TEACHERS' INSTRUCTIONAL PRACTICES WITHIN A CONNECTED CLASSROOM TECHNOLOGY ENVIRONMENT TO SUPPORT REPRESENTATIONAL FLUENCY

Yasemin Gunpinar
St. Catherine University
ygunpinar@stkate.edu

Stephen J. Pape
Johns Hopkins University
stephen.pape@jhu.edu

The purpose of this study was to examine the ways that teachers use Connected Classroom Technology (CCT) to potentially support achievement on translation problems that require moving between algebraic representations. Four mathematics classrooms were chosen based on their gain scores on pre- and post-test Algebraic translation problems. Two classrooms with the highest and the lowest gain scores were chosen among the classrooms with pre-test scores that were below 50%. This study used video-recorded observational data and found that teachers in effective classrooms created environments wherein students used multiple representations simultaneously and translated between representations through discussion. In contrast, teachers in less effective classrooms fostered environments wherein students used representations independently and missed opportunities to translate representations through discussion.

Keywords: Instructional Activities and Practices, Technology

Representational fluency is a cognitive competence that includes being able to interpret and construct representations as well as translate flexibly between them (Sandoval, Bell, Coleman, Enyedy, & Suthers, 2000). It is considered both a mechanism for supporting the development of deep conceptual understanding (Duncan, 2010; Pape & Tchoshanov, 2001) and a means of assessing conceptual understanding (Suh, Johnston, Jamieson, & Mills, 2008).

Representational fluency not only supports conceptual understanding but is also an essential component for problem solving (Nistal, Van Dooren, Clarebout, Elen, & Verschaffel, 2009). Students are more successful when they possess the ability to translate between representations as well as use multiple and non-symbolic representations (Brenner et al., 1997; Nathan & Kim, 2007). The education community continues to emphasize the importance and need for developing representational fluency (National Governors Association Center for Best Practices, Council of Chief State School Officers [CCSSO], 2010). However, students leave school without attaining representational fluency (Ainsworth, Bibby, & Wood, 2002; Herman, 2007; Knuth, 2000).

There are at least two factors that potentially support the development of representational fluency: communication and technology. Through communication, representational fluency may be supported by active engagement in discussions about interpretation, construction, evaluation, comparison, and generalization of representations (Warner, Schorr, & Davis, 2009). The development of representational fluency may be supported by allowing for quick access to multiple representations (e.g., symbolic, tabular, and graphical) through technology (Bieda & Nathan, 2009). Because evidence suggests that communication and technology may separately support students' developing representational fluency, the present study investigated instruction that is characterized by the use of Connected Classroom Technology (CCT) with the aim of examining the relationship between these instructional strategies and increasing such fluency.

Connected Classroom Technology

CCT are wireless communication systems that connect the teacher's computer and students' handheld technology (Pape et al., 2013, p. 169). These systems are designed to provide greater opportunities to discuss connections among multiple representations. Recent studies emphasize a sociocultural perspective that focuses on the relationship between learning opportunities and students' abilities to take advantage of these opportunities during learning (Gee, 2008). CCT

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provides at least two learning opportunities that support the development of students' representational fluency. These opportunities include "the mobility of multiple representations of mathematical objects" and "the ability to flexibly collect, manipulate and display to the whole classroom representationally-rich student constructions, and to broadcast mathematical objects to the class" (Hegedus & Moreno-Armella, 2009, p. 403). Jim Kaput once postulated, "wireless connectivity 'inside' the classroom would change the communicative heart of the mathematics classroom" (Hegedus & Penuel, 2008, p. 171). CCT's progression has potential to enact this transformation.

The Texas Instruments (TI) Navigator system includes two components to support representational fluency: Screen Capture and Activity Center. With Screen Capture, teachers can project a "snapshot" of students' calculators. This feature allows both teachers and students to compare the representations through productive discussion. With Activity Center, the teacher can project a coordinate plane on which students submit points, equations, and graphs. Another affordance is that it promotes examination and analysis of patterns as well as justification of mathematical generalizations (Hegedus & Moreno-Armella, 2009).

These components provide a context for effective classroom discourse because they are designed to publicly display multiple linked representations. The public display of students' mathematical constructions in conjunction with the communication of ideas and strategies fosters representational expressivity (Hegedus & Moreno-Armella, 2009). Also, the activities in the activity center with multiple representations may support translation between representations (Bostic & Pape, 2010), which are distinguishing characteristics of mathematical proficiency (CCSSO, 2010; Kilpatrick, Swafford, & Findell, 2001). Although teachers have found CCT to be an efficient means of instruction, there is little evidence that demonstrates or evaluates its effectiveness in relation to supporting representational fluency (Vahey, Tatar, & Roschelle, 2007).

Method

Teachers who participated in classroom observations during the third year of a four-year project were chosen for the present study. Forty of the 41 teachers' classroom observations served as the initial pool of observations with one being eliminated due to poor audio quality. Data sources consisted of classroom videos (typically two class periods), their verbatim transcripts, and algebra pre- and post-test. The length of each observation was between 48 and 97 minutes. To measure representational fluency, translation problems were extracted from the pre- and post-tests. Translation problems are those in which the initial representation (i.e., input) and the answer's representation (i.e., output) are different (Nathan, Stephens, Masarik, Alibali, & Koedinger, 2002).

There were two criteria for participant selection. First, classroom observations that focused on quadratic equations were considered. Second, classrooms with initial mean pre-test scores below 50% were considered. Among the classroom observations, the two teacher's classrooms with the highest gain (i.e., Ms. BW and Ms. MB) and the two teachers' classrooms with the lowest gain scores (i.e., Ms. MA and Ms. JR) were selected. Gain scores were calculated as the percentage of the maximum possible change (i.e., (Post – Pre)/ (Maximum Score (100) – Pre)). The cases were viewed without knowledge of effectiveness. Selected teachers' demographic information is listed in Table 1.

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Table 1: Teacher Participants Demographic Data

Classroom	Teachers	Year of CCT use	Undergraduate Major	Graduate Major	Years Teaching Experience
Effective	Ms. BW	2	Pre-Vet Med	Ph.D., Animal Feeding/Animal	13
	Ms. MB	3	Communication	MA, Journalism	3
Less Effective	Ms. MA	2	Mathematics	MA, Educational Computing	21
	Ms. JR	2	Mathematics	Curriculum and Instruction	20

This research employed a qualitative study design and analysis (Hatch, 2002) with the intent of providing contrasting or illustrative instances in instructional use of representations. To increase the credibility and validity of the conclusions, *usefulness, the chain of evidence, truthfulness,* and *reporting style* strategies were applied (Gall, Gall, & Borg, 2007).

Results

The teachers' practices in terms of how they supported students to engage with representations were explored in this study. Following the initial coding, the classes' effectiveness status was identified, and cross-case analysis of the two categories of cases were compared. Two major themes that potentially support representational fluency were identified. In each subsection, a theme and how it was practiced in both effective and less effective classrooms is described.

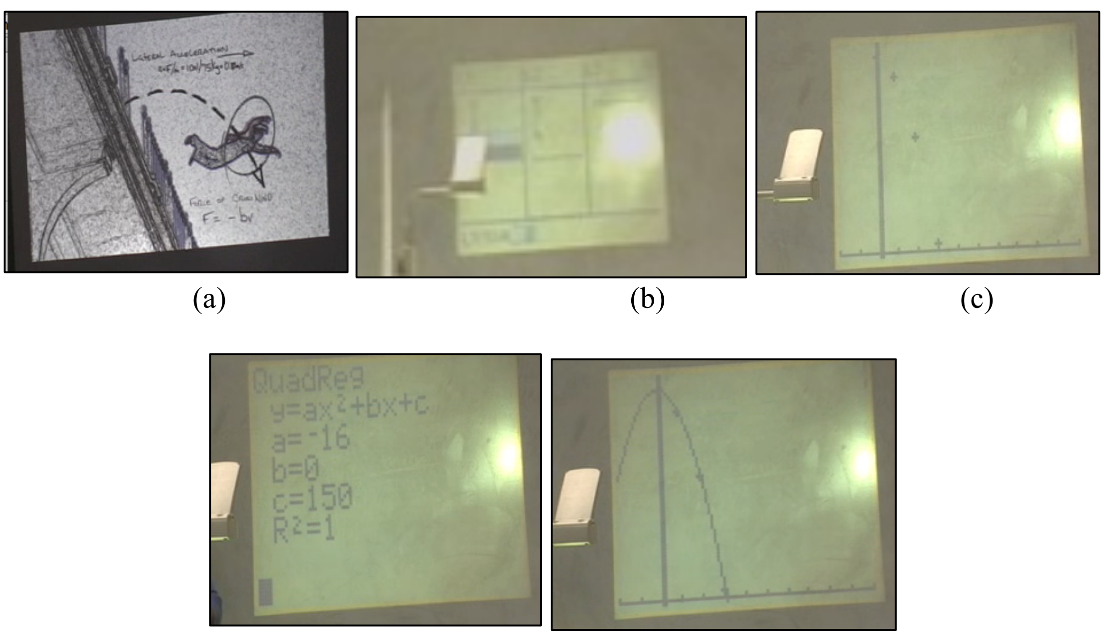
Using Different Translations

Unidirectional and bidirectional translations were observed during these classroom observations. Unidirectional refers to translating between different representations within the same activity (e.g., Symbolic1 → Graphical → Symbolic2) and bidirectional refers to translating between the same representations within the same activity (e.g., Symbolic1 → Graphical → Symbolic1). The students in all four classrooms frequently used different unidirectional translations. In effective classrooms, however, unidirectional translation was observed only once in Ms. MB's classrooms. All unidirectional translation sequences and the longest sequence of translations were observed in Ms. BW's classroom, an effective classroom. Many unidirectional translations were observed in Ms. MA's classroom video recordings, a less effective classroom. Although Ms. JR did not include translations in her first class period, the longest unidirectional translation among less effective classrooms was observed in her classroom. Although many unidirectional translations were observed in less effective classrooms, the students did not generally translate between representations; instead they observed their teacher's translations. Translations used in the unidirectional category are summarized in the Table 2.

Table 2: Unidirectional Translations

Classroom	# of Translations	Translation Sequence	Teacher Practiced
Effective	One	Graphical → Symbolic	Ms. MB
	Two	Symbolic1 → Graphical → Symbolic2	Ms. BW
		Tabular → Symbolic → Graphical	Ms. BW
	Three	Symbolic1 → Graphical → Symbolic2 → Verbal	Ms. BW
	Four	Symbolic1 → Graphical → Tabular → Symbolic2 → Graphical	Ms. BW
Less Effective	Two	Symbolic1 → Graphical → Symbolic2	Ms. MA
		Symbolic → Tabular → Graphical	Ms. MA
		Pictorial → Tabular → Graphical	Ms. MA
		Tabular → Symbolic → Graphical	Ms. MA
	Three	Symbolic1 → Graphical → Symbolic2 → Graphical	Ms. MA
	Four	Symbolic1 → Graphical → Tabular → Symbolic2 → Graphical	Ms. JR

In addition to unidirectional translations, one of the main features that differentiated the effective and less effective classrooms was the presence of bidirectional translations, which were observed only in the effective classrooms. Using four translations including bidirectional translation in Ms. BW's class might have improved her students' translation abilities because it includes many representations and translations (Figure 1).



(a) (b) (c)

(d) (e)

Figure 1. Screenshots from the students' work on the detective problem. This figure illustrates the use of (a) pictorial, (b) tabular, (c) graphical, (d) symbolic, (e) graphical representations, respectively.

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Table 3 displays the sequences of translations in each of the effective classrooms. Although, there was only one bidirectional translation in each of the effective classes, the activities in which these translations were observed occupied a substantial amount of class time.

Table 3: Bidirectional Translations

Classroom	# of Translations	Translation Sequence	Teacher Practiced
Effective	Two	Symbolic1 → Graphical → Symbolic1	Ms. MB
	Four	Pictorial → Tabular → Graphical1 → Symbolic → Graphical1	Ms. BW
Less Effective	None		

In this study cycling translations were observed where students iteratively translated between representations bidirectionally until they reached the correct solution. Both teachers in the effective classrooms provided tasks that allowed their students to view multiple representations simultaneously (Figure 2).

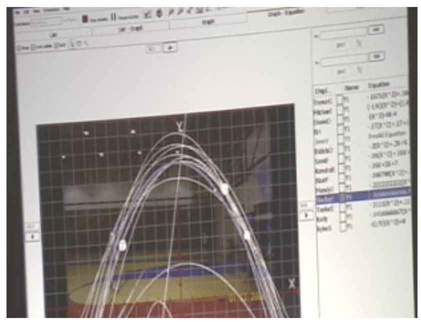

Figure 2. Using dynamic representations simultaneously. This figure demonstrates the cycling translation between graphical and symbolic representations.

These students saw that each modification they made to the coefficients simultaneously changed their graph. They could see the coordinates of a point on the parabola while changing the point's location. Finally, cycling translations were observed twice in effective classrooms: (a) two translations with one cycling, and (b) three translations with one cycling. On the other hand, in the less effective classrooms, a cycling translation (i.e., Symbolic → Graphical ↔ Symbolic) was observed only once in one of Ms. JR's class (Table 4).

Table 4: Cycling Translations

Classroom	# of Translations	Translation Sequence	Teacher Practiced
Effective	At least two	Symbolic → Graphical ↔ Symbolic	Ms. BW
	At least three	Pictorial → Symbolic → Graphical ↔ Symbolic	Ms. MB and Ms. BW
Less Effective	At least two	Symbolic → Graphical ↔ Symbolic	Ms. JR

These translations were not, however, used within a real-world context, and the students did not interpret the representations. Ms. JR also gave many hints during the cycling process, which potentially limited the students' independent thinking. An additional difference was the fact that representations were not generally dynamically linked within the less effective classrooms. The students in these classrooms typically explored representations independent of one another.

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Students perform better when they use more and multiple representations (Bostic & Pape, 2010; Herman, 2007; Nathan & Kim, 2007), when they have the ability to translate between representations (Brenner et al., 1997), and when they use non-symbolic representations (Suh & Moyer, 2007). Although students in both the less effective and effective classrooms used multiple representations, only the students in the effective classrooms generally used bidirectional and cycling translations. Since the new version of handheld calculators enables bidirectional translations (Özgün-Koca & Edwards, 2009) and repairing representations is a practice that supports learning (Warner et al., 2009), teachers can create environments where students have the opportunity to modify their representations until they arrive at the most accurate representation of their thinking. The students might be more successful if they can use bi-directional and cycling translations in conjunction with multiple unidirectional translations.

Scaffolding Translation through Teachers' Questioning

The teachers in the effective classrooms, Ms. MB and Ms. BW, asked questions to promote students' translation between representations. Ms. BW usually asked questions requiring short answers. When Ms. MB realized that her students seemed lost or confused or if she needed a student to clarify an answer, she would ask follow-up, open-ended, or hypothetical questions. Ms. MB invited all students to participate. She also encouraged them to share their solutions and opinions while solving the problems.

The teachers in the less effective classrooms, Ms. MA and Ms. JR, missed many opportunities to create discussion-rich environments. Also, by providing hints or asking questions that led to obvious generalizations, both teachers did not adequately challenge their students during problem-solving activities. Thus, the questioning techniques they used did not require the in-depth thinking that would encourage students to make translations between representations. In addition, Ms. MA and Ms. JR did not sufficiently interact with their students when they made mistakes. Instead, they quickly provided explanations. When discussing alternative ways of solving a problem, Ms. MA often started using her method without allowing her students adequate time to think for themselves.

Significance of the Study

This study provides a description of four teachers' practices and provides models for how teachers might construct their classroom to better promote their students' representational fluency abilities. They may offer examples to mathematics teacher educators when they prepare pre-service teachers or provide professional development for in-service teachers.

Teachers should be aware of their students' representational knowledge and seek technological or cognitive tools to visualize their students' thinking. Through the use of CCT such as the TI-Navigator, instructors can monitor and assess their students formatively to adapt their instruction based on their students' needs and misconceptions. Ultimately, teachers should create environments for students to interpret representations by linking them to real-world scenarios. Students should not only be able to see multiple representations on one screen but should also see the changes to a representation as its related representation (i.e., algebraic) is modified. Moreover, students should be provided more opportunities to make judgments about the accuracy of their representations and to change them as appropriate during problem-solving activities, which can be accomplished by including activities that require bi-directional and cycling translations. In addition, teachers should frame questions that facilitate students' developing understanding of representations and translations over time. One way to promote such sustained thinking is to foster discussion-rich environments.

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