

WHAT KNOWLEDGE AND SKILL DO MATHEMATICS TEACHER EDUCATORS NEED AND (HOW) CAN WE SUPPORT ITS DEVELOPMENT?

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Refocusing teacher preparation on the practice of teaching calls for new and different knowledge and skill. We describe the types of knowledge and skill that teacher educators need in order to teach a practice-focused mathematics methods course and report on one program's exploration of possible supports to develop this knowledge in teacher educators.

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In recent years, scholars and teacher educators have called for teacher preparation to focus more directly on the practice of teaching (Ball & Forzani, 2009; Grossman, Compton, Igra, Ronfeldt, Shahan & Williamson, 2009; Lampert & Graziani, 2009). They emphasize that preservice teachers (PSTs) need to learn to do key aspects of the work of teaching, rather than just talking about teaching or analyzing someone else's teaching. This call has spurred important changes in both the design and implementation of teacher education. For example, in some teacher education programs, course content has shifted to the teaching of specific "high-leverage practices" (Ball, Sleep, Boerst & Bass, 2009; Davis & Boerst, 2014; McDonald, Kazemi, & Kavanagh, 2013). Teacher education pedagogy is also shifting, in particular, to incorporate "pedagogies of enactment" (Grossman et al., 2009), which include "approximations of practice" such as coached rehearsals (Kazemi, Ghouseini, Cunard, & Turrou, 2015).

This shift toward practice-based teacher education occurring in many programs, combined with influxes of new mathematics teacher educators (MTEs), creates a need to focus on the preparation and professional learning of MTEs. The work of teacher education is complex. MTEs must support PSTs in developing deep and usable knowledge and skill. They need to understand the content themselves and in more specialized ways to support teacher learning (Superfine & Li, 2014), be able to manage the teaching of integrated content, and be able to provide specific and detailed feedback aimed at improving teachers' practice (Van de Ridder, Stokking, McGaghie, & Cate, 2008). Our teacher education program, like many, must prepare novice MTEs to *do* the work of teacher education while simultaneously supporting the learning of PSTs. This work is different in important ways from K-12 classroom teaching. Not only must MTEs be able to enact teaching practices such as eliciting student thinking or leading mathematics discussions, in practice-based teacher education MTEs must be able to decompose teaching practice (Grossman et al., 2009) and support PSTs' learning of these practices. The work of teaching PSTs to successfully engage in these practices entails being able to identify and talk about specific skills and techniques for carrying out the practice as well as appraising PSTs' enactment of the practice. This is different than successfully engaging in the teaching practice.

Less attention has been paid to the knowledge and skills that MTEs themselves need to teach teaching practice; however, the findings we do have suggest that MTEs may be underprepared. For example, in a study of 293 practicing university-based teacher educators, Goodwin and colleagues

(2014) found that most respondents felt underprepared for the work, reporting “happenstance in becoming engaged in teacher education” and a “lack of explicit development of teaching skills or pedagogies related to teacher educating” (p. 291). It is crucial to focus on the professional learning of MTEs for a number of reasons. Practice-based teacher education places increasing demands on MTEs, both in their instruction and their appraisal of PSTs’ skills. Second, in our context, our doctoral students will be among the next generation of MTEs, and we want graduates of our doctoral program to be prepared to support the learning of PSTs. Third, within our specific teacher education program, we seek ways to prepare and support MTEs to enact teacher education in ways that are aligned with the goals of the program. Across the board, MTEs’ skills matter for the outcomes of teacher education and such attention is crucial to achieving the desired outcomes.

This paper is focused around two fundamental questions in the preparation of MTEs. First, what knowledge and skill is demanded of MTEs to engage in practice-based teacher education? Second, what structures are useful for supporting novice MTEs’ learning of such knowledge and skill? We use the case of an elementary mathematics methods course to consider these questions.

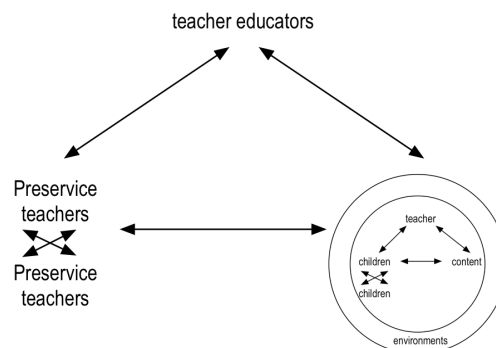


Figure 1. Instructional triangle for teacher education (D. L. Ball, personal communication).

Theoretical Framework

Teacher education is something that people *do*; it is not merely something to *know*. MTEs must use knowledge flexibly and fluently as they interact with PSTs, with the aim of helping PSTs become proficient with teaching. This interactive view of instruction can be portrayed using the “instructional triangle” (Cohen, Raudenbush, & Ball, 2003). Conceptualizing the work of teaching PSTs as interactions among teachers (teacher educators), students (preservice teachers), content (K-12 teaching practice), and environments (see Figure 1) has important implications for the identification of skills and knowledge that MTEs need to enact practice-based teacher education. For example, MTEs’ knowledge of content, particularly teaching practice, must go beyond being able to enact teaching practices to include being able to identify and decompose such practices, talk about the practices and ways of enacting them, and see different ways of enacting the practice, all of which must adhere to the articulation of the practice. Further, MTEs may need specialized knowledge of mathematics content for teaching teachers (Superfine & Li, 2014). They also need knowledge of their students including the skills that they bring to teacher education, the ways in which they are likely to interpret particular practices, and progressions of development with such practices. Further, because the environment is crucial to the work of teaching, MTEs must consider environments beyond their own classrooms such as field placement classes in which there are mentor teachers with particular orientations to teaching and supporting PSTs’ learning. Thus, teaching practice-based courses requires the integrated use of knowledge and skills in particular contexts of instruction.

Context

Over the last decade our teacher education program has engaged in a collective redesign of its undergraduate elementary teacher education program, centered on an effort to focus more directly on practices of teaching (see Davis & Boerst, 2014). The mathematics methods course has been collectively developed by a rotating group of members (see Ball et al., 2009 for a description of the collective work of the planning group) to focus on preparing PSTs to teach K-6 mathematics skillfully to their students. The course works on four teaching practices with a mathematics focus: (1) explaining core content; (2) leading discussions; (3) assessing students' knowledge and skills; and (4) planning instruction. The course develops PSTs' skill with these practices while simultaneously developing mathematical knowledge for teaching. In this paper, we examine the practice of explaining core content with a particular focus on representing and connecting mathematical ideas.

Explaining core content focuses on the work a teacher does to provide all students with access to fundamental ideas and disciplinary practices. There is much for PSTs to learn about this work, including: “strategically choosing and using representations and examples to build understanding and remediate misconceptions, using language carefully, highlighting core ideas while sidelining potentially distracting ones, and making one’s own thinking visible while modeling and demonstrating” (TeachingWorks, 2015). In teaching, explanations are often co-constructed with students; however, being able to co-construct an explanation requires knowing what is involved in building a mathematical explanation. Learning to do that oneself is an important part of understanding the practice of explanation well enough to help students learn to explain. We work on the practice of explaining core content across the course with a focus on explaining computational algorithms and their connections to representations in order to build PSTs' mathematical knowledge for teaching.

Mathematics Teacher Educator Knowledge and Skills for a Practice-Based Approach

The instructional triangle illuminates the complexity of the work of teaching PSTs. In this paper, we consider a subset of the knowledge and skill called for by these complex interactions – specialized knowledge of teaching, with a focus on the instructional practice of explaining core content with representations. We then turn to identifying teacher education practices and pedagogies that can be used to support PSTs' learning to teach.

Specialized Knowledge for Teaching Teachers: The Case of Explaining Core Content

To enact the type of practice-based approach to teaching “explaining core content” described earlier, MTEs need to draw on different types of specialized knowledge that could be viewed as parallel with types of mathematical knowledge for teaching (Ball, Thames, & Phelps, 2008), that is – common content knowledge, knowledge of content and students, and knowledge of content and teaching. However, the “content” in this case is the work of mathematics instruction, and specifically the work of explaining core content.

For MTEs, *common content knowledge* (CCK) can be viewed as the knowledge that mathematics teachers themselves hold. Consider the case of explaining subtraction with multi-digit numbers. The common content knowledge for teaching PSTs to explain multi-digit subtraction includes the mathematical knowledge for teaching (MKT) that would allow PSTs to engage in this instructional practice. This includes knowing which key mathematical ideas to highlight (e.g., importance of making equivalent trades), common student errors in a particular domain (e.g., challenges with regrouping across zero), and how different representations can be used to illustrate particular ideas (e.g., how bundling sticks can be used to show regrouping). Common content knowledge for MTEs might also include knowledge of the instructional practice itself, such as knowing characteristics of a good explanation (Leinhardt & Steele, 2005). In other words, MTEs need the knowledge that they are helping PSTs learn; however, just as teachers need more than common content knowledge of

mathematics (Ball et al., 2008), MTEs also need more specialized knowledge for teaching mathematics instruction.

MTEs would also need *knowledge of content and students* (KCS), which is characterized by knowledge of interactions between their students (in this case PSTs) and content, which in this example would be the instructional work of explaining core content. Knowledge of content and students includes knowing common errors that PSTs tend to make when engaging in the instructional practice. For example, when PSTs explain subtraction with regrouping using base 10 blocks, we have found that they often use language imprecisely when talking about “taking away” and “trading.” Additionally, PSTs often attempt to explain using base 10 blocks by first solving the problem with the materials and then showing the algorithm instead of conducting them simultaneously to make clear the meaning of the steps of the process. Knowledge of content and students also includes knowing what parts of an instructional practice tend to be most difficult for PSTs such as explaining the equivalency of trades when regrouping.

Another type of specialized MTE knowledge can be described as *knowledge of content and teaching* (KCT). This includes knowing the types of tasks and representations that are useful in helping PSTs learn a particular part of mathematics instruction. In the case of explaining core content, KCT includes knowing which mathematical content domains might be productive choices for PSTs, who are just beginning to learn this practice (e.g., explaining subtraction with regrouping is more accessible than explaining long division). KCT also encompasses knowing the characteristics of video examples that might be useful in illustrating aspects of a practice.

These illustrative examples highlight that, while MTEs need the same knowledge that skilled teachers need to engage in mathematics instruction, they also need an additional layer of knowledge that can be viewed through interactions inside an instructional triangle in which PSTs are the students and the content is mathematics instruction (See Figure 1).

Teaching the Instructional Practice of Explaining Core Content

In addition to specialized knowledge, a practice-based approach also demands new skills on the part of the MTE. To illustrate some of these new pedagogical skills, we consider two different pedagogies of practice that are used in our teacher education program to support PSTs in learning to explain core content as well as a practice of teacher education which has new demands inside of practice-based teacher education¹. These are not meant to be a comprehensive list of ways to support PSTs; rather, these serve as examples of the work that a practice-based approach demands on the part of the teacher educator and draws on the types of specialized knowledge illustrated in the previous section.

Modeling. In a practice-based approach to teacher education, “modeling by the MTE” is a key pedagogy (McDonald et al., 2013). This involves the MTE engaging in the instructional practice in front of PSTs to both demonstrate the practice and provide meta-commentary throughout to narrate and make visible the instructional work and decision-making. For example, to model the practice of explaining core content, a MTE could explain subtraction with regrouping using base 10 blocks and simultaneously comment on the instructional work (e.g., making a meta-comment about the decision of how to represent the numbers and base 10 blocks in parallel to highlight connections). This teacher educator pedagogy requires that the MTE be able engage productively in the practice of explaining core content, drawing on the common content knowledge of teachers described above; however, this pedagogy of practice also requires new and different skills not required by teachers themselves. In this case the skill of simultaneously engaging in a teaching practice while narrating and commenting on the instructional work involved. This additional layer involves skills such as deciding what to highlight (or not) about the instruction and how to describe the work in meaningful ways. These MTE skills draw on the specialized knowledge described above; for example, when modeling the

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

practice of explaining core content, a MTE might decide what to highlight based on their knowledge of what common errors PSTs tend to make when explaining that core content.

Rehearsal. Another pedagogy of practice that demands new work on the part of the MTE is facilitating coached rehearsals with PSTs (Kazemi et al., 2015). Consider the case of facilitating rehearsals of explaining core content. First, this requires the MTE to establish the norms and culture for working together on practice, as PSTs actually engage in explaining core content in front of the class. This likely builds on other relational work but adds new demands as the MTE asks PSTs to share their teaching publically. Another set of skills is required for facilitating the rehearsals themselves. For example, the MTE must decide when to interrupt a rehearsing PST, what to comment on in the performance, and how to coach (e.g., asking a question, making a suggestion, commenting on what was productive). These skills of providing in the moment feedback draw on a MTEs' specialized knowledge as they simultaneously analyze instruction in the moment and determine what might be most productive to coach on.

Providing feedback on pre-service teacher enactments. A teacher education practice used in our program to support PSTs to explain core content is the work of providing formative feedback on PST enactments). Similar to the work involved in facilitating rehearsals, this practice involves determining which parts of a performance to provide feedback on and how. This rests on the MTE being able to recognize productive, problematic, or incomplete aspects of an enactment and draws significantly on the types of specialized knowledge described earlier. Additionally, MTEs need to determine how to ground their feedback in whatever rubrics or decompositions of the instructional practice are used in the course. This work on the part of the MTE is different from giving feedback on PSTs' reflections about their practice (which is also important), as it demands that the MTE interact with and directly give feedback on enactments of practice, rather than on a PST's skill with analyzing and reflecting on his or her own practice.

Given the demands of the coordination of the specialized content knowledge and pedagogies of practice required to teach a practice-based mathematics methods course, we sought to design and study supports that would enable novice MTEs to build their knowledge and skills while simultaneously supporting their PSTs in learning to teach mathematics.

Structures for Supporting MTE Knowledge and Skills for a Practice-Based Approach

To explore the ways in which support could be provided to novice MTEs, our program uses a set of structures to build the specialized knowledge and pedagogical skills demanded in the teaching of a practice-based methods course. We first describe the overarching organization of the work and then closely examine three of the structures that supported our new MTEs.

Planning group participation

To teach the mathematics methods course, novice MTEs participate in a planning group. The group consists of experienced MTEs, many of whom are not currently teaching the course but are invested in either course development or MTE development, and novice MTEs. The group meets several times prior to the start of the course and then once per week throughout the duration of the course. The group is facilitated by a lead MTE who is an experienced MTE and is also responsible for teaching the lead section of the course. All members of the group observe the lead section with an eye to identified areas of focus. Following the observation of the lead section, members debrief the class with regard to the observation focus areas.

The goal of the group is three-fold. First and foremost, we seek to ensure that the course is designed and taught to consistently provide PSTs with learning opportunities that support their development as elementary mathematics teachers. Second, we provide opportunities for new MTEs to build specialized content knowledge and pedagogical knowledge to ensure that all PSTs are receiving instruction that will allow them to engage in the mathematics teaching practices. Third, we

use the teaching of the lead section to adjust the plan or the materials in response to what we learn from the first teaching of the shared plan.

Detailed lesson plans. Prior to each meeting, planning group members review detailed lesson plans and decompositions of practices involved in the upcoming class. The lesson plans include scaffolds to support MTEs in the areas of specialized knowledge, pedagogies of practice, and teacher education practices. An excerpt from a lesson plan is shown in Figure 2 to illustrate the level of detail provided. This plan supports MTEs’ knowledge of content and students by noting the areas in which PSTs may need support and explicitly naming what is to be worked on in the modeling. CCK is also supported through the description of the modeling and in the notes.

Activity/task	Detail	Notes
<p>Modeling the standard algorithm for subtraction with bean sticks: Comparison</p> <ul style="list-style-type: none"> Distinguish between the take-away and comparison meanings of subtraction Understand the meaning of the standard algorithm for subtraction and be able to explain why it works: Begin to problematize the language that is traditionally used with the standard algorithm (e.g., borrowing, you can't take a bigger number from a smaller number) Continue to develop skills with, and explicit knowledge of, what is involved in modeling computational procedures: e.g., physically placing materials; attending to language and using the language of the materials; coordinating use of the materials with talk and recording 	<p>Comparison interpretation: (15 minutes)</p> <p>Explain that we are now going to shift our focus to the comparison interpretation of subtraction. Model the standard algorithm again, with the same numbers (52 – 13), but this time using a comparison interpretation. Do NOT use the modeling checklist for this first performance.</p> <p>Elicit a few comments from the group.</p> <p>Ask interns to model the standard algorithm, with the same numbers (52 – 13), using a comparison interpretation. Suggest they start by laying out both 52 and 13. Encourage interns to use the modeling checklist.</p> <p>(If interns finish early, try another problem like: 60 – 27; 48 – 25 etc..)</p> <p>If time permits, invite an intern to come to and model for the group. Alternative, discuss questions/challenges which arose when working with a partner.</p>	<p><i>Modeling the comparison tends to be more complicated and it is important for interns to see a complete performance.</i></p> <p><i>The comparison can be modeled by matching, either by drawing lines or by removing the matching pairs. Both have issues to consider: When doing the matching by drawing lines, the answer may not be easy to find — the answer is the amount that remains unmatched. And with taking matches off the board you lose the original numbers in the problem.</i></p> <p><i>You might comment that comparison is typically more difficult for children.</i></p>

Figure 2. Lesson plan for modeling subtraction using the standard algorithm.

Planning group structure. Each meeting is structured to first provide MTEs an opportunity to debrief both the observation of the lead section and the teaching of their own section. These debriefs are focused on questions designed by the lead to highlight the key work of MTEs in this context. For example, the lead MTE might ask group members to observe a class session in which multiple pedagogies of practice are being used to identify the pedagogies and consider the features of each and how they are supporting PSTs’ learning. This type of question brings to the fore the questions that MTEs must consider when determining how to teach particular practices to PSTs as well as the key features of the pedagogies used.

Following the debriefing, MTEs engage in work to prepare for the next class. The structure of this work varies depending upon the mathematical content and required pedagogies and teacher education practices for the class. Structures include framing and walking through particular activities (e.g. setting up the goals and purpose of modeling the explanation of subtraction using a particular representation and talking about what should be highlighted for PSTs), providing space for discussion of questions that MTEs may have about the lessons, rehearsing sections of the lesson, and discussing annotated videos. Each of these structures was designed to support novice MTEs’ content knowledge and development of skill with teacher education practices and pedagogies of practice. The primary (noted by P) and secondary (noted by S) uses of each of the structures in shown in the table below. Descriptions of two key structures that are specifically designed to support new MTEs’ understanding of and ability to engage in pedagogies of practice and teacher education practices follow.

Table 1: Support structures and purposes for MTE learning (primary, P; secondary, S)

	Modeling	Rehearsal	Feedback	Specialized knowledge
Decompositions of practices	S		P	P
Lesson plans	S	S		P
Walk-throughs	P	P		P
Question sessions	S	S	S	P
MTE rehearsal	P	P		P
Annotated videos	S	S	P	P
Observation	P	P		P
Debrief	S	S		P

MTE rehearsals. MTE rehearsals are used in two different ways. To build skill in teacher education pedagogy of modeling, we have novice MTE rehearse the modeling that will take place in front of the PSTs. One experienced MTE runs the rehearsal while other MTEs participate in the role of PSTs. The experienced MTE pauses the rehearsal at strategic moments to support the novice MTE and the group as a whole in engaging in the metacognitive work of MTEs such as considering what should be highlighted (and how) for PSTs, at which moments meta-comments should be made for PSTs, and how best to represent the connection between the mathematical notation and the representations. This collective work provides support for MTEs in understanding the pedagogy as well as the pedagogical content knowledge and knowledge of content and students required to engage in the pedagogy with particular content.

We also rehearse running a rehearsal with PSTs. It is structured slightly differently, but provides many of the same supports. In rehearsal of rehearsal, an experienced MTE serves as the PST who is engaging in the practice of explaining core content, the novice MTE serves in the role of the MTE who is running the rehearsal with the PST, and another experienced MTE runs the rehearsal, giving feedback to the new MTE on their choices for pausing the rehearsal, the feedback they give to the PST, etc. In this case, the MTE serving in the role of the PST designs their performance to highlight common ways that PSTs approach explaining the content, including challenges with coordinating between representations and common language issues. This design provides opportunities for MTEs to develop their own KCS at the same time as developing their skill with the pedagogy of rehearsal.

Annotated videos. The work of providing feedback to PSTs involves not the ability to identify key parts of the practice, but an ability to align feedback with both the decomposition of the practice that is being used with the PSTs and with the PSTs' progression of development expected at the time of the feedback. One way to support novice MTEs with this work is through the use of annotated video. Videos are annotated by experienced MTEs to provide feedback to PSTs. Initially, novice MTEs watch the videos with annotations in an attempt to notice and justify on what and how the MTE provided feedback to the PST. This initial experience supports new MTEs in developing a sense of how to align feedback with the decomposition and how to choose what to give feedback on. Later MTEs watch videos without annotation then annotate the videos themselves as if they were giving feedback to the PSTs. Their annotation is then compared with the experienced MTE's annotation. Novice MTEs are provided with opportunities to discuss decisions that experienced MTEs made when providing feedback.

Discussion

The knowledge and skills of teacher educators are crucial for the success of teacher education. Our paper focuses on one particular element of content taught in an elementary mathematics methods course and unpacks the knowledge and skill that MTE need to support the learning of PSTs. We

describe several structures that we have found useful when supporting novice MTEs' skills and knowledge development. Importantly, our analysis shows that being an experienced K-12 classroom teacher is by itself insufficient for teaching practice-based teacher education courses. As a field, we must plan deliberately for the development of novice MTEs to realize the goals of practice-based teacher education. Further, although we focus on novice MTEs in this paper, because of the context of our work, we believe that experienced MTEs who have not taught practice-focused courses need to develop additional skills to enact practice-based teacher education. As a field, we must put increased focus on the preparation of teacher educators and this paper offers an analysis of one teacher education program's attempt to do so.

Endnotes

¹In our view, there is an important distinction between teacher education practices (such as eliciting PST thinking, assessing PSTs' work, providing feedback) and pedagogies of practice (such as rehearsal) that are activity structures in the context of teacher education. We take up this distinction in other work.

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