income area in the Midwest with a large proportion of English Language Learners. In the first year, we conducted the study during the first three months of the school year with 50 first graders (26 female; 24 male); however due to two students moving and one not completing the tests, we only present complete data from 47 students. In the second year, we conducted the study during the first three weeks of the school year with 45 kindergarteners (27 female; 18 male).

General Design

Each year, the study involved an experimental design, which included a pretest, stratified random assignment to control or experimental (“game”) group, intervention, posttest, and follow-up. We only present data from the pre- and posttest portions of the study. The design and materials replicated those used by Siegler and Ramani (2009) but included some modifications and additions to include a focus on negative integers. For the intervention, each participant worked with a researcher (professor or one of two graduate students) for three, 15-minute sessions. One of the graduate research assistants worked with four kindergarten students who benefitted from Spanish translation. During the first year there were 22 first graders in the game group and 25 first graders in the control group with complete data. During the second year, there were 23 kindergarteners in the game group and 22 kindergarteners in the control group.

Pre-test and Post-test Measures

The pretest and posttest were identical and conducted as individual interviews with the students; we did not provide specific feedback on their performance. Across the sections of the test, the problems used positive integers as well as negative integers with tasks involving counting, ordering integers, determining which integer was closer to or further from 10, and solving addition and

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subtraction problems involving positive and negative values (for further descriptions see Bofferding & Hoffman, 2014). We describe the two main measures of interest here.

First, on the integer identification task, we presented numerals on isolated pages in random order and asked students to identify integers from -10 to 10. Second, in the final section of the test, students were asked to place integers on number lines. Students completed a packet involving positive integers followed by one involving negative integers. Each page of the packet contained an empty number line 25.5cm long with two integers marked. On the first page of both packets, students were asked to put a pen mark where 0 would go, given the locations of -5 and 5. For the positive packet, the remaining pages contained empty number lines marked with 0 and 10. The placement of zero in the middle, i.e., leaving space for the negative numbers to the left, was an important feature. Students were asked to make a mark where a given integer should go a total of 18 times (1 through 9 in random order, twice). The researchers gave instructions such as, "If here is 0 [point to the middle] and here is 10 [point to the right], then make a mark on this line [motions to whole 25.5 cm line] where 6 should go." The negative number packet worked similarly, only with -10 marked on the left and 0 marked in the middle. Students were told to place the negative integers -1 through -9 on the respective pages.

Control Group

For their three sessions, the control group students rotated through three types of activities with the researcher. The first activity involved counting a collection of 1-10 items and counting backward as far as they could. No feedback was given on correctness. On the second activity, students put a set of six integer cards in order from least to greatest. For example, one set they ordered included the following integers: 2, 1, 0, -5, 10, and -8. After the students ordered the set, they were asked to show the least and the greatest. No feedback was given on the ordering or the identification of the cards. The last activity in the sequence was a game of memory where the goal was to match integers. Corrective feedback was given if students attempted to collect an incorrect match, but they were not told the names of the numbers.

Treatment (Game) Group

During each 15-minute intervention session, the experimental group played a board game against the researcher using a board labeled with the integers -10 to 10 (see Figure 1).

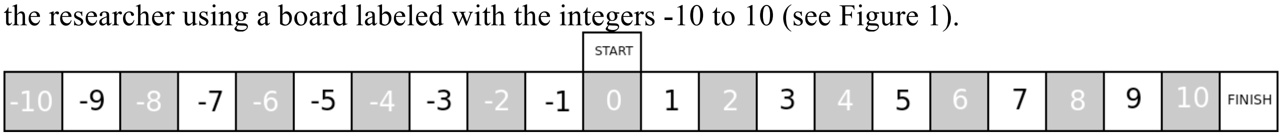

Figure 1: An illustration of the linear, numbered game board.

Players started by placing their tokens at zero, and the first player drew a card from a card deck. In the first version, all but one of the cards was labeled with a 1, 2, or 3. The remaining card contained the text, "All players go back to -10." When this card was drawn, the student had to count backward while moving the tokens back to -10. The researcher always stacked the deck so that this card would come up in the first few turns of the game, ensuring players would advance from -10 to 10 in each round. Players drew a 1, 2, or 3, moved their tokens that number of spaces, and named the numbers on the spaces they passed through. For example, if a player on -7 drew a "2," then she would move her token to -6 and say "negative six," then move her piece to -5 and say "negative five." The game ended when a player crossed 10.

During the third session, the card sending players back to -10 was replaced with a stack of cards marked with -2 or -4. Players began the game by drawing from this stack and counting backwards as they moved to -10. Once a player reached -10, on her next turn she would begin drawing with the deck containing positive numbers. From this point, play continued as normal, with the game ending once a player crossed over 10.Students played an average of 4 games in 15 minutes, and the researchers gave feedback (if needed) to correct the name of the integer that students landed on or correct the number of spaces they moved their game piece. Students had to repeat the correct name or counting sequence before the game play continued.

Analysis

Measurement

All items from the assessments were marked as correct/incorrect, except those in the number line estimation tasks. For the latter, we measured how far away from zero the student made a mark on the empty number line (to the nearest half-millimeter). We also gave the magnitude a sign, positive or negative (because zero was in the center of the line). While students were instructed to make a single, vertical line segment as their mark, some made several segments (by moving the pen rapidly up and down) or drew the numeral instead of a line segment. When measuring in these cases, we took the average of the left and right-most marks.

After one researcher completed the initial measurements for a set (e.g., measured one student's placements of positive numbers on the pretest), another researcher randomly checked five measurements. If there was disagreement on even one measurement, the second researcher checked all the measurements for that set. Lastly, a third researcher took measurements to resolve all disagreements.

Coding

To interpret the measurements, we created a four-tiered coding system. When dealing with only whole numbers as Siegler and Ramani (2009) did, it was sufficient to use two quantitative measures. The R^2 values measured the degree to which the placements were linear, and the slope of the regression line measured whether increases in the numbers to be placed resulted in a proper increase in the placements. When negatives were introduced, a complication was added. Students not only had to space the numbers evenly (high R^2value) and with equal spacing (slope near one), but they also had to know on which part of the number line to make the placements. As an example, consider a student who counted from the left (at -10) when marking positive numbers. The R^2 and slope could be exactly one, but the student would have major errors as 1 would end up at -9, 2 at -8, 3 at -7, and so on. To capture errors such as this, we added a third quantitative measure: numbers placed on the wrong side of zero. For students to show great understanding in their placements, they needed to have high R^2 values, a slope near one, and few numbers on the wrong side of zero.

To make this systematic, we created codes for four levels of understanding. A student with Level 3 understanding had an R^2value ≥ 0.90, a slope of 1 ± 0.3, and at most one value placed on the wrong side of zero. A student with Level 2 understanding did not show Level 3 understanding and had an R^2value ≥ 0.80, a slope of 1 ± 0.8, and at most two values placed on the wrong side of zero. Level 1 understanding meant not fitting into the higher levels and an R^2value ≥ 0.60, a positive slope, and at most four values placed on the wrong side of zero. Finally, Level 0 understanding was for students who did not fall into any of the higher levels. The cutoffs for these levels evolved after familiarizing ourselves with the data, including looking at scatterplots, regression lines, and using qualitative codes.

Comparing Groups

Our primary hypothesis was that the game groups would make significantly more gains in their ability to place integers on an empty number line. We operationalized this using the level-of-understanding codes described above. Specifically, we hypothesized that the mean increase in level of understanding would be significantly higher for the game group both in kindergarten and in first

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grade, and both with positives and negatives. We were most cautious in our hope with the kindergarteners' performance with negatives, especially if they lacked the ability to correctly identify negatives. In addition to using inferential statistics to test the above hypotheses, we also sought qualitative patterns in the data to motivate fuller explanations and future research.

Results

Identifying Negative Integers

On the pretest none of the kindergarteners in either groups could identifying any of the negative integers; instead they ignored the negative signs and either identified the positive numeral or said random number names. On the posttest, none of the kindergarteners in the control group were able to identify the negative integers. However, six students (26%) in the game group could correctly identify the majority of them. In first grade, three students (12%) in the control group and four students (18%) in the game group were able to identify negative integers on the pretest. By the posttest, eight students in the control group (32%) and 21 (95%) students in the game group did so.

Number Line Estimation

Overall, the kindergarteners showed low levels of proficiency at placing integers on an empty number line. Despite the fact that six students in the game group had success identifying negative integers on the posttest, none of the students showed Level 2 or 3 understanding according to our coding system (see Table 1). Similarly, there was limited success with positives in the game group; only two students achieved Levels 2 or 3. No students in the control group for kindergarten moved above Level 0 for positives or negatives. While several kindergarteners' R^2 values improved, they often had a tendency of placing the numbers on the wrong side of zero (see Table 2 for an example).

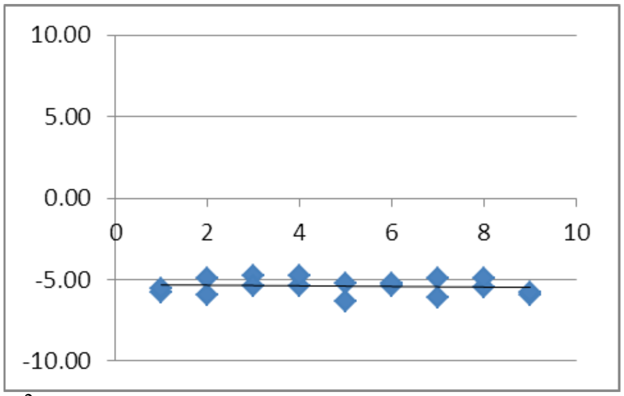
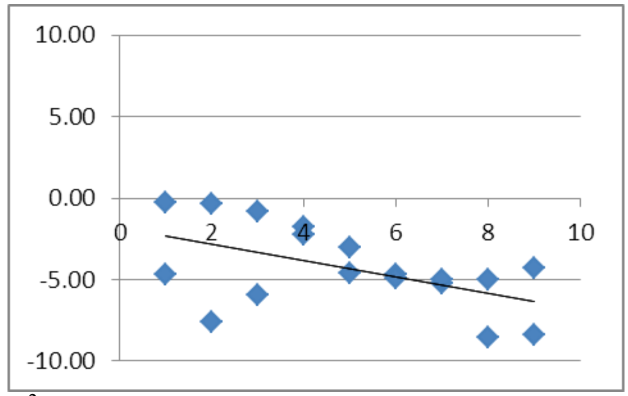
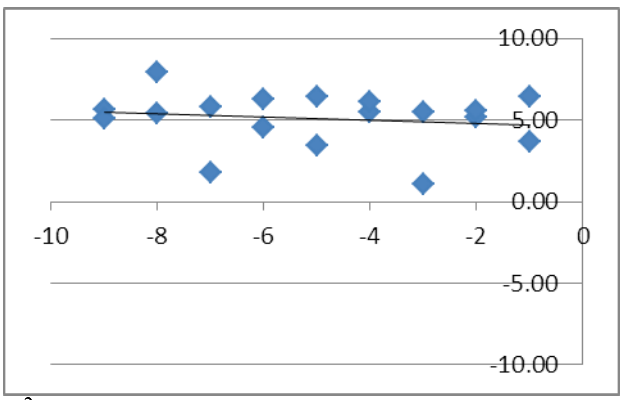
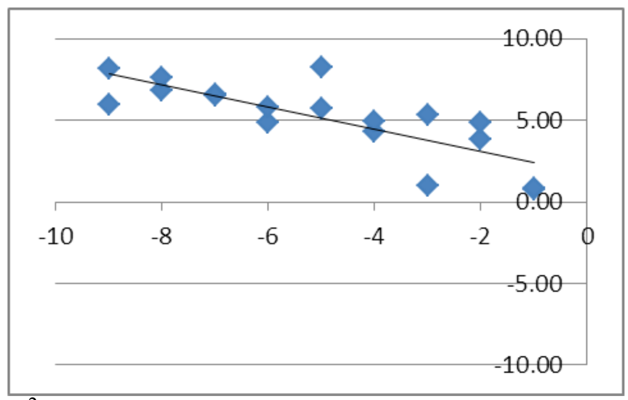
Table 1: Students' Levels of Number Line Estimation on Pre- and Posttest

	Kindergarten Control (N=22)		**Kindergarten** Game (N=23)		**1st Grade** Control (N=25)		**1st Grade** Game (N=22)	
Level	Pre	Post	Pre	Post	Pre	Post	Pre	Post
Positive								
3	0	0	2	2	7	6	2	3
2	0	0	0	0	6	4	6	6
1	1	0	1	3	3	2	1	2
0	21	22	20	18	9	13	13	11
Negative								
3	0	0	1	0	4	3	3	7
2	2	0	1	0	6	4	5	6
1	1	0	0	2	2	3	3	1
0	19	22	21	21	13	15	11	8

The first graders performed better than the kindergarteners in every way. There were seven students who achieved Level 3 with negatives on the posttest and six who achieved Level 2. Thus, well over half (59%) showed high levels of proficiency. With the positive integers, nine students achieved Levels 2 or 3 (41%). Even the control group experienced success: ten students achieved Levels 2 or 3 with the positives (40%) and seven with the negatives (28%).

To make the comparison between the groups more rigorous, tests of four a priori hypotheses were conducted using Bonferroni adjusted alpha levels of .0125 per test (.05/4). The four hypotheses consisted of checking for significant differences between the mean change in level of understanding, pretest to posttest, for the positive and negative integers, crossed with the two grade levels. Results

Table 2: A Kindergartener's Number Line Estimation Data (Level 0) on Pre- and Posttest

Student G02, Pretest Positive Values	Student G02, Posttest Positive Values
	
R^2 = .01; Slope = -.02, all with wrong sign	R^2 = .28; Slope = -.50, all with wrong sign
Student G02, Pretest Negative Values	**Student G02, Posttest Negative Values**
	
R^2 = .02; Slope = -.10, all with wrong sign	R^2 = .62; Slope = -.68, all with wrong sign

indicated that the mean change in level of understanding was not significantly different with the positives for the kindergarteners between the control group (*M* = -0.05, *SD* = 0.21) and the game group (*M* = 0.09, *SD* = 0.29). Also for the kindergarteners, the mean change was not significantly different with the negatives between the control group (*M* = -0.23, *SD* = 0.61) and the game group (*M* = -0.13, *SD* = 0.46). Likewise, there was no significant difference seen in the first graders with the positives between control (*M* = -0.32, *SD* = 0.95) and game (*M* = 0.18, *SD* = 1.01). However, there was a significant difference seen the mean change with respect to the negatives in first grade between the control (*M* = -0.24, *SD* = 0.72) and the game group (*M* = 0.55, *SD* = 1.18), $t(34) = -2.70$, $p = .011$. Therefore, the intervention, i.e., playing the linear board game significantly impacted participants' ability to place negative integers on an empty number line.

Students who were less successful on the number line estimate task fell into two major groups. One set of students spaced out the numbers along the entire line, ignoring that 0 fell in the middle of the line. Therefore, these students had close to half of their points fall on the wrong side of zero (see Figure 2).

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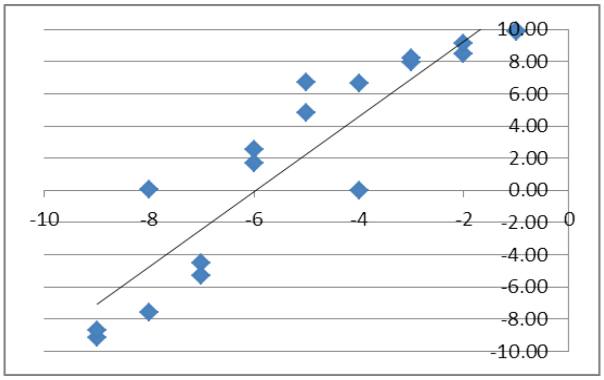

Figure 2: An illustration of data spanning the whole number line when placing -9 to -1.

A second set of students placed numbers in two to three similar locations, regardless of the number shown, as if they split the number line into small and large or small, medium, and large. Therefore, their points formed distinct clusters along the line (see Figure 3). Sometimes, these students started at 0 and counted up to place numbers 1-5 or started at 10 and counted down to place numbers 6-10, which accounted for the clustering.

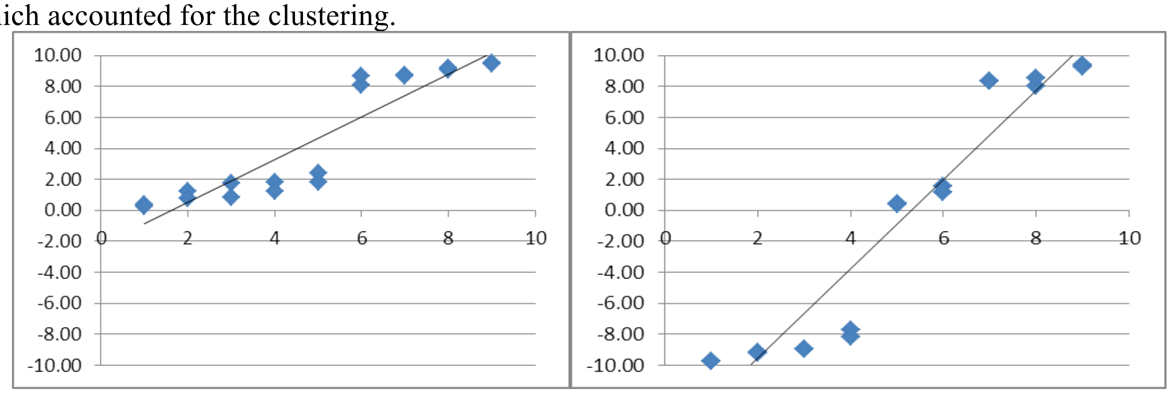

Figure 3: Illustrations of data chunked along the number line when placing 1 to 9.

Conclusions and Implications

Based on the results, we conclude that playing the board game did not help kindergarteners develop a mental number line including negative numbers. Although they started to space out their placement of the integers, they frequently placed numbers on the opposite side of zero. More surprising, they did not improve on placing the positive values on the board as was found in previous studies with preschoolers (Ramani & Siegler, 2008; Siegler & Ramani, 2009). A likely reason for this is that students were given space to mark positive numbers before 0 (as opposed to having zero at the edge of the page). Therefore, they often chose to mark 0 and 1 near the left edge of the paper, to the left of zero, at the beginning of the line. This suggests that as students learn about positive numbers, they need opportunities to see zero in other locations than just at the edge of the paper, and also suggests that Ramani and Siegler's (2008) results may overestimate students' abilities. Because the kindergarteners here learned about positive numbers *and* negative numbers, it is also possible that the kindergarteners had too much to learn compared to children in Ramani and Siegler's study (2008), and the time spent on negatives might have taken away from time needed with positive numbers. Alternatively, providing a longer intervention may lead to a stronger effect for both positive and negative numbers.

On the other hand, the first graders in the game group benefitted from playing the board game. Almost all of the students were able to identify negative numbers on the posttest and a significant number were able to estimate the placement of all integers on the number line fairly well. These

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results suggest that students are more likely to develop a mental number line that includes negative numbers if they already have a whole number mental number line.

Finally, students' placement of the numbers suggests a few areas to focus on in instruction. Students had an inclination to take up as much space as they were given, spacing out the positive numbers across both negative and positive parts of the number line (and similarly for negative numbers). Further, they often started counting from the very left of the page, rather than attending to the given points. When introducing and using visual aids such as the number line in the classroom, teachers should present numbers in multiple formats (not always starting at the left of the paper) and talk about numbers on either side of key reference points, such as zero. Presenting number lines with different numbers marked and with different scales may help students attend to the relevant features of the number lines and placement of numbers.

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