

# A STUDY ON TEACHING GROUP AND SUBGROUP CONCEPTS

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## ABSTRACT

The aim of this study is to investigate the effectiveness of computer-based/assisted teaching method using ISETL programming language on students' understanding of subgroup and group concepts. The data was collected from a group of students by means of written assesments and clinical interviews. Analysis of student's written responses and clinical interviews seems complexities in their understanding of group and subgroup concept.

**Key words:** ISETL (Interactive SET Language), Learning Group, Learning Subgroup

## GRUP VE ALTGRUP ÖĐRETİM KAVRAMLARININ ÖĐRETİMİ HAKKINDA BİR ÇALIŐMA

### ÖZET

Bu çalışmanın amacı, ISETL programlama dilinin kullanıldığı bilgisayar destekli öğretim yönteminin öğrencilerin altgrup ve grup kavramları ile ilgili başarılarına etkisini incelemektir. Çalışmanın verileri yazılı ve klinik görüşmelerle toplanmıştır. Öğrencilerin yazılı ve klinik görüşmelerinin analizi sonucu grup kavramını anlamada karmaşıya yaşadıkları görülmüştür.

**Anahtar kelimeler:** ISETL, Grup Öğrenimi, Altgrup Öğrenimi

## INTRODUCTION

Abstract algebra has traditionally thought to be the most difficult course because of abstract nature of the course, and particularly many students have a shock in group theory.

The concept of group is an example of a new mental object the construction of which causes fundamental difficulties in the transition from school to university mathematics. As Robert and Schwarzenberger (1991) point out, one root of this difficulty is historical and

epistemological: "the problems from which these concepts arose in an essential manner are not accessible to students who are beginning to study (and expected to understand) the concepts today (Nardi, 2000).

There has been a large number of research on the learning and teaching of group theory. Some would like to make modern algebra and group theory more visual (Bardzell and Shannon 2002, Cox, Little and O'shea 1997). They use some structures to visualize certain concepts from abstract algebra including

groups, subgroups, quotient groups, and automorphism.

There are a number of studies to integrating technology into the abstract algebra and group theory class. For example Rainbolt (2001) used GAP software as a tool in abstract algebra class. Keppelmann and Webb (2000) used Finite Group Behavior (FGB) program and they discuss the features and philosophy of FGB. Cannon and Playoust (1995) used the Magma computer algebra system in their class. Scientists such as Selden and Selden (1978), Hazzan and Leron (1999) described misconceptions in abstract algebra.

However, the current trend in pedagogy in abstract algebra has focused on the learning perspective. And an important attack on this approach has been made by Ed Dubinsky and his colleagues. According to this approach, learning is central and the research focuses on how students learn mathematics.

In fact, replacing the lecture method with constructive, interactive methods involving computer activities and cooperative learning can change radically the amount of meaningful learning achieved by average students (Leron and Dubinsky, 1995).

An individual's knowledge of the concept of group should include an understanding of various mathematical properties and constructions independent of particular examples, indeed including group consisting of undefined elements and a binary operation satisfying the axioms (Dubinsky et al. 1994).

I used ISETL programming language in this study. ISETL has several important properties. It is designed as a tool for teaching and learning mathematics, particularly abstract algebra. ISETL's syntax resembles to standard mathematical notation.

## METHOD

The students used in this study were 15 students enrolled in abstract algebra course designed for the professional teaching of

mathematics program in Atatürk University of Turkey.

I used the text "Learning Abstract Algebra with ISETL" by Ed Dubinsky and Uri Leron (1994). The course ran for 6 weeks with 4 hours of lectures per week. The data was collected from two main sources: Written assessment and clinical interviews.

Near the end of the course, a written assessment was given to determine the students' understanding of group and subgroup concepts. The questionnaire was given in Appendix.

Based on the results of the written assessment, 9 students were selected to discuss their results. While selecting students to interview, special attention was given to interviewing some students who gave correct, partially correct and incorrect answers on the assessment. The interviews are about 15-20 minutes in duration.

## FINDINGS and DISCUSSION

My analysis focused on the following issues: Group and Subgroup.

### Group Concept :

Table 1. Scores on Group

| Part 1 | Correct Answer | Partially Correct | Incorrect Answer | No Response |
|--------|----------------|-------------------|------------------|-------------|
| 1      | 14             | 1                 | 0                | 0           |
| 2      | 9              | 4                 | 1                | 1           |
| 3      | 10             | 1                 | 4                | 0           |
| 4-A    | 15             | 0                 | 0                | 0           |
| 4-B    | 15             | 0                 | 0                | 0           |
| 5      | 8              | 4                 | 0                | 3           |

Many students had no difficulty describing the group concept, that is, a set with a binary operation and four axioms. But only one student wrote commutative property as a group property.

As stated above, abstract group is a conceptual object like other concepts of abstract algebra. When students try to understand this unfamiliar concept, they can make some

mistakes. For example they see group as a set; because, the set concept is familiar for students. And they do not take a consideration operation.

In some cases group operation changes from an unfamiliar one to a familiar one. In the following excerpt, Ali considered that  $Z_7 - \{0\}$  is not a group.

*If  $Z_7 - \{0\}$  is a group then it has to have all of axioms of groups, which means it has to be closed, it has to be associative, it has to have identity and inverse.  $Z_7 - \{0\}$  consists of  $\{1,2,3,4,5,6\}$  and it is not a group. Because it is not closed. For example  $3 * 4 = 12$ . 12 is not in this set. Hence  $Z_7 - \{0\}$  is not a group.*

Ali considered the operation of  $Z_7 - \{0\}$  as an ordinary multiplication. Ali was not aware of the operation.

Some students confused global properties of an operation on a set. Sometimes associative, commutative are inherited from a larger group. These properties do not change in a smaller set. Some students confused this shortcut and they considered that the operation was inherited from a larger group. Sevgi considered that  $Z_7 - \{0\}$  is a group. Because she thought that integers are a group under ordinary multiplication.

*$Z_7 - \{0\}$  is a group. Because integers are a group with ordinary multiplication, operation in  $Z_7 - \{0\}$  is inherited from integers.*

Sevgi considered the operation as ordinary multiplication. She didn't know that it required particular attention on the operation. However she didn't know that integers were not a group under multiplication.

**Subgroup Concept:**

Table 2. Scores on Subgroup

| Part 1 | Correct Answer | Partially Correct | Incorrect Answer | No Response |
|--------|----------------|-------------------|------------------|-------------|
| 1      | 15             | 0                 | 0                | 0           |
| 2      | 8              | 0                 | 0                | 7           |
| 3      | 7              | 1                 | 1                | 6           |
| 4      | 7              | 3                 | 5                | 0           |
| 5      | 6              | 3                 | 6                | 0           |

All students in this study described subgroup concept correctly. Some students considered that every subset was subgroup. For example Cemal said that  $\{0,1\}$  was a subgroup of  $Z_8$ .

*A subgroup of  $Z_8$  is  $\{0,1\}$ . Because this set is a subset of  $Z_8$  and it is a group.*

Cemal did not consider that a subgroup was an independent group. He firstly considered a subgroup as a subset.

Perhaps the most common confusion was in question "Is  $Z_3$  a subgroup of  $Z_6$ ?" (see Hazan, Leron, 1996). In the following excerpt, Gül tried to determine whether  $Z_3$  is a subgroup of  $Z_6$ . She said:

*"Well. One minute. A subset of a group is a subgroup if it has all of the group axioms. This subset has to be closed, it has to be associative, it has to have an identity, an inverse.*

*" $Z_3$  is a subset of  $Z_6$ . The elements of  $Z_3$  are 0,1,2.  $Z_3$  has to be closed. It is already closed."*

*(Writes as follows:*

$$1+1=2, 2 \equiv 1 \pmod{3}, 1 \in Z_3$$

$$2+2=4, 4 \equiv 1 \pmod{3}, 1 \in Z_3$$

$$1+2=3, 3 \equiv 0 \pmod{3}, 0 \in Z_3$$

*Hence it is closed.)*

Gül made several errors in her attempt to verify group axioms. First she stated:

$$1+1=2, 2 \equiv 1 \pmod{3}, 1 \in Z_3$$

In addition to confusion about modular arithmetic, she automatically considered the subgroup operation as modulo 3.

Our students could not see the operation in  $Z_3$ . Many knew that  $Z_3$  with addition mod 3 and  $Z_6$  with addition mod 6 was a group. But when I asked this question (Is  $Z_3$  a subgroup of  $Z_6$ ?), the students exhibited disequilibrium.

## CONCLUSION

My primary concern in this study was how the students learned and understood group and subgroup concepts.

I strongly believe that students' active involvement is essential to understanding these concepts. I used ISETL for this purpose. ISETL can be a useful tool in teaching and learning abstract algebra. But the students need more time.

I feel that students acquire a deeper understanding of these two concepts.

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## APPENDIX:

### *Written Assessment Questions*

#### *Part 1:*

1. What is a group?
2.  $G = \{(a,b): a,b \in \mathbb{R} \text{ (Real numbers) and } a \neq 0\}$ . Let  $(a,b) * (c,d) = (ac, bc+d)$  in  $G$ . Show  $(G,*)$  is a group.
3. Is  $(\mathbb{Z}_7 - \{0\}, *_7)$  a group? ( $\mathbb{Z}_7 - \{0\}$  consists of  $\{1,2,3,4,5,6\}$  and operation is multiplication mod 7)
4. Consider the group  $(\mathbb{Z}_9, +_9)$ , consisting of the set  $\{0,1,2,3,4,5,6,7,8\}$  and the operation of addition mod 9.
  - a. What is the identity of this group?
  - b. What is the inverse of 5 in this group?
5. Let  $(G,o)$  be a group with identity  $e$ . Show that if  $x^2 = x \circ x = e$  for all  $x$  in  $G$ , then the group is abelian.

#### *Part 2:*

1. What is a subgroup?
2. If  $H$  and  $K$  are subgroups of  $G$ , then prove that  $H \cap K$  is a subgroup of  $G$ .
3. The center of a group  $G$  is the set of elements  $Z = \{x : xg = gx \text{ for all } g \in G\}$ . Show that  $Z$  is an abelian subgroup of  $G$ .
4. Find a subgroup of  $\mathbb{Z}_8$ . Explain your answer.
5. Is  $\mathbb{Z}_3$  a subgroup of  $\mathbb{Z}_6$ ? Why?