FACTORS RELATING TO THE DEVELOPMENT OF MATHEMATICS FOR TEACHING

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This study investigated the effect of different aspects of mathematics knowledge for teaching on performance in upper elementary mathematics methods courses. In the Ontario (Canada) context, prospective teachers have been, until the 2015-2016 school year, able to obtain a Bachelor of Education (BEd) degree in as little as eight months after a different undergraduate degree, with most students taking no university mathematics courses whatsoever prior to the eight month BEd. We examined the effect of performance in a new course on mathematics for teaching as well as an exam in mathematics content, on performance in the methods course. Mathematics knowledge for teaching, as measured by the final grade in the mathematics for teaching course, was found to be a significant predictor of first semester methods course performance, with general content knowledge being less important.

Keywords: Mathematical Knowledge for Teaching, Teacher Education-Preservice, Teacher Knowledge

Introduction

At the time of this study, prospective teachers at our Ontario university would enroll in a one-year Bachelor of Education (BEd) program in order to be certified to teach. This program would occur after the candidates had completed another unrelated degree, such as a BA in History or English. The majority of the prospective teachers in our program have not taken any mathematics courses at the university level prior to entering the BEd program. Within our program, prospective teachers would graduate with a BEd after taking only one thirty-six hour mathematics Methods course. Since specialised mathematical knowledge of teachers has been directly linked to student achievement (Baumert et al., 2010), developing prospective teachers' mathematical understanding, in particular the understanding needed for teaching, is an important aspect of the education program we offer. With the limited number of hours we have to build these understandings, we have been concerned with what program developments we could put in place to increase the opportunity for knowledge development. One of the changes has been a Competency Exam taken by all prospective teachers in September at the beginning of their BEd year. This exam tests only content knowledge found in the elementary curriculum (Ontario Ministry of Education, 2005). More recently, a second program change was undertaken, which supports some of the candidates and is the focus of this research.

At our institution, there is one other way that prospective teachers can achieve their degree: through the Concurrent Education program. This means that they are accepted into the BEd program and their other degree at the same time. This allows for some education courses to be taken earlier, however the majority still occur in the one-year BEd program taken after their first degree is completed. The extra time, however, has allowed for us to institute one thirty-six hour Mathematics for Teaching course prior to the Methods course in the BEd program. The effect of this addition is that these students now have double the hours in mathematics courses particularly aimed at mathematics knowledge needed for teaching. Although this alternate route is an option for our prospective teachers, the majority of prospective teachers, however, do still enter the eight month BEd program after completing their first degree and thus miss the first Mathematics for Teaching course.

We were interested in the effects of these options on performance in the Methods course, hence,

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this study examines the effects different types of mathematics knowledge (as developed through these various components of our program) have on the prospective teachers' understandings of mathematics as needed for teaching. Specifically, we examined the influence that the performance in the Mathematics for Teaching course, and the scores on the Mathematics Competency Exam, have on performance in the Methods course.

Framework

Since mathematics knowledge for teaching is not "just" a knowledge of subject matter (Baumert et al., 2010; Kajander, 2010; Silverman & Thompson, 2008), it is important that teacher knowledge includes a knowledge of students and of teaching mathematics. Simply stating a procedure or procedural steps would not be enough to show mathematical reasoning and support a claim when teaching (Ball & Bass, 2000); indeed mathematics for teaching is often described as including 'more' than the ability to perform standard procedures (Silverman & Thompson, 2008). In many descriptions of teachers' mathematical knowledge, the content a teacher needs may be something that goes beyond, or is somehow distinct from, what any non-teacher studying mathematics would need (Baumert et al., 2010; Ma, 1999). It has been further argued that such knowledge differs from the knowledge incoming prospective teachers typically possess (Chamberlin, Farmer, & Novak, 2008; Davis & Simmt, 2006; Kajander, 2010).

In our recent work we have sought to determine, describe, and unpack specific content pieces that we feel are critical to teacher mathematical capacity. In particular, we have found the models and modelling approach described in Lesh and Doerr (2003) fundamentally helpful in this regard. It is this approach that frames the content areas and approaches chosen for inclusion in the course textbook which was written especially for our Mathematics for Teaching course (see Kajander & Boland, 2014), and which guides the course activities. Developing a deep conceptual sense of elementary mathematics content based on models, relationships, and connections, and all interconnected with reasoning which might be applicable to classroom discourse, form some of the goals of our elementary mathematics courses for teachers. In particular, we strive to focus on the development of "how to gradually decompose and unpack the mathematical rules and operations through the use of representations, and knowledge of how to use representations to develop generalizations" (Mitchell, Charalambous, & Hill, 2014, p. 55).

The Mathematics for Teaching (MKT) model proposed by Ball and her colleagues (Ball, Thames, & Phelps, 2008) makes a distinction among different kinds of mathematical understandings held by teachers, such as the descriptions of 'pedagogical' and 'specialised' knowledge. While at times we find these distinctions have been blurred in our own work, they can be helpful in providing broad ways to describe different components of teacher knowledge. For simplicity in describing our current data, we make use of the terms from three of the categories of the MKT model, specifically, common content knowledge, specialised content knowledge, and the over-arching category of pedagogical content knowledge.

Ball, Thames, and Phelps (2008) begin their discussion of the different types of mathematics knowledge with an examination of common content knowledge (CCK). Based on their research, this knowledge is described as the general mathematics knowledge that any person studying mathematics would know. The Mathematics Competency Exam in our program is designed to test basic elementary school mathematics performance to the eighth grade level. However, the exam does not include items related to the use of models, multiple methods, or connections among ideas, all of which we see as crucial to teachers' knowledge. Rather, the test draws directly from our provincial grade eight mathematics curriculum document (Ontario Ministry of Education, 2005). Hence, for the purposes of the current analysis, we use the common content knowledge acronym, CCK, to refer to the grade in the September writing of this test (the 'Competency Exam'), as we feel it represents knowledge that all school students might be expected to develop. A sample item from the

Mathematics Competency Exam is provided in Figure 1.

A small 1 m^2 garden is to be planted with herbs. One-fourth of the garden will be chives and one-fourth parsley. Of the remaining portion of the garden, one-third will be basil, one-third will be thyme, and the rest will be sage. What fraction of the total garden is sage?

Figure 1. Sample Question from Mathematics Competency Exam.

The next piece of the MKT model that is pertinent to our study is that of specialised content knowledge (SCK; Ball, Thames, & Phelps, 2008). As described in the MKT model, this knowledge is special knowledge required of teachers, and goes beyond what an individual studying mathematics would require. This knowledge, however, does not depend on knowledge of students or knowledge of teaching; it is indeed ‘mathematical’. Final course grades in the Mathematics for Teaching course were used as a measure of SCK. While a thorough analysis of every examination item from this course with the MKT framework has not been conducted, the overall descriptions from the literature of important elements of SCK do align well with our course objectives and assessment items, hence a global alignment is arguable. Our course focuses, for example, on models and reasoning about problems that would be important to teachers while in the field, however does not focus on pedagogy or knowledge of students. A sample item from the final course exam is shown in Figure 2.

The following pattern is made of toothpicks:

n: 1 2 3

- State a formal or function rule using “n” as the picture number, which determines the number of toothpicks needed for each picture. Redraw the third picture in the above pattern and use it to show how your rule relates to the pattern.

- State a different formal or function rule which relies on a different geometric way of grouping and counting the same pattern of toothpicks. Again redraw the third picture in the above pattern and use it to show how the new rule relates to the pattern.

Figure 2. Sample Item for Mathematics for Teaching Course.

Lastly, we look at one final aspect of the MKT model: pedagogical content knowledge (PCK). This aspect of the model includes sub parts of knowledge of content and teaching, and knowledge of content and students (Ball, Thames, & Phelps, 2008). Based on the MKT model, PCK includes knowledge of models and reasoning in relation to student understanding, including knowledge of typical student errors and so on. To measure this attribute, we used the grade in the Methods course from the fall semester course exam. Items on this exam include extensive use of models and reasoning (as do the Mathematics for Teaching course items), but also items related to typical student errors, misconceptions, and other topics drawn from the MKT model categories of PCK. Thus we use the over-arching name of PCK to refer to the aggregate of our items of this type, as found in the fall Methods course exam. Figure 3 provides a sample item from the Methods course exam.

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Consider this problem for the questions below: $1\frac{1}{3} \div \frac{1}{4}$

- Student A believes the answer is $5\frac{1}{3}$.
- Student B believes the answer is $5\frac{1}{12}$

Is either student correct? If so, which student is correct?

Using reasoning and models, show the thinking of each student that may have resulted in their solution for the question.

Figure 3. Fall Methods Course Exam Item.

Methods

Our study sought to further enhance our understandings of what and how previous experiences in mathematics for teaching, as grounded in the MKT (Ball, Thames, & Phelps, 2008) model, might support further development at the methods course level. We used a quantitative method design to search for relationships between the different factors in our program. We were interested in whether the kinds of specialised content knowledge taught in our Mathematics for Teaching courses were helpful in supporting the development of pedagogical content knowledge in our Methods course. We also examined if there were effects from the scores on the Competency Exam on the development of pedagogical content knowledge. For ease and familiarity, we have named the three variables using the MKT descriptors, as just explained. However, agreement with our naming is not required to examine the results – the names of the variables we term CCK, SCK and PCK could simply be replaced by the course components they respectively represent, which are the Competency Exam grade, the Mathematics for Teaching course final grade, and the Methods course first term exam grade.

Participants

All participants were prospective teachers in the thirty-six hour upper elementary mathematics Methods course. All participants in the Methods courses were asked to participate in the study, and most did (N=71 each year for each of two years). In both years, data were collected from all participants who volunteered during the Methods course. In particular, we were interested in whether each participant had previously taken the new Mathematics for Teaching course (which could have been taken in any previous year since 2010-2011), and if so, how well they did in it.

Descriptive statistics were first used to examine the variables PCK and CCK by separating the groups based on whether or not participants had enrolled in the Mathematics for Teaching course (the final grade in which was recorded as the SCK score). As mentioned, the grade on the Mathematics Competency Exam was used as a measure of CCK, and grades on the Professional year Methods course fall exam were used to measure PCK. Further quantitative analysis was conducted to explore the relationships of the two independent variables SCK and CCK, to see the impact on PCK. Relationships among these three variables were explored using independent t-tests as well as regression analysis. We used independent sample t-tests to compare the PCK as well as the CCK to determine differences between the two subgroups (those with SCK and those without). A regression analysis was also conducted on those who had taken the new Mathematics for Teaching course (SCK) to determine the effect of SCK and CCK on the fall methods exam (PCK).

Since all sections of the Methods course during the three year period were taught by the same instructor (first author) in a similar manner, and all of the previously taken Mathematics for Teaching courses had also been taught by a consistent instructor (second author), we chose to combine the quantitative data sets for both years. In total then, we collected performance data from a total of 142 prospective teacher participants, drawn from two cohorts of 71 participants. Of these 142 prospective teachers, only 22 of the prospective teachers had enrolled in the Concurrent Education program and therefore only these 22 had taken the Mathematics for Teaching course.

Results

We began by examining the data collected from the 142 participants in our pool of prospective teachers. As mentioned, 22 of these had taken the Mathematics for Teaching course, and 120 had not, meaning they were registered in the stand-alone eight month BEd program. Thus the subgroup with an SCK score had 22 participants. Of the 142 examined in total, 10 (only 1 from the SCK sub-group) chose not to take the September write of the Competency Exam so did not have a CCK score. Of the 132 participants who took the Competency Exam (21 with a SCK score), 66 of them did not reach the 75% required “pass” score. Of this 66, 10 were in the SCK subgroup. All of the participants had a PCK score since all were enrolled in the Methods course.

Table 1: Percentage Means by Subgroup

Subgroup	CCK Mean score (%) (Competency Exam)	PCK Mean score (%) (Methods course exam)
SCK	74.867 (N=21)	76.500 (N=22)
No-SCK	68.622 (N=111)	67.700 (N=120)

Initial observations showed that the participants with SCK, meaning they had taken the Mathematics for Teaching course, did better in both CCK and PCK (see Table 1). Since 10 of the SCK group did “fail” the CCK (“Competency Exam”) portion, further analysis was performed on examining the participants in the SCK sub-group (see Figure 4) to explore this situation further. Only 21 participants are included in the graph since 1 did not have a CCK score. Prospective teachers are given the option of challenging the Competency Exam in September of their BEd year. They are encouraged to take the exam at this time in order to set goals for areas of content that they need to work on prior to taking the exam in March where a score of less than 75% would mean they do not get their BEd degree. Most prospective teachers do choose to the take exam in September for this reason.

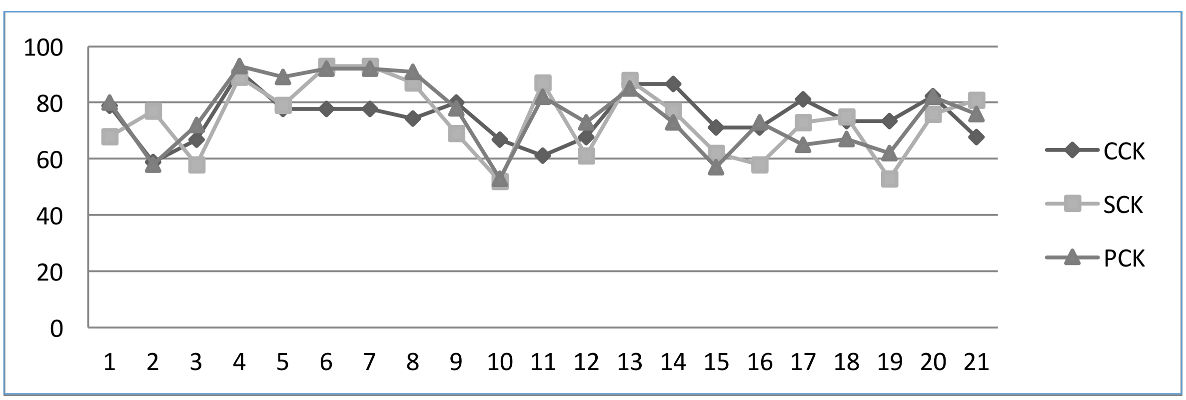

Figure 4. Graph of SCK Participants’ Percentage Scores, N=21.

Examining the graph of the performance of each of the 21 participants for whom we had all three scores, suggests that the scores for SCK and PCK were more closely clustered for more of the participants than the CCK scores. (The lines joining the points are illustrative only; the actual dataset is comprised only of the three scores for each separate person (numbered 1 to 21)).

When examining the relationship for participants with and without SCK on the fall Methods course exam (PCK), Levene’s Test for Equality of Variances showed there was no significant

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variance between the subgroups. Therefore, an independent t-test was run on the data as well as 95% confidence intervals (CI) for the mean difference. It was determined there was a significant difference in results of PCK between those who had taken the Pre-professional year Mathematics for Teaching course (SCK) and those who had not ($t(138) = -2.472$, $p = .015$) with a difference of -8.196 (95% CI, -14.752 to -1.64). When subsequently examining the relationship for participants who had taken the new Mathematics for Teaching course (SCK) and those who had not, on the Mathematics Competency Exam (CCK), Levene's Test for Equality of Variances determined there was significant variance ($p = .010$) between the subgroups. Therefore, an independent t-test was run on the data and equal variances were not assumed, as well as 95% confidence intervals (CI) for the mean difference was also determined. It was determined there was a significant difference in results on the Competency Exam (CCK) between those who had taken the new Mathematics for Teaching course and those who had not ($t(55.006) = -2.581$, $p = .013$) with a difference of -2.813 (95% CI, -.629 to -4.997).

Table 2: Regression Model

Predictor of Methods course exam	Beta	Sig.
Competency Exam (CCK)	.265	.099
Mathematics for Teaching course (SCK)	.662	.000

R = .797, R squared = .634, N = 21

The regression model showed a highly statistically significant ($R^2 = 0.632$, $p = .000$) prediction of the fall Methods exam (PCK) when examining the participants who had taken the new Mathematics for Teaching course (SCK). It was determined that 63% of the variance on the fall exam (PCK) could be explained by the two factors. For this model, SCK ($\beta = 0.0662$, $t(18) = 4.333$, p=.000) significantly predicted scores on the fall Methods exam (PCK), while CCK ($\beta = 0.265$, $t(18) = 1.737$, $p = .099$) was not significant. Scores in the new Mathematics for Teaching course (SCK) had a greater impact on the fall Methods exam (PCK) scores (see Table 2).

Discussion

The data may shed some further light on the impact and importance of different aspects of mathematics knowledge on the development of pedagogical content knowledge. While one perception might be that learning about contexts, models and reasoning in elementary mathematics might best be done in methods courses (rather than 'mathematics' courses), we continue to argue for the crucial need for such learning during *mathematics* courses for teachers, in order to better support what can be subsequently achieved during methods courses. The current data may support such an assertion. Indeed, Mitchell, Charalambous, and Hill (2014) argue that an understanding of models and reasoning is an area particularly in need of support for many teachers.

The results suggest that the Competency Exam, used in the current analysis as a measure of CCK, had a relatively weak impact on performance at the Methods course level. We argue that "knowing the curriculum", particularly the more computationally-related aspects, is far from sufficient for teaching. Certainly, our previous qualitative observations of participants during the Methods course suggest that the more specialised the background of the participants, the greater the strides they may be able to make in terms of PCK development (Holm & Kajander, 2012). Without such conceptual understanding, learning about structuring a lesson to explore mathematical concepts is impossible; teachers are left with a rule-based treatment as their only lesson option. Indeed, we have found that prospective teachers without previously-developed knowledge of how to understand and represent mathematical content (SCK) are so significantly distracted by the mathematical challenges that they are often less involved in the pedagogical conversation taking place. Hence we find that while more knowledgeable Methods course participants are having discussions regarding lesson design, student responses, typical errors, and so on, the participants with weaker SCK are typically focused mainly on the mathematical concepts, missing out on the other ideas. This suggests that those with weaker SCK may well have reduced opportunities to develop PCK, and aligns with the results presented here. Hence the current results accurately reflect our anecdotal experiences while teaching the Methods course, regarding the importance of SCK in pedagogical development during elementary teacher education.

The study results suggest that the impact of a specialised course in mathematics for teachers, (SCK), does impact Methods course performance (arguably termed PCK here) in a highly significant way, and thus underscore the need for continued emphasis on *specialised* mathematical experiences for teachers. Generally, participants who did well in the Mathematics for Teaching course tended to do well in the Methods course, and additionally were also observed informally to demonstrate deeply pedagogical understandings during Methods course classes. In particular, several participants who had taken the Mathematics for Teaching course prior to beginning the Methods course and had done very well, both began and ended the Methods course in a very strong position. As course instructors, we observed generally that evolving confidence in the specialised mathematics seemed tightly aligned with pedagogical mastery. This data provides further support of the mounting evidence that it is the more conceptual and specialised mathematical knowledge that supports better teaching rather than only more general mathematics knowledge. The results also align with the Baumert et al. (2010) study, which found that the degree of teaching-specific mathematics knowledge of early secondary level classroom teachers made a measureable difference on student achievement, while general mathematics background of teachers did not.

At the institution where this study took place, there is a continued sense by non-mathematics education faculty and administration that specialised content knowledge, such as knowledge of models and reasoning, should be contained in Methods courses, rather than being thought of as 'mathematics'. Based on the MKT (Ball, Thames, & Phelps, 2008) model, and our own data, we continue to argue for the need for *mathematical* experiences specialised to teaching, in order to support richer pedagogical development. The data presented here contribute to such a stance.

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