PRESERVICE TEACHERS' CONCEPTION OF EFFECTIVE PROBLEM-SOLVING INSTRUCTION AND THEIR PROBLEM SOLVING

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Despite the importance of teachers' conception of effective problem-solving instruction, limited attention is given to this area in the current literature. In this study we examined 96 preservice teachers' (PSTs) views on effective problem-solving instruction and how their conceptions are related and reflected in their views on problem solving and problem solving performance. Analyses of survey responses revealed that our PSTs seem to develop narrow views on effective problem-solving instruction. In addition, we found a positive association between PSTs' conceptions of effective problem-solving instruction and problem solving. However, no such connection exists between PSTs' views problem-solving instruction and their performance.

Keywords: Teacher Beliefs, Problem Solving, Teacher Knowledge, Teacher Education-Preservice

Introduction

Preparing effective teachers of mathematics who promote students' conceptual understanding and problem-solving abilities is one of the most urgent problems facing teacher educators (Morris, Hiebert, & Spitzer, 2009). It is recognized that the quality of problem-solving instruction matters the most for improving students' mathematical abilities (e.g. Stigler & Hiebert, 1999). However, there has not been a clear agreement about what can be counted as effective problem-solving instruction (Krainer, 2005). The interpretation of a problem-solving lesson as 'good (or effective)' or 'bad (or ineffective)' is a value-loaded judgment. In addition, the notion of problem solving has been used with multiple meanings that range from "working rote exercises" to "doing mathematics as a professional" (Lesh & Zawojewski, 2007). Thus what is meant by effective problem-solving instruction is often subject to interpretation in a particular context and ill-defined in the literature, which suggests the importance of clarifying the meaning of effective problem-solving instruction in mathematics education.

Although several researchers focused on students' and teachers' perspectives of mathematics classes includes their meanings of "effective teaching", "good teaching", "good teacher", "good class", or "model class" (Kaur, 2008; Li, 2011; Seah & Wong, 2012; Cai & Wang, 2010), limited attention is given to preservice teachers' (PSTs) conception of effective problem-solving instruction in the current literature in the US context. The purpose of this study is to explore PSTs' conceptions of problem solving and effective problem-solving instruction and to investigate any relationship that might exist among PSTs' conceptions of effective problem-solving instruction, problem-solving, and their problem solving performance. In exploring the relationship between PSTs' conception of effective problem-solving instruction and their problem solving abilities, we specifically focus on fraction topics because it is often reported that not only students but also teachers have difficulties in understanding fractions and fraction operations (NRC, 2004; Son & Crespo, 2009. The research questions that guided this study are: (1) What are the characteristics of PSTs' thinking about effective problem-solving instruction and problem solving?; (2) Is there any relationship among PSTs' conceptions of effective problem-solving instruction, problem solving, and their problem solving performance?; and (3) What are the PSTs' views on what it takes to develop effective problem-solving instruction?

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Theoretical perspectives

What constitutes effective problem-solving instruction?

Problem solving is a powerful vehicle for students' mathematical learning (NCTM, 2000). Schroeder and Lester (1989) identified three types of teaching approaches to problem solving that have been emphasized at different periods of time in mathematics education: (1) teaching *for* problem solving, (2) teaching *about* problem solving, and (3) teaching *through* problem solving. Each of these perspectives offers different affordances. The first approach involves teaching skills or abstract concepts first and then students apply the learned skills or concepts to solve the given problems. The second approach indicates teaching students the process of problem solving or strategies for solving problems explicitly. In the book of *How to Solve it* (1945), George Polya generalized the four steps that can be used regardless of subject matters—(1) identifying a problem, (2) designing a strategy, (3) implementing, and (4) looking back. Teachers explicitly teach the aforementioned four-step process with strategies for problem solving (i.e., approaching methods to a problem). The third perspective includes classroom instruction where students learn mathematical concepts through real contexts and problems, which helps students build meaning for the concepts before moving to abstract concepts (Boaler, 2008; NCTM, 2000, 2014).

Various criteria can be used in specifying the features of effective problem-solving instruction. Drawn from Stanic and Kilpatrick (1989), we believe that *problem solving as art* should be a goal of effective problem-solving instruction. According to Stanic and Kilpatrick, three different meanings were attributed to the notion of problem solving in mathematics education-- *problem solving as means to a focused end, problem solving as a skill,* and *problem solving as art.* Different from the first and second perspectives where problem solving is viewed as a means to practice skills or as one skill taught in school mathematics, problem solving should be viewed as an act of discovery through creative use of mathematical thinking. Thus, among the three types of teaching approaches by Schroeder and Lester (1989), we consider the third approach--*teaching mathematics through problem solving* as effective problem-solving instruction (NCTM, 2000, 2014).

Research on teachers' conceptions of problem solving and effective instruction

Prior research has documented that teachers' beliefs and conceptions about the subject and its teaching interact and influence mathematics teachers' planning and delivery of instruction which may impact student achievement (Koehler & Grouws, 1992). However, research directly addressing the issue of 'good (or effective)' classroom instruction from teachers' perspectives is a relatively new endeavor in mathematics education (Cai, Kaiser, Perry, & Wong, 2009). In particular, limited attention is given to the issue of 'good (or effective)' problem-solving instruction from teachers' perspectives in the current literature in the US context.

Kaur (2008, 2009) carried out a series of studies in Singapore where 8th grade students were asked to describe the qualities of a "good mathematics class" and the "best mathematics teachers". Kaur concluded that "good mathematics teaching in Singapore is student-focused" (2009, p. 346). However, Shimizu (2006), who investigated Japanese students' perceptions on good mathematics lessons, reported that Japanese students consider that a mathematics class is good when there is a "whole class discussion" (2009, p. 316). In general, the research presented herein shows that the views of teachers and students regarding "good teaching" or "effective teaching" vary in relation to multiple factors.

Using the three meanings of problem-solving by Stanic and Kilpatrick (1989) and the three teaching approaches to problem solving by Schroeder and Lester (1989), we explored PSTs' conceptions of problem solving and effective problem-solving instruction.

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Methods

96 PSTs from two different university sites – one from a large northeastern university and the other from a large southwestern in the US – were invited for this study. Participants majored in elementary education and they were either in their sophomore, junior or internship year. A written task was used for the study, which consists of two parts (see Fig. 1).

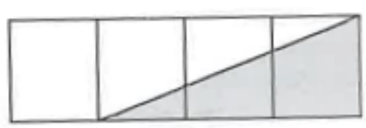
Part 1: Please answer the following questions in as much detail as possible.

1. When people say problem solving, what does the word "problem solving" mean to you?
2. What do you believe constitutes effective problem-solving instruction?
3. What skills are necessary to create effective problem-solving instruction?
4. How do you believe the skills necessary for teachers to create effective problem-solving instruction develop?

Part 2: Solve the following problems.

1. At both Rivers High School and Mountainview High School, ninth graders either walk or ride the bus to school. 6/7 of the 9^{th} grade students in Rivers High School ride the bus, while 7/8 of the 9^{th} grade students in Mountainview High School ride the bus. If there are 40 9^{th} grade students who walk at Rivers and 25 9^{th} grade students who walk at Mountainview, in which school do more students ride the bus? In which school do a greater fraction of the students ride the bus? Explain your strategies or solutions as much as in detail.
2. For each picture shown below, (i) write a fraction to show what part is shaded. For each picture, (ii) describe in pictures or words how you found that fraction, and why you believe it is the answer.

(1)



(2)

3. Merlyn spends \$60 of her paycheck on clothes and then spends 1/3 of her remaining money on food. If she had \$90 left after she buys the food, what was the amount of her paycheck? Explain your solution method as much as in detail. You may use representations (e.g., diagrams, rectangles, number line etc.).

Figure 1: Main task of this study.

For the analysis of PSTs' written response to problem solving and effective problem-solving instruction, we used an inductive content analysis approach (Grbich, 2007). We initially organized raw data into an Excel spreadsheet, read all of the responses. PSTs' responses to the notions of problem solving and effective problem-solving instruction were categorized based on themes emerging as researchers read multiple cases. Then we explored the subcategories under each analytical aspect according to the framework (e.g., Table 1). Finally, we interpreted the data quantitatively and qualitatively (Creswell, 1988).

For the problem solving task, we first created a rubric based on correctness of PSTs' responses to each item and their problem solving process and assigned a score for each item. To examine relationship among PSTs' conceptions of problem solving, effective problem-solving instruction, and

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their problem solving performance, we ran SPSS statistical program (i.e., chi-squared tests and ANOVAs).

Summary of selected findings

PSTs' conceptions of effective problem-solving instruction and problem solving

To investigate PSTs' conception of effective problem-solving instruction, we reviewed their responses and classified the responses into four aspects based on common themes (see Table 1). Among the four aspects, the first one is the *purpose* aspect of problem-solving instruction (i.e., what is a good problem solving lesson aimed at?). The second aspect is *problem features* (i.e., what is considered as a good problem for problem-solving instruction?) and third one is *problem solving steps* aspect (i.e., what step(s) are/is required for problem-solving lesson?), which involves four steps to solve a problem such as identifying a problem, planning a strategy, carrying out and looking back. The last one is *teaching* aspect (i.e., what instructional strategies or teaching practice are needed for effective problem-solving instruction?).

Table 1: Four aspects of PSTs' conception of a good problem solving lesson and frequencies

Category	Sub-category	# of PSTs	Relation to 3 approaches of teaching PS
1. Purpose aspect (28)	To find a good solution	7	For
	To develop critical/ creative/ logical /reflective thinking (cognitive aspects)	16	Through
	To develop a good understanding of mathematics	5	Through
2. Problem aspect (41)	a. Word problems	2	For/Through
	b. Real-life problems	2	Through
	c. Problems that allow students to use their prior knowledge	5	Through
	d. More practice problems that allow students to apply the same technique	6	For
	a. Problems that require different strategies/multiple solutions	13	Through
	b. Problems that require explanations	0	Through
	c. Problems that require various representations	3	Through
	d. Problems that require creativity	1	Through
	h. Problems that are not overwhelming/not too difficult	9	For
3. Problem solving steps aspects (42)	a. Structuring a lesson based on all four problem solving steps	14	About
	b. Identify problem	12	About
	c. Devise a strategy	3	About
	d. Carry out	2	About
	e. Look back	11	About
4. Teaching aspect (78)	a. Emphasizing different/multiple ways of solving a problem	26	Through
	b. Allowing students to share and discuss their ideas	8	Through
	c. Making sure if students understand the topic	7	Through
	d. Giving examples about how to solve	6	For
	e. Giving definitions	2	For
	f. Providing hands-on manipulatives	1	Through
	g. Giving enough time to work on problems	5	For/Through
	h. Providing a direct and clear direction and structure	15	For/About
	i. Engaging students in solving a problem mentally	6	Through
	j. Lessons that are interesting to students	2	Through

Note. Majority of PSTs addressed multiple categories. These responses were coded in multiple categories as long as the categories were present in their written responses.

Out of the four aspects, the most popular category is teaching aspect, followed by problem solving steps aspect, problem aspect, and purpose aspect. Among PSTs who mentioned about the teaching aspect, 26 PSTs emphasized that different/multiple ways of solving a problem is important for effective problem-solving instruction. However, interestingly, many PSTs out of the 26 PSTs also considered that it is important to provide a direct and clear direction (15 PSTs) or give examples about how to solve a problem (6 PSTs). This finding suggests that our PSTs perceived the value of multiple solutions, but they believed that they could teach the different/multiple solutions through a *direct instruction* rather than through student-centered discussions. After identifying the four aspects, we collectively considered them to categorize PSTs' conception of effective problem-solving instruction into the three groups by referring to Schroeder and Lester's (1989) identification. Out of 96 participants, 42 participants considered effective problem-solving instruction as teaching *about* problem solving, 23 participants as teaching *through* problem solving, and 31 participants as teaching *for* problem solving. This finding indicates that despite the consistent emphasis on teaching *through* problem solving in current mathematics education, a large portion of our PSTs still did not have a clear view of teaching *through* problem solving.

In a similar way to what we analyzed for PSTs' conception of effective problem-solving instruction, to explore their conception of problem solving, we reviewed their responses and classified the responses into four aspects: problem aspect, process aspect, purpose aspect, and knowledge/skills/ability required. Out of the four aspects, the purpose aspect is the most frequent, followed by process aspect, and problem aspect and only a small number of PSTs considered the aspect of knowledge/skills/ability required for problem solving (Son, Lee, & Arabeyyat, 2015). Based on this analysis, we categorized PSTs' conception of problem solving into three groups by referring to Stanic and Kilpatrick's (1989) identification. Out of 96 PSTs, 55 PSTs considered problem solving as *means* to a focused end, 27 PSTs as a *skill*, and 14 PSTs as *art* of discovery.

Relationship between PSTs' conceptions and their mathematical performance

A chi-squared test showed that there is a positive relationship between PSTs' conception of problem solving and their conception of effective problem-solving instruction, $\chi^2 = 16.888$, $df = 4$, $p = 0.002$. That is, PSTs who perceived problem solving as *means* to a focused end seem to consider effective problem-solving instruction traditionally in that their views on effective instruction were categorized into "teaching *for* problem solving". In addition, the results for ANOVAs revealed that there is a significant difference of mean scores concerning problem solving competence among groups of PSTs who perceived different views on problem solving, $F(2, 75) = 3.292$, $p = .042$. PSTs who perceived problem solving as *art* showed highest mean scores in the problem solving tasks, followed by PSTs who with problem solving as *means* to a focused end (see Table 2).

Table 2: Results from ANOVAs test

		Sum of Squares	df	Mean Square	F	Sig.
1	Between Groups	349.936	2	174.968	3.297	0.042
	Within Groups	3980.0120	75	53.067		
	Total	4329.949	77			
2	Between Groups	25.768	2	12.884	0.216	0.807
	Within Groups	2743.049	46	59.631		
	Total	2768.816	48			

Note: 1 = PSTs' views on problem solving; 2 = PSTs' views on effective problem-solving instruction

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However, there was no such connection between PSTs' conception of effective problem-solving instruction and their mathematical performance, $F(2, 46) = 0.216$, $p = .807$. Appendix A presents PSTs' mathematical competence in the problem solving task, focusing on PSTs who perceived problem solving as art.

What knowledge and skills are needed for effective problem-solving instruction?

When the PSTs were asked to indicate types of skills necessary for creating effective problem-solving instruction, our PSTs pointed out not only knowledge and disposition but also skills such as problem solving skills, teaching skills, and lesson design skills (see Table 3). Out of the three big categories, the most popular category is skills, followed by knowledge and dispositions.

Table 3: PSTs' report on knowledge and skills needed for *effective problem-solving instruction*

Knowledge and skills for good lessons	
1. Knowledge	*72*
1.1 Knowledge about content.	22
1.2 General pedagogical knowledge.	6
1.3 Knowledge about problem solving steps.	11
1.4 Knowledge on different solving strategies.	18
1.5 Thinking (critical, organized, creative, thinking)	15
2. Skills	*79*
2.1. Problem solving skills	39
2.1.1. Identifying or understanding questions/problem	18
2.1.2. Breaking problem into easier steps	2
2.1.3 Organizing or logical skill	6
2.1.4 Thinking backward/Reflect or check answers	1
2.1.5 Following the 4 steps (Work on the process behind problem solving)	4
2.1.6 General good problem solving skill (Not specified)	8
2.2. Teaching skills	30
2.2.1 Unpacking knowledge/Step by Step direction	3
2.2.2 Explaining/ Articulating	10
2.2.3 Answering students' diverse questions	3
2.2.4 Attending to students' thinking or work (Noticing skill)	6
2.2.5 Engaging/Motivating students	4
2.2.6 General teaching skill (Not specified)	4
2.3. Lesson design skills	10
2.3.1. Lesson planning	4
2.3.2 Creating a good problem	6
3. Attitude and disposition	*50*
3.1 Patient	15
3.2 Open-mind	12
3.3 Creative	18
3.4 Collaborative	2
3.5 Efforts or working hard	5

When we further explored PSTs' perception of what it takes to develop effective problem-solving instruction, ten categories emerged shown in Table 4. A large portion of the PSTs considered that necessary skills for effective problem-solving instruction are developed through teaching experience (31) or problem solving itself (44).

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Table 4: Resources for developing skills for good problem solving lessons

Category	Total
1. By practicing problem solving or solving problems to be a good problem solver through mastering problem solving skills	44
2. Experience or trial and error (e.g., teaching and making lesson plans)	31
3. Developing dispositions (e.g., creativity, open-minded, compassionate toward kids)	12
4. Time	11
5. By learning knowledge or skills (e.g., different problem solving methods)	10
6. By understanding students' work or working with students	6
7. Teacher education program (e.g., by taking classes)	5
8. Working with other teachers	4
9. Finding resources (e.g., standard, book, videos)	3
10. By observing experienced teachers, experts, or mentor teachers	1

Discussion and Implications

This study contributes to the current literature on problem solving and the knowledge base of teacher education. In particular, this study has implication for teacher educators working to design mathematics education courses for PSTs, as well as for researchers interested in furthering understanding of teachers' knowledge, beliefs, and problem solving strategies. The findings of this study suggest that teacher educators need to find a better way to help PSTs perceive problem solving as *art* and effective problem-solving instruction as teaching mathematics *through* problem solving. One approach would be: Have PSTs experience three different perspectives of teaching mathematics and compare affordances and limitations of each approach. Then teacher educators need to give PSTs more opportunities to experience teaching *through* problem solving in their mathematics methods courses where PSTs engage in mathematical modes of thought by analyzing and interpreting the problems (Son, 2013; 2016). Future studies need to be done with different research tools and in multiple contexts, possibly using interviews or observations to provide more detailed explanations for teachers' responses. Furthermore, intervention studies that experiment with these suggestions are needed to find a better way to support PSTs' conceptions regarding problem solving, problem solving lessons, and their problem solving abilities.

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