PROFILES OF RESPONSIVENESS IN MIDDLE GRADES MATHEMATICS CLASSROOMS

Jessica Pierson Bishop
University of Georgia
jpbishop@uga.edu

Hamilton Hardison
University of Georgia
ham42@uga.edu

Julia Przybyla-Kuchek
University of Georgia
jep94142@uga.edu

In this paper we consider how mathematics instruction that values, attends to, and builds on students' mathematical ideas is realized through discourse. We describe interactions that build on students' thinking and in which students help to determine the direction of mathematics lessons as responsive. Using a framework we developed to characterize the responsiveness of mathematics interactions, we report the variation in responsiveness across seven middle grades classrooms by describing (a) students' mathematical contributions, (b) the moves teachers enact in response to these contributions, and (c) how these two components interact. We found that there are multiple ways to be responsive to student thinking.

Keywords: Classroom Discourse, Instructional Activities and Practices

In this paper we consider how mathematics instruction that values, attends to and builds on students' mathematical ideas is realized through discourse. Discourse by its nature is responsive and relational (Bahktin, 1986; Halliday, 1978). Our goal is to consider a particular feature of discourse rooted in Bakhtin's notion of dialogism and Halliday's interpersonal metafunction—what we have termed responsiveness. We describe classroom interactions that build on students' thinking and in which students help to determine the direction of mathematics lessons as responsive. *Responsiveness* to students' mathematical thinking is a characteristic of interactions wherein students' mathematical ideas are present, valued, attended to, and taken up as the basis for instruction. Interactions can be more and less responsive, and in this proposal we document multiple profiles of responsiveness across middle grades mathematics classrooms. Our proposal addresses the conference theme of *Questioning Borders* by addressing issues of access and participation. In particular, the ways in which conversants are responsive to each other allows them to participate in certain ways, to take on different roles and identities, and ultimately affects how classroom participants engage with the mathematics at hand.

Theoretical Framework & Literature Review

To understand another is a responsive act. One must engage with another's idea and respond to it in order to understand it—though the manner and quality of engagement can vary widely. Bakhtin (1986) explains it as follows: "The fact is that when the listener perceives and understands the meaning (the language meaning) of speech, he simultaneously takes an active, responsive attitude toward it. He either agrees or disagrees with it (completely or partially), augments it, applies it, prepares it for execution, and so on" (p. 68). If understanding is responsive, then so too is speaking (and other forms of communication). In his discussion of speech genres Bakhtin says, "Utterances are not indifferent to one another, and are not self-sufficient … Every utterance must be regarded primarily as a *response* to preceding utterances" (p. 91). We focus on responsiveness because participating in mathematical discourse is inherently a responsive act, for both listeners and speakers. Specifically, we are interested in the degree of responsiveness to students' *mathematical ideas* within classroom settings and the extent to which utterances mutually acknowledge, take up, and reflect an awareness of student thinking. Below we share a brief overview of research related to responsiveness—primarily studies that focus on student thinking—and synthesize findings in order to situate our study.

Broadly speaking, research related to responsiveness can be categorized into two main types of

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

studies: (a) descriptive studies of classrooms in which students' mathematical thinking is either already prevalent or is developed within a classroom over time, and (b) studies identifying features of instruction that are positively related to mathematical proficiency. The first group of studies takes as a given the desirability and effectiveness of instruction that incorporates student thinking and considers what this type of instruction looks like. Some studies look at specific teacher moves such as a probing sequence of questions (Franke et al., 2009) or a reflective toss (vanZee & Minstrell, 1997). Other studies develop broader frameworks to describe the classroom contexts and features of instruction that influence or constrain whether, how, and in what ways teachers explore student ideas in their teaching. For example, Leatham, Peterson, Stockero, and Van Zoest (2015) developed a framework to identify instances when it might be productive to pursue students' mathematical ideas in-the-moment. Their focus is how to identify a particular type of student contribution—a *M*athematically Significant Pedagogical *O*pportunity to build on *S*tudent *T*hinking or *MOST*. Additionally, research on the construct of teacher noticing has explored requisite knowledge and skills teachers need to be responsive to student thinking. In particular, Jacobs, Lamb, and Philipp (2010) developed a framework for teacher noticing comprised of three interrelated components that has students' mathematical thinking as its foundation. Before a teacher can incorporate student thinking into instruction, she must first *attend* to student ideas and *interpret* their significance before *deciding how to respond*.

The second group of studies identifies features of instruction related to responsiveness that are positively related to improved mathematical proficiency. Many of these studies use quantitative tools to provide evidence that the presence of student thinking is an effective feature of instruction based on a positive relationship between student thinking and outcome measures such as achievement scores, problem solving, and improved dispositions toward mathematics (Carpenter et al., 1989; Ing, et al., 2015; Nystrand et al., 1997). For example, in their work investigating what practices 'press' students for conceptual mathematical thinking, Kazemi and Stipek (2001) found that press for conceptual understanding was positively correlated with students' understanding of fractions. They identified features of discourse that were present in high press classrooms including engaging students in explaining, justifying, verifying, and arguing about their own and their peers' thinking.

Across this literature we see a vision of mathematics instruction that values students' mathematical ideas and seeks to incorporate those ideas into instruction in productive and meaningful ways. Some of the research focuses on discursive moves that can be enacted in-the-moment. Other research considers either how to develop classrooms that value students' thinking or the skills needed to respond to students' thinking. Moreover, researchers have indicated that this type of instruction is not commonplace and is challenging to enact (Pimm, 1987). Our goal in this proposal is to develop a framework to characterize responsiveness in mathematics classrooms and use it to describe the variation in responsiveness across classrooms. The research question guiding our study is, In what ways are middle grades mathematics classrooms responsive to students' mathematical ideas during whole-class discussions?

Methods

Participants and Data Collection

The data in this report are part of a larger study investigating characteristics of productive mathematics discourse in grades 5–7. Participants include teachers and students in seven classrooms across three U.S. states. Participating teachers were recommended by district personnel, faculty researchers, and professional development providers based on the teacher's reputation for using problem solving and discussion regularly during instruction. Our participants were five fifth-grade teachers, one sixth-grade teacher, and one seventh-grade teacher. All were certified in either elementary or middles grades education with 7 to 30 years of teaching experience. For each teacher, we videorecorded and transcribed an introductory lesson on fractions. We chose fractions because it is an important topic and spans the middle grades required content. Lessons were filmed at various points throughout the school year based on when fractions were introduced and ranged in length from 40 to 95 minutes. In our analysis, we used both the video recordings and transcripts, which allowed us access to gestures, student written work, and other non-verbal communication in the classroom.

The Development of a Coding Framework for Responsiveness

We developed a coding framework for responsiveness in whole-class mathematics discussions (Przybyla-Kuchek, Hardison, Bishop, 2015) using the constant-comparative method. The unit of analysis is a segment, which we define as a series of turns of talk with a common focus (e.g., activity or strategy). Our framework comprises two components: (1) students' mathematical contributions and (2) the moves teachers enact in response to these contributions (Figure 1). In this section, we describe the ordered levels for each component of our framework.

<table>
<tr><td colspan="3"></td><td colspan="3">Teacher Moves</td></tr>
<tr><td colspan="3"></td><td>Low</td><td>Medium</td><td>High</td></tr>
<tr><td rowspan="3">Student Contributions</td><td>None</td><td>Minimal</td><td></td><td></td><td></td></tr>
<tr><td rowspan="2"></td><td>Considerable</td><td></td><td></td><td></td></tr>
<tr><td>Substantive</td><td></td><td></td><td></td></tr>
</table>

Figure 1. Whole-class responsiveness framework.

Student contributions. Our framework includes four levels of students' mathematical contributions: none, minimal, considerable, and substantive. We define a segment as *None* if it contains no mathematical student contributions (e.g., teacher monologue). *Minimal* segments are dominated by students performing routine calculations, recalling facts, and providing short responses to known-information questions. *Considerable* segments are those in which students share their strategies or other mathematical ideas without justification. Unlike minimal segments, considerable segments contain evidence that students have opportunities to make sense of mathematical content and to share their ideas. *Substantive* segments, like considerable segments, are characterized by students discussing their mathematical ideas; however, in substantive segments, student contributions also include providing justifications, making generalizations, or participating in mathematical argumentation. When considering these elements of students' contributions, we focus on the structure of these contributions rather than on correctness (from our own perspectives). A segment may be characterized by substantive student contributions even if it contains individual turns of talk that might be described as minimal or considerable.

Teacher moves. Our framework includes three levels of teacher moves that reflect the extent to which students' mathematical contributions are made public, taken up, and serve as the basis for instruction: low, medium, and high. In *Low* segments, teachers do not use students' mathematical contributions as the foundation for instruction; low teacher moves include, brushing-off, evaluating, and not reacting to students' contributions. In *Medium* segments, teachers focus on (a) understanding and highlighting individual students' thinking by revoicing student ideas or asking probing questions, or (b) asking classmates to engage momentarily with particular student ideas by asking other students to correct, evaluate, or indicate whether their thinking aligns with another student (e.g., "Who used the same strategy?"). However, teachers do not focus simultaneously on both (a) and (b). In *High* segments, teachers simultaneously focus on student thinking and explicitly direct students to engage significantly with the mathematical ideas of others. High moves include requesting comparisons across student contributions, taking up a student-posed problem as a whole-class activity, asking

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

students to restate or apply another's strategy, and inviting students to ask questions of their peers. As with student contributions, teacher moves are coded holistically at the segment level.

Analysis

Prior to coding each introductory fraction lesson, a member of the research group watched the videorecording and partitioned the lesson transcript into segments. We did not code segments consisting of entirely nonmathematical content (e.g., discussions of norms, transitional time). At least two members of the research group coded each lesson independently using the videorecording and segmented transcript. Each segment was first assigned a student contribution code. Segments without mathematical student contributions (i.e., those coded as none) were not assigned a teacher moves code as there were no student ideas to which teachers could respond. Segments containing mathematical student contributions (i.e., those coded as minimal, considerable, or substantive) were also assigned one of the three teacher moves codes. Sets of independent codes were compared for each lesson, and coding discrepancies were discussed until the coders achieved consensus on a final set of codes for each lesson. In summary, each segment was characterized by a combination of codes corresponding to exactly one of the ten empty cells depicted in Figure 1; we refer to these ten coding combinations as compound codes.

Because lessons varied in terms of minutes of whole-class instruction and segments were defined by shifts in common focus, there was variation in both the number of segments per lesson and number of seconds per segment. Thus, to investigate trends and variation in responsiveness across lessons and classrooms, we weighted each segment's student contribution, teacher move, and corresponding compound code (e.g., considerable–low) according to the instructional time in seconds corresponding to each segment. We then calculated the percentage of whole-class mathematical discussion time accounted for by the respective code.

Illustrating the Framework with Excerpts

In this section we discuss three excerpts from the lessons we analyzed to illustrate different levels of student contributions and teacher moves described in the framework above (See Figure 2). In Excerpt 1, Teacher MA's 6th graders are exploring fractions using a number line on the whiteboard. After a student subdivides the interval from zero to one into 24 parts, Teacher MA asks her students about the representation. The student contributions provide little evidence of students sharing their ideas and consist entirely of short responses to known-information questions wherein the students are attempting to match particular responses predetermined by the teacher. Thus, this segment is characterized by minimal student contributions. For teacher moves, Teacher MA implicitly evaluates students by echoing students' responses and recording them on the whiteboard. When Tucker's response, "three," breaks from the format of the other student responses, Teacher MA corrects his response by providing additional information, "three twenty fourths." Thus, Excerpt 1 illustrates a segment characterized by low teacher moves. Excerpt 2 occurs in Teacher EC's 5th grade classroom, where Leena presents her strategy for determining how much candy each child would receive if five children shared eight candy bars. The student contributions provide evidence of students sharing their mathematical ideas and making sense of mathematical content. While Leena's description contains a hint of justification (e.g., "…*since* there are five students, I split it into five…"), the collective student contributions are best characterized as strategy sharing. By requesting that another student restate Leena's strategy, Teacher EC simultaneously focuses on Leena's thinking and asks other students to engage with her thinking in a nontrivial manner. Throughout the interaction with Jordan, Teacher EC asks additional questions that engage Jordan with Leena's strategy. Excerpt 2 illustrates a segment characterized by high teacher moves.

Excerpt 1: Minimal-Low	**Excerpt 2: Considerable-High**	**Excerpt 3: Substantive-Medium**
Tchr MA: So, Mitchell, what's this first line represent? *Mitchell:* One twenty fourth. [As students respond, Teacher MA writes labels on the number line.] *Tchr MA:* One twenty fourth. Victoria, the second line. *Victoria:* Two twenty fourths. *Tchr MA:* Um, Tucker, what's the next line? [overtalk] *Tucker:* Oh yes. [overtalk] Uh, three? *Tchr MA:* Three twenty fourths. Manny? *Manny:* Four twenty fourths. *Tchr MA:* Ida, the next line. *Ida:* Five twenty fourths. *Tchr MA:* Five twenty fourths. Uh, Nancy the next line. *Nancy:* Six twenty fourths. *Tchr MA:* Cam. *Cam:* Seven twenty fourths. *Tchr MA:* Judy. *Judy:* Eight twenty fourths. *Tchr MA:* Ron. *Ron:* Nine twenty four. *Tchr MA:* Gloria. *Gloria*: Ten twenty fourths. *Tchr MA:* Claire. *Claire:* Eleven twenty fourths. *Tchr MA:* Lily. *Lily:* Twelve twenty fourths. *Tchr MA:* Luke. *Luke:* Thirteen twenty fourths. *Tchr MA:* Uh, Jay. *Jay:* Uh, fourteen twenty fourths. *Alex:* Fifteen twenty fourths	*Leena*: I drew eight candy bars and since there are five students, I split it into five, so there's fifths. And then I colored in each color equals one person. So there's one fifth from every candy bar and all together that equals eight fifths and turned into a mixed number that's one and three fifths. *Tchr EC*: Very nice, very nice. So you broke each, oh, can someone explain what she just said? What did she just tell us? Anyone can put it in their own words. What did Leena just tell us? Can someone put it in their own words? Jordan? Want to give it a shot? *Jordan*: Um *Tchr EC*: I'll help you if you get stuck. Go ahead. What was the first thing that Leena did? *Jordan*: She drawed the eight candy bars. *Tchr EC*: She drew eight candy bars right. Okay, continue. *Jordan*: And then she *Tchr EC*: So she drew the four, eight candy bars and then what'd she do? *Jordan:* And she took, and then she got, then she like all *Jordan*: She divided them into five groups. *Tchr EC*: She divided them into five, groups or five pieces, yeah. *Jordan*: And then she gave each kid equal amounts but, um, *Tchr EC*: How much did she give each kid? *Jordan*: An equal amount. *Teacher EC*: An equal amount, okay. And what was the equal amount that she gave to each person? *Jordan:* Eight fifths.	*Tchr LE*: Mia said she's adding an eighth here and an eighth here and she got two sixteenths. Right Mia? [Mia nods] *Tchr LE*: Okay. Um, Jack. *Jack*: I always, uh, thought that you couldn't add up the denominators. *S48:* Yeah *S49:* Yeah *Tchr LE*: Why not? *Jack*: Uh, because it's *Jonathan*: The number would. The number would get lower. *Jack*: Oh yeah. The – it would get smaller. Because *S50*: The fraction *Tchr LE*: What it? What is it? *Jack*: The denominator. Like the whole number. Fiona: Fraction (inaudible) smaller. *Tchr LE*: But how does that relate to my picture? Well two sixteenths is the same as one eighth. Does – does my fraction get smaller? *Class*: No. *Joseph*: When you add eighths, it wouldn't go up. It would – you would go to a whole number so you wouldn't add – you would just like add, um, the – the top. You wouldn't add the bottom cause then you would never get a whole number. *Tchr LE*: What do you mean I'd never get the whole number? *Joseph*: Cause it would just keep adding on. *S51*: Yeah the denominator would [overtalking] *Jonathan*: Yeah it would just keep getting lower and lower and lower. [overtalking] *Joseph*: The denominator would keep getting. [ovetalking] *Gracie*: number but actually it would be getting smaller [overtalking] *Tchr LE*: Oh. If you kept adding the denominators then it would just – you'd never reach a whole.

Figure 2. Three excerpts illustrating the responsiveness framework.

In Excerpt 3, Teacher LE asks 5th graders to write equations related to their solutions for an equal sharing problem. When Teacher LE asks students to share their equations, Mia asserts that one-eighth plus one-eighth is two-sixteenths. The students in this excerpt participate in mathematical argumentation as they try to determine the validity of Mia's assertion. Jack generalizes the situation

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

to adding any denominators rather than considering only the particular equality in question. Throughout the segment, students collectively develop a fairly sophisticated informal proof by contradiction. Consequently, excerpt 3 is characterized by substantial student contributions. Teacher LE initially revoices Mia's claim. Her subsequent moves are predominantly probing questions rooted in understanding particular students' contributions (e.g., "What do you mean I'd never get the whole number?"). Although students engage with others' ideas throughout the excerpt, Teacher LE's moves do not direct students to engage with the ideas of others. As such, Excerpt 3 illustrates a segment characterized by medium teacher moves.

Findings and Implications

We now describe the findings from our analysis of the responsiveness of mathematics classroom discourse. We consider general trends across classrooms, variability in responsiveness both across and within classrooms, and the ways in which student contributions and teacher moves interacted in our data set. In some of the participating classrooms, the majority of the lesson was spent engaging in whole-class mathematics discussions, whereas in other classrooms students spent significant amounts of time in small groups or doing individual work. Across our classrooms, the percent of time spent in whole-class mathematics discussions ranged from 23 to 74% (mean of 49%). The analyses that follow are *only* for time spent in whole-class discussion. Figure 3 displays the distribution of the student contributions and teacher moves as a percentage of time spent during whole-class mathematics discussions in the different categories of our framework.

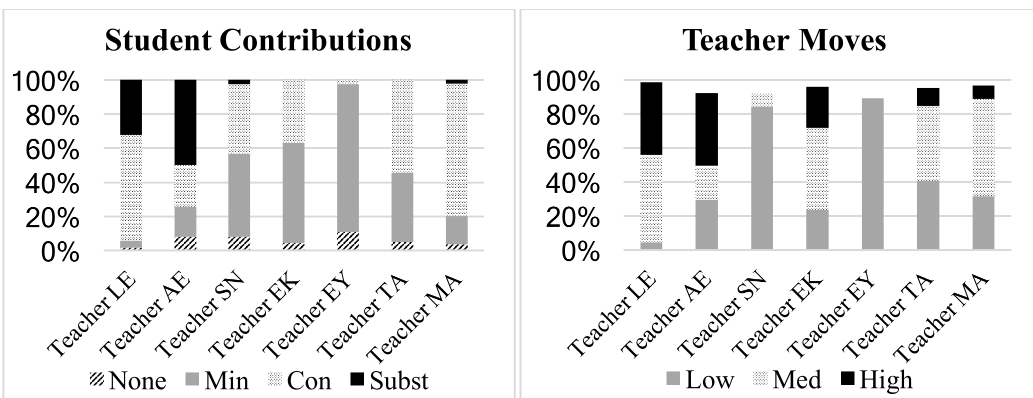

Figure 3. Percentage of time spent across the responsiveness framework categories.

We found that whole-class discussions were not dominated by teacher monologues, but that over 90% of the time students made mathematical contributions during whole-class discussions. In all but one of the classrooms, more than one-third of the time students made substantive or considerable contributions; and in four classrooms, over half the time students made substantive or considerable contributions. In most classrooms, students had opportunities to solve problems and explain, justify, or generalize their thinking during whole-class discussions. Moreover, in five of the classrooms, more than half of the teacher moves were medium or high level. These teachers not only created opportunities for students to engage with mathematics, but in many of the whole-class discussions they focused on and took up students' mathematical contributions. In addition to analyzing the student contributions and teacher moves independently, we also considered how our two framework components worked together during whole class discussions. Teacher AE's responsiveness profile is seen in Table 1.

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

Table 1: Responsiveness Profile for Teacher AE based on percent of time during whole-class discussions

			Teacher Moves			
			Low	Medium	High	*Sums*
Student Contributions	None	Minimal	17.8%	0%	0%	*17.8%*
	8.1%	Considerable	0%	2.5%	21.7%	*24.2%*
		Substantive	7.5%	17.4%	20.8%	*45.8%*
		Sums	*25.3%*	*20%*	*42.5%*	

Note that the most common compound code in Teacher AE's classroom was a considerable student contribution paired with high-level teacher moves, occurring just over one-fifth of the time during whole-class discussions. The second most common combination was substantive student contributions paired with high-level teacher moves, also occurring about one-fifth of the time. Thus, in roughly 40% of the time spent in whole-class discussion, Teacher AE supported students' engagement with their classmates' mathematical ideas in order to make sense of, explain, justify, critique, exemplify, or generalize. However, we also see a large percent of minimal-low interactions and segments of no student contributions (i.e., None) in this classroom.

After creating a similar table for each teacher, we found that for four of the classrooms, over 50% of the time spent in whole-class discussions involved considerable or substantive student contributions and medium or high teacher moves (see the shaded cells in Table 1). Each of these four combinations requires students to engage in important mathematical activities and uses the resultant student ideas as the basis for instruction. We noticed that in *all* classrooms, time was spent during whole-class conversation in minimal–low interactions (ranging from 4.4% to 87% of the time). This suggests to us that interactions at the lower levels of our framework are not necessarily negative, but that they play a role in whole-class discussion. However, we believe it is problematic if the majority of time spent in whole-class discussions falls into this category.

We also found that the responsiveness of classroom interactions varied. For example, in Figure 3 we see that the proportion of time teachers responded with high-level moves ranged from 0 to 46.5%, and the proportion of time teachers responded with low-level moves ranged from 0 to 89%. There was also large variability in student contributions with ranges from 3 to 78% and 0 to 46%, respectively, for considerable and substantive student contributions. Moreover, five teachers enacted teacher moves at all levels of the framework and four classrooms had student contributions at all levels suggesting that not only is variation in responsiveness present across classrooms but also within classrooms.

In summary, our data indicate that the kinds of instruction advocated for in existing literature is possible (e.g., Franke et al., 2009; Kazemi & Stipek, 2001; NCTM, 2014). In the interactions we analyzed we found that, for large portions of whole-class discussions, student ideas can be used to drive instruction and that there are multiple ways to be responsive to student thinking. We also found that mathematical interactions were not always in the highest categories for teacher moves and student contributions. Thus, at times, it seems appropriate and necessary to have teacher moves and student contributions at the lower levels of the framework. Additionally, our coding framework was able to adequately capture variation in responsiveness in middle grades mathematics classrooms. Given the variation present in our data, we encourage a variety of student contributions and teacher moves during whole-class discussions wherein a significant proportion of whole-class discussions are characterized by considerable or substantive student contributions and medium or high teacher moves.

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

In this paper we considered one aspect of mathematics classroom discourse, responsiveness to students' mathematical ideas, but acknowledge that this focus provides a narrow view of mathematics classroom discourse. We believe that there are other important discursive features that can and should be analyzed, and which would showcase our participating teachers differently. In the future we hope to apply our framework to additional lessons in our data and incorporate analyses of other aspects of classroom discourse.

Acknowledgments

This material is based upon work supported by the National Science Foundation under grant number DRL-1265677. Any opinions, findings, conclusions, and recommendations expressed in this material are those of the authors and do not necessarily reflect the views of NSF.

References

Bakhtin, M.M. (1986). *Speech genres and other late essays*. Austin: University of Texas Press.

Carpenter, T.P., Fennema, E., Peterson, P.L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499–531.

Franke, M.L., Webb, N.M., Chan, A.G., Ing, M., Freund, D., & Battey, D. (2009). Teacher questioning to elicit students' mathematical thinking in elementary school classrooms. *Journal of Teacher Education, 60*(4), 380.

Halliday, M.A.K. (1978). *Language as social semiotic: The social interpretation of language and meaning*. London: University Park Press.

Ing, M., Webb, N.M., Franke, M.L., Turrou, A.C., Wong, J., Shin, N., & Fernandez, C.H. (2015). Student participation in elementary mathematics classrooms: the missing link between teacher practices and student achievement? *Educational Studies in Mathematics*, *90*(3), 341–356.

Jacobs, V.R., Lamb, L.L.C., Philipp, R.A. (2010). Noticing of Children's Mathematical Thinking. *Journal for Research in Mathematics Education*, *41*(2), 169–202.

Kazemi, E. & Stipek, D. (2001). Promoting conceptual thinking in four upper-elementary mathematics classrooms. *The Elementary School Journal*, *102*(1), 59-80.

Leatham, K. R., Peterson, B. E., Stockero, S. L., & Van Zoest, L. R. (2015). Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. *Journal for Research in Mathematics Education*, *46*(1), 88–124.

National Council of Teachers of Mathematics. (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: Author.

Nystrand, M. Gamoran, A., Kachur, R. & Prendergast, C. (1997). *Opening dialogue: Understanding the dynamics of language and learning in the English classroom.* New York: Teachers College Press.

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms.* London: Routledge & Kegan Paul.

Przybyla-Kuchek, J., Hardison, H., & Bishop, J.P. (2015). A framework for teacher responsiveness. In T.G. Bartell, K.N. Bieda, R.T. Putnam, K. Bradfield, & H. Dominguez, (Eds.), *Proceedings of the 37th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (p. 1176). East Lansing: MI: Michigan State University.

van Zee, E. & Minstrell, J. (1997). Using questioning to guide student thinking. *The Journal of the Learning Sciences*, *6*(2), 227–269.