CRITICAL MATHEMATICS EDUCATION: EXTENDING THE BORDERS OF MATHEMATICS TEACHER EDUCATION

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This study describes efforts at two institutions to integrate critical pedagogy within the context of two mathematics content and pedagogy courses for K-8 pre-service teachers (PSTs). The purpose of the curriculum within these courses was to focus PSTs' attention on how issues pertaining to social justice may be taught within mathematics contexts. The desired goals were for PSTs to: 1) come to appreciate individuals within their own communities as valid practitioners of mathematics; and 2) come to understand the responsibility that they, their school districts, curriculum developers, and others bear to ensure that all students have equitable opportunities to learn mathematics.

Keywords: Equity and Diversity, Teacher Education-Preservice, Curriculum

Critical Mathematics Education: Transforming Ideology

Mathematics is often taught, and learned, within the confines of a classroom where, in the classical model of mathematics education, the teacher holds the power to teach, and the student receives the information being taught. Mathematics is mostly perceived and presented as an elite body of knowledge far removed from the "lives and ways of living of the social majorities in the world" (Fasheh, 2000, p. 5). Yet, mathematics is woven within all aspects of our world and should be taught, and engaged with, through the lens of a more global perspective. There is a need to institute change in teacher education to provide a counter-narrative to traditionally-held perceptions of mathematics and its pedagogy. Mathematics education instruction needs to be constructed so that individuals may develop both cognitive and social understandings of their world (Wood, 2001). According to Bigelow and Peterson (2002):

> [we need to hold in] our minds and in our classrooms the big global picture. The world is a web of relationships. To be truly effective, every effort to make a difference needs to be grounded in that broader analysis. (p. 7)

Skovsmose (1985) states that when implementing critical education pedagogy, two criteria-subjective and objective, need to be used when selecting problems for the classroom. The subjective requires that the problem appears relevant to the students and within their conceptual understanding. The objective requires using data and detail to view an existing social issue unbiasedly to build deeper understanding. The integration of mathematics and social justice provides the potential to have meaningful conversations about issues impacting local communities, as well as the world, and prepares individuals for citizenship.

This research study inquires how we can teach mathematics that helps learners attain the content proficiencies, but beyond that, equips them with the empathy, consideration, and skills necessary to appreciate all individuals that comprise their community and to see these people as capable of learning and doing mathematics. We seek to merge the research domains of mathematics and critical mathematics education to design courses that question and extend the borders of academic mathematics to include mathematics for social justice. In this paper, we describe curricular efforts specific to two teacher education courses that address goals for PSTs to: 1) come to appreciate individuals within their own communities as valid practitioners of mathematics; and 2) come to understand the responsibility that they, their school districts, and others bear to ensure that all students have equitable opportunities to learn mathematics.

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Methodology

Setting and Participants

Two mathematics teacher education courses were taught by the authors at two distinct four- year institutions, one in the midwest and the one in the southeast U.S. PSTs pursuing licensure to teach K-8 enrolled in these courses. The first class is an elective class for elementary PSTs that focuses on issues of social justice in a mathematics education context. This class focused on assignments that required PSTs to explore social justice problems involving mathematics in a global context. The second class was a required mathematics content class for elementary PSTs. The second class included components focused on projects aimed at developing PSTs' awareness of individuals within their communities doing mathematics in local contexts. The key course goals, course activities, and outlines of the course structures were designed collaboratively.

Research Methods

Quantitative. There were two aspects to the methodology of the study. First, to gain insight into PSTs' views concerning multiculturalism and diversity, we administered the scale, *Teachers Sense of Responsibility for Multiculturalism and Diversity (TSR-MD)* (Silverman, 2009) to both classes as a pre-post test to determine if the different approaches to class activities led to changes in the PSTs' scores on this instrument. The instrument requires students to respond to a set of Likert scale questions. The Wilcoxon signed-rank test was conducted on each of the questions in the scale. The significant results with their effect sizes are discussed below.

Course 1 (n = 23, (Spring and Fall 2015) was the elective course aimed at social justice in the context of mathematics education. The possible responses for Questions 1 – 74 were:
1 = Strongly disagree, 2 = Disagree, 3 = Moderately disagree, 4 = Moderately agree, 5 = Agree, 6 = Strong agree. Initially, some students seemed to interpret Question 1, *In general, race is unimportant to me*, as "it would be best not to see race in teaching". On the post-test, students interpreted this question differently, beginning to develop a novice understanding of why a teacher should see and accommodate racial diversity. A Wilcoxon signed-rank test determined that there was a statistically significant decline in the number of PSTs who responded positively to this statement on the post-test (Mdn = 3.0) compared to the pre-test (Mdn = 5.0), z = -2.06, p = .039, r = .43. For Question 8, *In general, schools are responsible for addressing the differences among races*, there was a significant increase in the number of students on the post-test (Mdn = 4.0, s = 1.3) who saw schools as being responsible for addressing difference between the races, as compared to the pre-test (Mdn = 4.0, s = .96), z = 2.03, p = .042, r = .42. Regarding Question 10, *My students' economic class plays a role in my teaching*, there was a significant change in how PSTs perceived how students' economic class should affect their teaching as evidence in the post-test (Mdn = 2.0) compared to pre-test (Mdn = 4.0). There appeared to be a shift toward thinking that economic class should not impact teaching and learning, which seemed somewhat paradoxical given the class focus. Test results were significant, z = -2.09, p = .037, r = .43. On Question 11, *It is my responsibility to ensure various economic classes are represented in my teaching*, the post-test (Mdn = 5.0, s = .1) compared to the pre-test (Mdn = 5.0, s = .89) showed more positive than negative differences indicating that PSTs were assuming more personal responsibility for ensuring that various economic classes were represented in their teaching, z = 2.07, p = .039, r = .43. Regarding Question 20, *Gender is relevant to learning*, post-test results (Mdn = 5.0, s = 1.3) compared to pre-test results (Mdn = 5.0, s = 1.5), yielded significance, z = 2.41, p = .016, r = .5, pointing to an increase in PSTs viewing gender as being important for teaching and learning. For Question 21, *Curriculum developers are responsible for ensuring various genders are represented in the content curricula*, the post-test results (Mdn = 5.5, s = .87) in relation to the pre-test (Mdn = 5.0, s = 1.11) showed significant changes in scores demonstrating that PSTs placed more responsibility on curriculum developers to represent genders in the curriculum (z = 2.14, p = .032, r

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= .45). Question 27 asked PSTs to respond to: *It is my responsibility to ensure various faiths are represented in my teaching*. The Wilcoxon signed-rank test indicated a significant increase in PSTs assuming responsibility for representing different faiths in their teaching (post-test, *Mdn* = 6.0, *s* = 1.07; pre-test, *Mdn* = 5.0, *s* = 1.3) with $z = 2.40$, $p = .017$, $r = .5$. For Question 29, *Curriculum developer are responsible for ensuring various faiths are represented in the content curricula*, post-test (*Mdn* = 5.0, *s* = 1.3) compared to pre-test (*Mdn* = 4.0, *s* = 1.4) indicated that PSTs came to place more responsibility on curriculum developers to address diversity of faiths in the curriculum ($z = 1.99$, $p = .047$, $r = .42$).

In the second course, a required mathematics content class for elementary PSTs (n = 14, Spring 2015), we also observed some significant results on the TSR-MD (Silverman, 2009). However, all of these results should be approached with great caution, given the small sample size. For Question 7, *Various races need to be represented in teaching only if students from those races are present in the classroom*, responses on the post-test (*Mdn* = 4.0, *s* = 1.7) compared to pre-test (*Mdn* = 2.0, *s* =.1) were interesting as all of the changes (*n* = 8) were positive, with no negative changes and 6 ties in rank. PSTs seemed to register a belief that different races should be portrayed on the basis of who was in the class, which was counter to the goals of the class. Test results were significant, $z = 2.54$, $p = .011$, $r = .68$. Regarding Question 49, *In general, culture is unimportant to me*, test results were significant with $z = 2.5$, $p = .01$, $r = .67$. The post-test results (*Mdn* =2.0, *s* = 2.0) compared to the ranks of the pre-test (*Mdn* = 1.0, *s* =.61) indicated a change in how PSTs regarded culture in teaching. For Question 50, *My students' culture plays a role in my teaching*, post-test ranks (*Mdn* = 5.0, *s* =.96) compared to pre-test ranks (*Mdn* = 6.0, *s* = .5) showed a slight increase in the PSTs' understanding of their responsibility in including culture in their teaching (0 positive differences, 7 negative differences, and 6 ties) with a significant test result ($z = -2.5$, $p = 0.14$, $r = -.67$). For Question 53, *Curriculum developers are responsible for ensuring that various cultures are represented in the content curricula*, the post-test ranks (*Mdn* = 5.0, *s* = .83) compared to the pre-test (*Mdn* = 5.5, *s* = .65) indicated a slight increase in PSTs' beliefs that curriculum developers are responsible for including various cultures in the curriculum ($z = -2.33$, $p = .02$, $r = -.63$). (For Questions 75-89, the Likert responses were: 1 = Nothing, 2 = Very little, 3 = Some, 4 = Quite a bit, 5 = A great deal). On Question 75, *How much can you do to ensure diverse students learn about their own cultural heritage in your class?* the post-test ranks (*Mdn* = 4.0, *s* = .61) versus the pre-tests (*Mdn* = 4.0, *s* = .62) showed five positive differences, zero negative differences, and nine ties. The significance test was significant with $z = 2.23$, $p = .03$, and $r = .60$. Finally, for Question 76, *How much can you do to teach your students about cultural conflic*t? The post-test results (*Mdn* = 5.0, *s* = .76) compared to the pre-test results (*Mdn* = 4.0, *s* = .83) showed six positive differences, zero negative differences and eight ties, indicating a modest increase in PSTs assumption of responsibility for teaching students about cultural conflict. The test was significant, $z = 2.33$, $p = .02$, and $r = .63$.

The differences in the number and type of significant results for the two courses can be accounted for, in part, by the differences in the foci of the two courses and the emphases of the assignments within the courses, as explained below. However, we also have concerns that the wording of the TSR-MD may have sometimes confused the PSTs. In order to explore some of the results that are confounding given the goals of the courses, follow-up interviews to clarify PSTs answers would be helpful.

Qualitative. We used narrative inquiry as a research method (Creswell, 2008) to organize, present, and analyze the qualitative data, which involved a detailed examination of the field texts (e.g., course activities, projects) produced by the PSTs. In choosing this method, we "adopt a particular view of experience as phenomenon under study" (Connelly & Clandinin, 2006, p. 375). Like other qualitative research methods, well defined boundaries cease to exist in narrative inquiry. In our narratives, we consciously choose and present data that will best help us address the central goal of this paper. The two teacher education courses shape the contextual setting and the students

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(PSTs) enrolled in these courses are an integral part of the study. We inquire into our practical and professional experiences grounded in this context, enact dual roles as teacher educators and researchers, and retell a story that unravels connections between theoretical ideologies and practical enactments. The story is structured around a chronology of events that include: a) course design and enactment; b) documentation of field texts; and c) our analyses and interpretation of field texts in relation to the chosen theoretical domain.

The course designs aimed to establish an equitable community of mathematics learners as they explored ways to better understand mathematics and the potential it holds to explore social justice issues. The key course goals were to help PSTs: a) begin to build a critical understanding of the world; b) enlist mathematical tools that support and promote critical understanding of social issues; and c) understand that a global perspective helps us to know ourselves, our community, and our fellow human beings. As members of a community of learners, PSTs were required to engage in class activities, listen to and learn from their colleagues, and respect the opinions of other participants. To establish specific connections to critical mathematics education, participants were asked to think more deeply about the types of tasks that are usually presented in mathematics curriculum; to attend to and explore the connections between a given mathematical task and the local and global contexts. Course content was delivered through practical and research components. As part of the practical component, each week, PSTs participated in at least one content exploration activity set in a social, cultural, and political context. As part of the research component, PSTs were required to identify and investigate a social practice and establish connections to school mathematical topics.

We highlight two course activities (field texts) collected from PSTs work from the two courses. The first activity, required participants to use the Gapminder software tool (www.gapminder.org) and thus help them probe their beliefs and perceptions of the developed and the developing nations of the world. This activity enabled participants to uncover, understand, and question their perceptions about various nations. In particular, they were asked to complete the following tasks:

1. Identify a research question.
2. Explore: a) Describe how the data was analyzed using the Gapminder software. b) Interpret the graphs and tell the story. c) Discuss plausible reasons that will explain the trends that were noted in the graph.
3. Reflect: In light of the presented data and its analysis, reflect on the research question. a) Suggest possible actions that could be initiated by various agencies to address the global issue. b) As a citizen of the world, how do the findings this concern you? What can you do about it? c) Discuss ways in which prospective teachers can engage students in a deeper analysis of global or local issues using the lens of statistics and through the use of the Gapminder software.

We provide one PST's response here.

> PST Research Question: What is it that causes some countries to have such a high child mortality rate, when others don't even have to think about the possibility of their children not making it? I decided to analyze three countries from different locations all over the world: the United States of America, Colombia, and Botswana; all three are on different continents with different ways of life, and overall very diverse economic levels. I was curious to find why with the modern medicine we have is there still a reason to have any child mortality at all.

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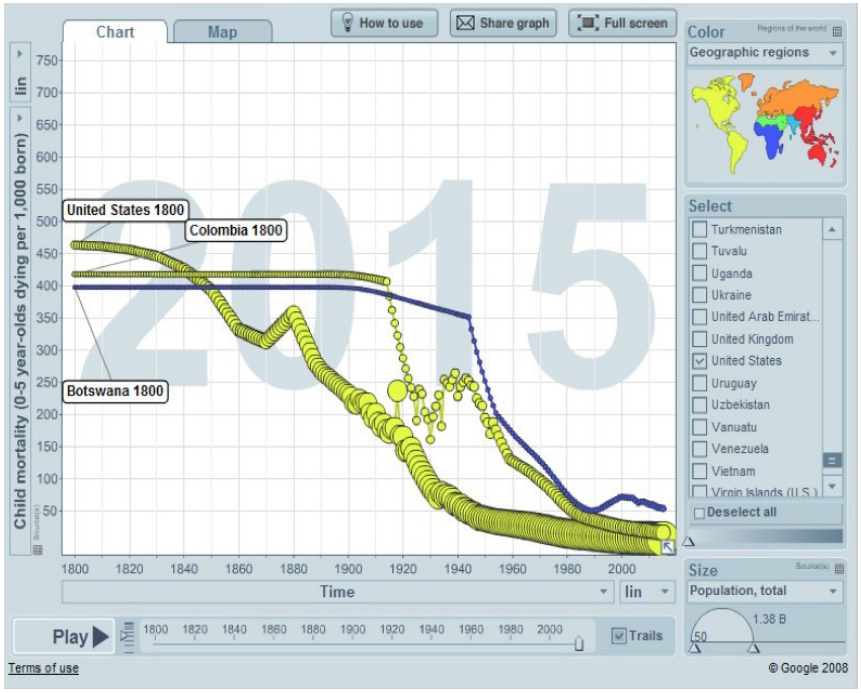

Figure 1. Gapminder Graph: Children Mortality -USA, Colombia and Botswana.

Interpretation: After looking at all three graphs and countries I've learned a lot about what child mortality means in relation to the world around it. It's so interconnected with the rest of the countries issues, no matter if it's what society thinks of those around them or major issues such as the government changing power or parties, this is always being affected, like a ripple effect. It was horrifying to see the starting numbers and think about what those parents had to go through if their child played a part in that statistic, but I did find it comforting to see that even the countries I thought would still be very bad have improved so very drastically. I did catch myself losing touch with what the statistic meant though, sometimes when I would write out the numbers it was like I was almost desensitized with what that number was saying. The gravity of children dying is intense and I think sometimes when we're shown so many numbers like that it's hard to remember what they are truly portraying to us.

Reflection: I think that national agencies could initiate programs educating not only the adults of these countries but the children on health and medicine and prevention. This would help ensure that when these children grow up they know even more about how to help their own children, because if no one learns how to prevent it, these health care issues or diseases will evolve till they can't be controlled at all. I also believe that any action we do or someone does to help these people, needs to be done with these people, they aren't less than us or weaker, they are equals to those helping them.

As a citizen of the world these findings concern me that not all countries have access to the medicine, healthcare, or even knowledge about these issues, and for some to have had them for so long and not reached out to help makes my stomach hurt. ... I understand that the whole situation is more complicated than that involving the governments but I think that anyone can relate to seeing children and the thought of them not making it till they are 5, not even starting kindergarten, is horrifying.

I believe that teachers could integrate things like Gapminder into the classroom in many ways. It's a great way for children and students to start seeing what these numbers actually are and how

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they compare to what we have as Americans. It could definitely show them what they take for granted everyday but I also believe in order for them to truly dive deeper into what this mean and talk about the statistics in a local, national, and global level, they need to fully understand what each means and that it isn't just a number, we can't let them desensitize themselves to what they are seeing or they will never understand the weight that these numbers hold.

Through the Gapminder exploration, Skovsmose' criteria of subjective and objective problem posing related to critical mathematics pedagogy are engaged. Students were able to define a problem, that they deemed important to society, and then utilize unbiased information provided through Gapminder to identify issues that influence the overall trends depicted in data.

An activity from the second course required participants to select a topic and complete a project emphasizing that mathematics is a human activity. PSTs identified a personally meaningful everyday activity, collaborated with individuals who were insiders to this activity, and reflected on questions such as: What counts as knowledge in school mathematics? Who are viewed as experts of knowledge? Whose mathematical expertise is valued? Project examples of some projects include firefighter's mathematics, lunchroom mathematics, mathematics of curve stitching, mathematics of a seamstress, and mathematics of a construction worker.

We provide a brief overview of one PST's research project titled *My mother, a lunch lady and our mathematics*. Sam, a prospective middle school teacher, chose her mother, Pam, as her inspiration and resource. Pam has worked in the school lunch industry for seven years and has progressed from an assistant cook, to head cook at a private school, and currently is the assistant cook at a public elementary school. Her role is to help develop lunches on a monthly basis, help prepare the daily lunches, and help serve the food as the students arrive. In collaboration with her mother, Sam developed a mathematical modeling task situated in the school lunchroom context.

Ms. Pam is a head cook in a school cafeteria. She must follow a set of guidelines to ensure that students receive the right amount of each food group per day. There is also a regulation that lunches must be within a certain calorie range.

4. Create a weekly lunch plan that Ms. Pam would approve.
5. On a given day, how much of each food type would Ms. Pam have to prepare?

Sam enacted this activity with a group of 5th graders. We present 5th graders' work specific to question (1) and Sam's commentary. After beginning work on the assigned task, students realized that they needed additional information regarding nutritional and serving size regulations, which Sam provided. Figure 2 captures students' initial responses to the two questions (first model).

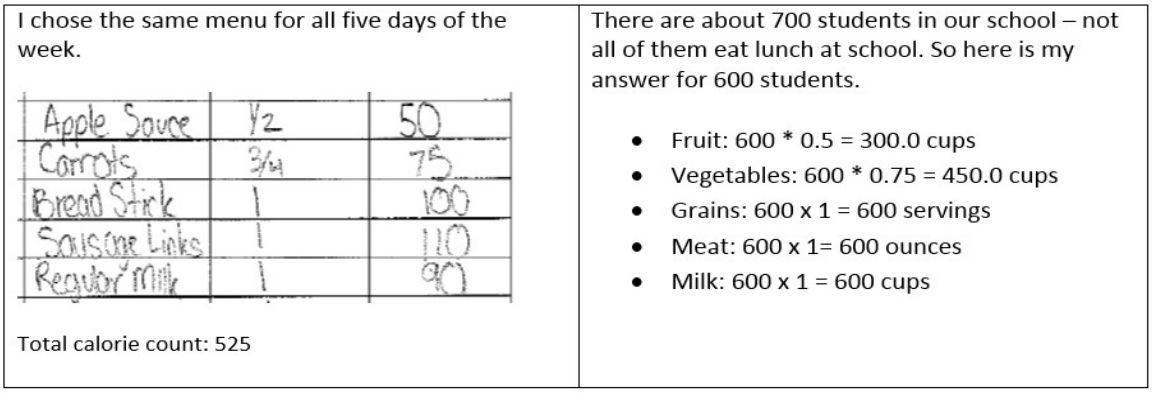

Figure 2. Lunch Menu - Trial One.

Upon a review of this work, Pam noted that this simplistic plan neither fulfilled the nutritional requirements nor the recommended serving size regulations. She challenged Sam and the students to experiment with different menu options, interpret their solutions in the context of the problem, and to be explicit in communicating their thinking. Students created another model, this time using five different menu options, one for each day of the week (see Fig. 3).

Food type	Fruit			Veg			Grains			Meat			Fluid			Total Calorie count
	Name	*SS*	*CC*	*Name*	*SS*	*CC*	*Name*	*SS*	*CC*	*Name*	*SS*	*CC*	*Name*	*SS*	*CC*	
Mon	Apple sauce	1/2	50	Carrot	3/4	75	Bread	1	100	Sausage	1	110	Reg. Milk	1	90	**425**
Tue	Banana	½	75	Peas	3/4	60	Roll	1	65	Ham burger	1	185	Choc. Milk	1	90	**570**
Wed	Grapes	½	60	Broccoli	3/4	80	Bun	1	150	Grilled chicken	1	120	Str. milk	1	90	**500**
Thu	Orange	½	45	Green beans	3/4	65	Hotdog bun	1	125	Breaded chicken	1	160	Reg. milk	1	90	**485**
Fri	Apple juice	½	45	Green beans	3/4	65	Bread	1	70	Ham burger	1	185	Choc. milk	1	90	**455**
Weekly serving		2 1/2	Weekly serving		3 3/4	Weekly serving		5	Weekly serving		5	Weekly serving		5		

Figure 3. Lunch Menu - Trial Two.

However, as they revisited the task guidelines, students realized that Ms. Pam would not approve their lunch menu. They realized that they still had not attended to the nutritional requirements and the serving size regulations. In the students' next iteration, they made three changes: a) Make changes to the grain / meat item so as to bring the serving size up; b) Make no changes to the milk/ veg. items since they conform to the serving size regulation (if possible, add fruit); and c) All changes must increase the calorie count but not exceed 700. Students realized that a revised model with these changes would certainly get Ms. Pam's approval. At this point in her project work, Sam began to notice how her mother emerged as a prominent figure in this discussion. Sam began questioning her initial beliefs about who can be considered as sources of mathematical knowledge. She began to understand the meaning of learning as, "a truthful collaboration in which all parties come both as learners and as resource" (2, 1998). Sam noted, "Through my interactions with my mother, I learned the lunchroom is full of hidden mathematical concepts. She helped me realize the potential for using them to design math activities." Sam also began to realize that while her mother did do meaningful mathematics, and appeared to be very confident about her knowledge of her practice, "she seemed to dismiss this knowledge as if there was nothing to it." Sam began to see that this dismissal was not warranted. Sam could instead see that her mother, a working class lady, was able to build on her lived experiences, to create a mathematical task that challenged the students. Sam explained, "Upon reflection, I see that I have been oblivious to my own mother's expertise because I truly did not believe that she is a source of knowledge. I have grown confident in teaching modeling to children as I no longer believe that these are abstract concepts for only a select group of students."

Implications for Mathematics Teacher Educators

Both the quantitative and qualitative analyses indicate preliminary results that suggest that through a deliberate scaffolding of course activities and projects, mathematics educators can help PSTs learn to appreciate individuals within their own local communities as practitioners of real mathematics and can leverage the expertise of these individuals in their own classrooms. This may

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build confidence within diverse groups of students that they, too, can engage in meaningful mathematics and can build self-confidence within the community about what they have to offer students, mathematically speaking. The results also speak to the power of using sources of real-world data, such as Gapminder, to help PSTs learn to use mathematical tools to explore problems of a global nature and make data-driven recommendations to help solve them. The results suggest that there are concrete ways that we can help PSTs learn to use mathematics to act locally and globally and become willing to assume responsibility for doing so.

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