FROM SPEAKING TO WRITING: THE ROLE OF THE REVERSAL POETIC STRUCTURE IN PROBLEM-SOLVING

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Speakers in conversation typically repeat and modify earlier comments. In mathematics conversations, these repetitions, or poetic structures, can facilitate the collaborative discovery of mathematical relationships. A close analysis of 90 turns of an algebraic problem-solving conversation reveals eight types of poetic structures. This report summarizes the general results of the analysis and highlights the role of a particular type of poetic structure, the reversal. In a reversal poetic structure, two elements switch places syntactically, for example, subject and predicate or adjective and noun switch roles. Students working with no teacher intervention used the reversal poetic structure as they rearranged variables and coefficients in their verbal method, and as they transitioned from a spoken method to written mathematical notation. This analysis highlights the ways in which speaking supports mathematical thinking.

Keywords: Classroom Discourse, Problem Solving, Algebra and Algebraic Thinking

Introduction

Speakers of all languages repeat each other. Because repetition is pervasive in daily speech (Du Bois, 2014), it can contribute to mathematical problem-solving conversations. This paper highlights ways in which particular types of repetition—poetic structures—facilitate students' mathematical learning. Poetic structures occur when speakers repeat the grammatical structures of phrases spoken before, perhaps changing words or small aspects of grammar. This paper reports on the role of a particular poetic structure, the reversal, in an algebraic problem-solving session. In this mathematical conversation, the reversal poetic structure was associated closely with the transition from verbal problem-solving to written mathematical notation.

This study contributes to research on language as a resource for mathematical learning, which developed from studies of multilingual classrooms (e.g. Barwell, 2015; Planas & Setati-Phakeng, 2014). Language-as-resource research is concerned with issues such as code-switching, the influences of educational policy on classroom communicative practice, and language as a resource in formal vs. informal mathematics discourse. Collaborative learning pedagogies rely fundamentally on students' linguistic interaction. It seems intuitively correct that small changes to previous statements can account for collective mathematical constructions. Poetic structure analysis grounds this intuition in observable conversational actions. Because repetition is so commonplace, its analysis can deepen our understanding of the ways in which language facilitates learning in multilingual classrooms, or in monolingual classrooms in any language.

Theoretical Foundation

Dialogic syntax, an emerging research focus in linguistics, forms the theoretical foundation for this analysis (Du Bois, 2014; Sakita, 2006). Dialogic syntax recognizes that as speakers repeat prior statements—their own or those of others—they reproduce syntactic arrangements that create meaningful relationships across sentences and across speakers. Hearers decode and respond to the meanings that are created at these structural levels beyond the sentence. For example, in the hexagon task described in this paper, Sheila's *minus 2…times 2* is recast in Joseph's clarifying question:

78 *S:* So number of hexagons would be 4 times 6 minus n minus 2. So 4 times 6 would be 24. Number of hexagons would be 1, 2, 3, 4. 4, uh, times 2.

79 *J:* Times 2 or minus 2?

The verbs *minus* and *times* shift within Sheila's and Joseph's comments separately, and are repeated across their comments, while retaining focus on the direct object of *2*. This example highlights the ways in which speakers use repetition to negotiate mathematical conjectures. Dialogic syntax proposes this coordination as a "new, higher order linguistic structure…the coupled components recontextualize each other, generating new affordances for meaning" (Du Bois, 2014, p. 360). The field of dialogic syntax holds this exchange as a new type of grammar, in which linguistic structure is no longer confined to the sentence or to a single speaker.

Du Bois (2014) provides a useful review of the theoretical antecedents of dialogic syntax, which draw from a wide range of fields, including linguistics, anthropology, literary theory, and cognitive science. He identifies four foundational themes, some of which resonate with prior research in mathematics education. The first theme, parallelism, refers to the concrete repetitions within nearby utterances. In the example above, Joseph's *minus 2* is parallel to his *times 2*, and both are repetitions of the endings of Sheila's sentences. Staats (2008) highlights ways in which these parallel, poetic structures can express both inductive and deductive mathematical reasoning. Oslund (2012) uses poetic structures in teachers' narratives to trace their shifts across fraction metaphors that were familiar or unfamiliar to their students.

Underlying grammatical parallelism is the principal of indexicality, or the capacity of language to refer to or point to other words and to elements of the situational context. Indexical words like *this*, *that*, and variable names like *n* have been associated with mathematical activities such as generalization and collaborative learning (Barwell, 2015; Radford, 2003). Parallelism occurs when units larger than a word—*times 2*—"point to" corresponding units like *minus 2*, creating bundles of indexicality.

Du Bois' second theme, analogy, refers to the meanings created through manipulation of similar units. For example, *times* and *minus* are alternatives within the frame of mathematical operations. The third theme, priming, is the experimentally-measured tendency to repeat lexical or syntactic units (see Staats & Branigan, 2014 for a discussion of applications and limitations within mathematics education research).

The fourth theme, dialogicality, has received slightly more attention in mathematics education research. Barwell (2015) following Bakhtin (1981), discusses three orientations of dialogicality: multivoicedness, multidiscursivity, and linguistic diversity. The first of these, multivoicedness, recognizes that all speech has a history. Speakers recast words and meanings from their past interactions each time they talk. This paper provides an analysis of multivoicedness in a mathematical problem-solving session. Overall, then, dialogic syntax is a new framework for mathematical education research, but through its interdisciplinary character, it shares theoretical antecedents with research on language as a resource for mathematical learning.

Participants and Task

Sheila and Joseph are undergraduate students who participated in a paid problem-solving session outside of class that was audio- and video-recorded. They had recently completed a university class in precalculus. Their task was to find an equation for the perimeter of a string of *n* adjacent hexagons, arranged so that pairs of interior sides are removed from the perimeter. They worked for about 40 minutes without any teacher intervention; about nine minutes of the conversation are analysed here. The task includes diagrams for hexagon strings for n = 1 to n = 4 hexagons, shown in Figure 1. The task also includes a table of values for n = 1 to n = 5 hexagons and the corresponding perimeter. A correct answer is $p = 4n + 2$. The task was based very closely on a proposed measure of readiness for undergraduate study (Wilmot, Schoenfeld, Wilson, Champney, & Zahner, 2011, p. 287).

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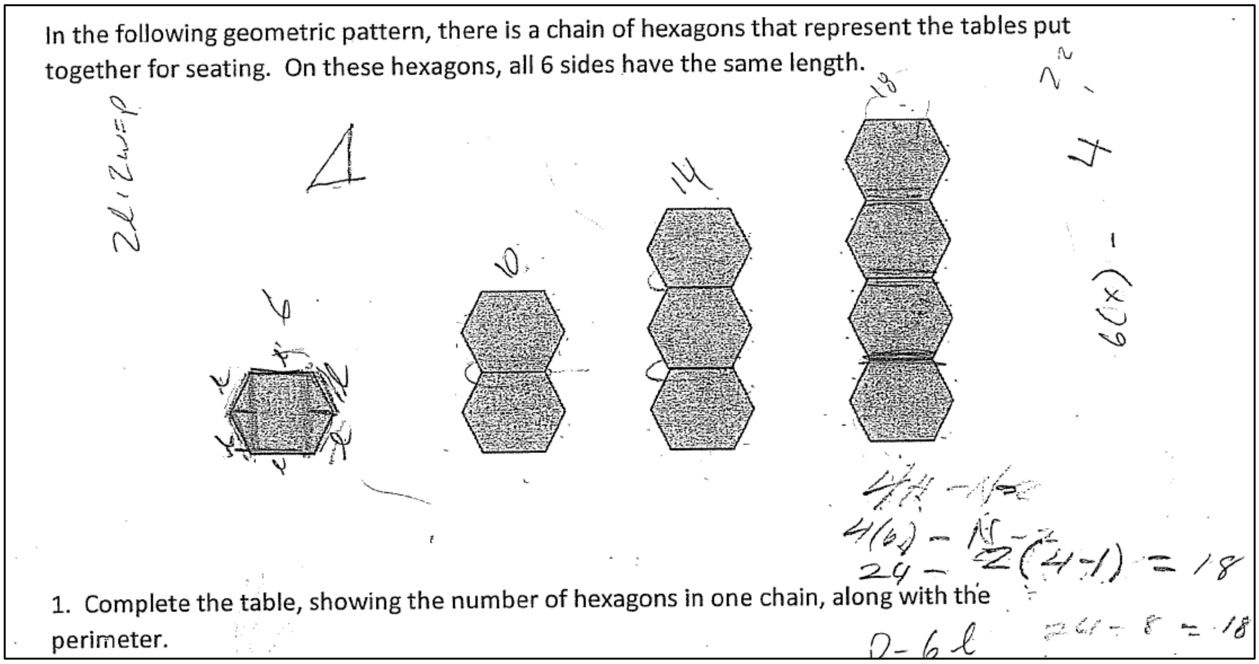

Figure 1. Task diagrams for $n = 1$ to $n = 4$ hexagons.

Methods for Identifying Poetic Structures

The first 90 turns of the conversation were coded using a spreadsheet to note the ways in which a phrase formed a poetic structure with a previously spoken phrase. It was necessary to develop a coding protocol because a phrase can repeat elements of several previous phrases. The coding approach relied on a combination of close attention to the syntax of poetic structures and grounded theory coding to iteratively improve the choices about what phrases counted as repetitions of prior statements (Charmaz, 2006). The resulting system was comparatively conservative. In Gries (2005), for example, any repetition of syntax counts as repetition, even if all the words change. The phrase *3 times 2* would be considered a repetition of the phrase *4 minus 1*, because both involve a subject-verb-object construction. However, mathematics education audiences are concerned with language that facilitates mathematical learning. To better focus on continuity of mathematical topic, then, two phrases had to share syntax and at least one word in order to be considered a repetition. When multiple previous utterances could have been the foundation of a repetition, I chose the most recent one. Overall, then, this method undercounts poetic structures in comparison with related linguistics research, because it prioritizes continuity of mathematical topic.

Each repetition was classified as either *Internal* or *Across*. An internal repetition refers to repetitions within a single turn at talk by a single speaker. An across repetition refers to repetition involving two distinct turns at talk. An across comment could refer to a comment that the same speaker said in one of her previous turns at talk, or to a comment from the other speaker. I recorded the most recent turn in which the phrase occurred, even if this was within the same speaker's conversational turn; what the earlier phrase was; whether there was a change in speaker; and whether the phrase was a nearly-perfect duplicate of the previous line or a transformation of it.

I separated the conversation into four episodes, each representing a mathematical insight that the students achieved together. In episode 1, turns 1- 28, Sheila and Joseph filled a table of values on the task sheet for n = 1 to 5 hexagons and the corresponding perimeter. In episode 2, turns 29-58, they determined that they should calculate perimeter rather than area. In episode 3, turns 59-71, they initiated the idea that the shared interior sides of the hexagon strings required them to subtract two, but they did not resolve how many times to subtract two. In episode 4, turns 72-90, they expressed a

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correct verbal method, and wrote a formula in which both H and N stand for the number of hexagons, #H(6) – 2(N – 1) = .

Results: Types of Poetic Speech

A close analysis of 90 turns at talk shows that poetic structures occurred very frequently. Coding produced just over 50 across repetitions and just over 55 internal repetitions. Out of 90 turns at talk, about 30 were very short comments, such as *Okay*, *Yup* or *Oh, okay, there we go*, which were too short or non-mathematical to produce poetic structures under the coding protocol. On average, then, there were about 1.75 poetic structures per substantial conversational turn.

The analysis revealed eight types of repetitions that contributed to the discovery of mathematical relationships. There were in addition poetic structures of that didn't fall into a clear type. The types were: List, Echo, Comparison, Contrast, Interposed List, Consolidation, Expansion and Reversal. Elsewhere, I describe the ways in which these eight poetic structures allowed Sheila and Joseph to gradually construct a method for solving the hexagon problem through repetitions, with small changes, of recent statements (Staats, 2016). Here, I briefly describe each type of poetic structure and I discuss in more detail the role of the reversal poetic structure.

In Episode 1, echoes and lists predominated. Sheila and Joseph began their exploration by counting the sides of the n = 1 to n = 4 hexagon diagrams. As Sheila counted the perimeter of the n =1 diagram with: *1, 2, 3, 4, 5, 6. 6,* she used a list—from 1 to 6—and an echo in the form of the second "6." At turn 15, Joseph also used echoes and lists: *So, this would be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 10.* The students began to fill the table for number of hexagons and the perimeter. An *Interposed List* at line 24, allowed Sheila to coordinate the independent and dependent variables: *so we're just putting in the 1 to 6, 2 to 10, 3 to 14.* At line 27, Joseph used a *Comparison* poetic structure, *22 for 5*, to extend the interposed list and to help complete the table of values.

In episode 2, Joseph suggested drawing additional interior line segments so that they could use the geometry of triangles and the formula P = 2L + 2W. Sheila suggested focusing on the exterior perimeter instead, and comments in line 52:

> 52 *S:* Complete the table showing the number of hexagons in 1 chain along with the perimeter. So then we're counting all the sides, so it'd be 6L. For 2 it'd be 1, 2, 3, 4, 5, 6, 7, 8, 9, 10. 10L.

This is a *Consolidation* poetic structure, because a list from a previous turn, *1, 2, 3, 4, 5, 6, 7, 8, 9, 10*, has been fit inside a previous comment from turn 24: *2 to 10*. Consolidation is similar to another poetic structure in this conversation, *Expansion*, which first occurs are line 62:

> 62 *S:* Uh, so this would be 6L. 6. And then this would be 10L minus 2. Minus 2. This would be 2, 4 minus 4. This would be 6. 18L. So the total number of sides minus 2 on this side. So it'd be, uh, 6. 6, and then this would be, uh, 12 minus 2. So.

Here, there is a poetic structure repetition of the form /*this would be 6L*/*this would be 10L…/This would be 2, 4 minus 4*/ *This would be 6. 18*/. Here, the short list *2, 4* and a moment later, *6*, were the first attempts to count the interior sides. This is an expansion of the comment /*this would be 6L*/*this would be 10L…/* with a new list. Expansion differs from consolidation because expansion introduces a new element that had not been spoken before.

At turn 79, the *Contrast* poetic structure occurred, which we saw above, when Joseph clarified, *Times 2 or minus 2*?, The final type of poetic structure for this conversation, the reversal, is discussed more thoroughly below.

It is important to note that each of the eight poetic structure types is a discursive move that could easily occur in a non-mathematical conversation. Contrast could occur, for example, as *steamed rice or fried rice?* A comment *Mark has some advice for you* could prompt the reversal: *Well, I have*

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some advice for Mark! Because these poetic structures are all general discursive options, when they occur in mathematics conversation, they help us identify moments when language is a resource for mathematical learning.

Discussion: The Role of the Reversal Poetic Structure

Different types of poetic structures emerged and were prominent in different episodes of the hexagon conversation. In Episode 1, for example, many of the poetic structures were simple ones—echoes or lists. The reversal poetic structure emerged in Episodes 3 and 4. In a reversal poetic structure, two elements switch places syntactically. For example, subject and predicate or adjective and noun switch roles, as in *10 minus 12* versus *12 minus 10* or *negative two* versus *two negatives*. Reversal occurred in six turns, 62, 75, 76, 78, 81, and 90. Sheila and Joseph tended to use the reversal poetic structure as they arranged variables and coefficients in their verbal method, and as they transitioned from a spoken method to written mathematical notation.

Reversal first occurred as Sheila's internal repetition in turn 62, when she began to count the interior sides of the hexagon diagrams. This was the first time either student mentioned "minus" or subtraction.

62 *S:* Uh, so this would be 6L. 6. And then this would be 10L minus 2. Minus 2. This would be 2, 4 minus 4. This would be 6. 18L. So the total number of sides minus 2 on this side. So it'd be, uh, 6. 6, and then this would be, uh, 12 minus 2. So.

Sheila mentioned the perimeter first for the n = 1 and 2 diagrams: *6L*, and then *10L minus 2.* Then she mentioned the interior sides for the n = 3 diagram: *This would be 2, 4 minus 4.* The reversal occurred next, at the n = 4 diagram, when she switched the order of the interior sides and the perimeter: *This would be 6. 18L.* This reversal of the previous comment seems to represent Sheila's shift from a quantity that she understands well—perimeter—towards the quantity that she needs to understand better. The reversal seems to facilitate her focused attention on a new pattern in the diagram.

In turn 75, there is an across reversal from turn 72. In turn 72, Sheila commented on *total number of hexagons times six.* In turn 75, Joseph switched the order to place the 6 first: *So it'd be like 6 times x per se number of hexagons.* The phrase *number of hexagons* had been spoken before, but only while reading from the task page. Sheila's *total number of hexagons* at 72 was the first time that the *number of hexagons* was used as an independent phrase for conjecturing. Joseph's reversal at turn 75 was notable because he proposed shifting to a more standard way of speaking and writing, in which coefficients precede variables. As he said this, he wrote on the same paper as Sheila was working on: 6(x) —.

At turn 76, Sheila does not take up Joseph's suggestion immediately, but she does use reversals in turns 76 and 78 as she grapples with the n = 4 hexagon case.

76 *S:* Uh, divided, um, um, 4. 4 minus, minus n 2. Uh, like n would be the total number. 1, 2, 3, 4, number of cases. So, like in statistics the number of cases would be n and that would be your number minus 2. So 4 h, 4 h would be your number of hexagons. [At turn 77, Joseph responds: Yup.]

78 *S:* So number of hexagons would be 4 times 6 minus n minus 2. [Here, she wrote: 4(6) – N – 2]. So 4 times 6 would be 24. Number of hexagons would be 1, 2, 3, 4. 4, uh, times 2.

Sheila uses an internal reversal at 76 by switching the variable *n* with its description, number of cases: *n…would be the total number…number of cases would be n.* Her final comment in turn 76: *4 h, 4 h would be your number of hexagons* was reversed at 78 so that it began with hexagons: *So number of hexagons would be 4.* A moment later, but still at 78, Sheila incorporated Joseph's *6 times* from line 75, which is an across repetition: *So number of hexagons would be 4 times 6…*and she

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began to build the subtraction of the interior sides, though inaccurately at this point: *So number of hexagons would be 4 times 6 minus n minus 2.*

In turns 75 to 78, there are reversals across the word *times*, and there are reversals of the variable *n* and the description of the variable. A possible interpretation of these reversals is that they represent some tension between topic of focus of each speaker. Joseph's reversal helped him propose the beginning of a formula that was written in a standard format, but that leaves the variable, *number of hexagons* in the terminal position. In contrast, Sheila is working on how to start with the number of hexagons—at this moment, the n = 4 case—and perform calculations on n = 4 in order to verify the perimeter of 18. As a verbal representation, saying *number of hexagons* first stabilizes the quantity for further conjecturing and calculation. Sheila used a couple of reversals to get the 4 into a convenient position to calculate with it. Brian was moving towards a standard form of writing mathematics, but Sheila was still focused on working out the algebraic relationships regarding the interior sides.

In this passage, then, we can hear the transition from reading the task (the phrase *number of hexagons),* to proposing a written form; (*6 times x per se number of hexagons*); to writing it as 6(x) —; to recasting it in an easier verbal form for conjecturing (*So number of hexagons would be 4 times 6 minus n minus 2*). Analysis of poetic structures, and especially the reversal poetic structures, helps to highlight these transitions across several ways of experiencing and representing mathematics.

The reversal poetic structure was also associated with writing in turn 81. Sheila and Joseph were still working on the n = 4 hexagon case. They knew that six times four yields 24, that they must subtract pairs of interior sides, and they must get 18, but they didn't know how many twos to subtract. Here, Sheila subtracted 8 instead of 6 to account for the interior sides.

80 *S:* Negative 2 would be negative 8. So 24 minus 8 is how much?
81 *J:* Wouldn't it be 24 plus 8 because there's these two negatives? I'm sorry. There's these two negatives.

The reversal in line 81 is that Joseph's *two negatives* reverses Sheila's *Negative 2* in turn 80. In this phase of the conversations, the students were working on the written statement, where both H and N stand for the number of hexagons: 4H — N — 2. There is some ambiguity because there were several erasures and the angle of the video recorder did not capture each moment well. However, it is clear that Joseph used a reversal to focus Sheila's attention on a problem with the written notation.

The final reversal occurred in line 90. In line 90, Sheila first articulates a correct method for the task. By the end of line 90, Sheila had written: 24 — 2(4 — 1) = 18.

90 *J:* So this would be 2, 4, 6. So that would be 1, 2, 3, 4, 6. So let's see, 1, 2, 3. So number of insides, 4, so 4 minus 1 times, uh, 4 minus 1, so this would be 2 into 4 minus 1 equals, right? So that would be 3. 3 times 2 would be 6. 6 from 24 is 18, right?

As Sheila said the phrase *2 into 4 minus 1 equals*, she wrote 24 — 2(4 — 1), so it seems reasonable to consider that for Sheila, the word *into* meant *times.* The reversal poetic structure occurred when Sheila's phrase *so 4 minus 1 times* was reversed so that *4 minus 1* took the second position after the operation *times* or *into*: as *2 into 4 minus 1*. The *4 minus 1* reversed position from beginning to end of its phrase. This reversal poetic structure co-occurs with Sheila's decision to write the two before the parentheses of (4 — 1), and so this is a third case in which reversal facilitated the transition of a verbal method into a standard written format.

Conclusion

Each time we speak, we cross a border. We acknowledge the common ground of previous statements and offer, through shifts in our sentences, a pathway forward. Applying this perspective to mathematical conversation can highlight the small collaborations—equally linguistic and mathematical— that must take place as mathematical insight develops. In this conversation, poetic structure reversals helped students shift from verbal and computational methods into standard ways of writing mathematics. The reversal poetic structure may prove to be concerned with positioning important mathematical entities, and therefore, it may facilitate movement between different mathematical representations, ones that are read, spoken and written. The theoretical framework for this analysis, dialogic syntax, is an interdisciplinary, border-crossing perspective that provides several rich directions for future research in mathematical discourse. Dialogic syntax draws attention to the subtle precision of informal language in supporting mathematical discovery.

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