"RELATIONAL" EQUITY: ELEMENTARY STUDENTS CO-CONSTRUCT A SOCIAL-MATHEMATICAL POWER DYNAMIC DURING COLLABORATIVE ENGAGEMENT ON EQUIVALENCE TASKS

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We report a qualitative analysis of elementary school students engaged in collaborative problem solving involving mathematical equivalence tasks. We build on previous research showing that students often use strategies based on either operational or relational understandings of the equal sign. We closely analyze three cases and identify nuanced aspects of the social interaction that influence whether and how students develop and use operational or relational strategies toward a final solution. Students' demonstrated understandings of the equal sign during collaboration aligned with those identified in past research. We argue that a social-mathematical power dynamic was co-constructed in each of the dyads, and the ways students navigated that dynamic affected the quality of individual engagement and therefore learning.

Keywords: Cognition, Elementary School Education, Equity and Diversity, Problem Solving

Previous Literature on Mathematical Equivalence and Study Objectives

Understanding that the two sides of an equation must represent the same quantity is crucial for the development of algebraic reasoning. However, many children struggle to understand the equal sign as a *relational* symbol that connotes this equality relationship. Instead, children aged 7-11 often define the equal sign in *operational* terms, describing it as a cue to add all the numbers in the problem (McNeil, 2007; Perry, Church & Goldin-Meadow, 1988). Additionally, when children of this age solve equivalence problems such as 3 + 4 + 5 = 5 + __, they often add all of the numbers together and mark 17 as the final answer (McNeil, 2007; Perry et al., 1988).

One key to solving mathematical equivalence problems correctly may be noticing the atypical location of the equal sign in these problems. When children aged 7-11 are asked to reproduce equivalence problems from memory, they often place the equal sign at the *end* of the problem rather than in the *middle* (e.g., a child will recall the problem above as 3 + 4 + 5 + 5 = __; McNeil & Alibali, 2004). This particular pattern of errors may be due to the fact that children in the United States rarely practice arithmetic problems that do not take the typical form $a + b = c$ (McNeil et al., 2006). Extensive practice with this problem format may lead children to expect that the equal sign will always appear at the end of a problem (McNeil & Alibali, 2004). Accurate encoding of the location of the equal sign in mathematical equivalence problems is closely related to using strategies based on a relational understanding of the equal sign to solve these problems correctly (e.g., Crooks & Alibali, 2013; McNeil & Alibali, 2004).

Although the association between accurate encoding of the equal sign and use of correct strategies to solve equivalence problems is well documented, no research to date has investigated how noticing the location of the equal sign leads to the development of improved problem-solving strategies within a naturalistic setting. The current study is a secondary analysis of video data from a larger, ongoing study of children collaborating to solve equivalence problems (Brown & Alibali, 2015). Our goal was to identify nuanced aspects of social interaction that affected whether noticing the location of the equal sign influenced how children solve the problems. We present a qualitative micro-analysis of a series of selected transcript segments.

Wood, M. B., Turner, E. E., Civil, M., & Eli, J. A. (Eds.). (2016). *Proceedings of the 38th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. Tucson, AZ: The University of Arizona.

Methods

The data reported here were drawn from a larger project involving 38 pairs of second and third grade friends. Each pair participated in one 45-minute session in which they completed a pretest, collaboration episode, and posttest. During the pretest, participants worked individually to solve four mathematical equivalence problems and to complete tasks assessing their understanding of the equal sign as a relational symbol. Following this pretest, the pair worked together to solve two additional equivalence problems. The collaboration episode was filmed for later transcription and analysis. The posttest was similar to the pretest and was used to assess whether the children had developed any new strategies during the collaboration episode.

This report focuses on the collaboration episodes of three pairs of children. None of the six children solved any of the pretest problems correctly. Most of the incorrect strategies they used at pretest were consistent with an operational understanding of the equal sign. Our goal is to explore what happens when children notice the location of the equal sign in the problems. We selected one pair in which the children noticed the equal sign and began to generate a relational strategy, but did not use this correct strategy to solve the problems. We also selected two pairs in which the children noticed the equal sign, generated a relational strategy, and used this correct strategy. However, the latter two pairs differ greatly in the quality of their collaboration and in the amount of learning. Comparing these pairs illuminates how social-mathematical asymmetry can influence the outcomes of peer collaboration (Gutiérrez, in press).

Results

In the following sections, we present data from three pairs of children. Each pair was instructed to work together to solve two mathematical equivalence problems, but to write down their final answers and indicate their certainty about those answers by themselves. Children had a large sheet of paper on which they could write during their collaboration, and the problems were presented on smaller sheets. For each case, we first provide a brief overview to highlight certain points of the interaction, anticipating the transcription and line-by-line analysis that follows. (Note: due to space constraints, we provide partial transcriptions for some of the pairs.)

Pair 1: The case of Elsa and Morgan

Problem 1 (8 + 5 + 4 = 4 + ___). In this excerpt, Elsa attempts to influence the pair's work and control the environment from the outset. For example, in transcript Lines 1–4, Morgan moved the problem sheet closer to them but Elsa moved the problem to a different location. Morgan momentarily spoke up (Line 5), suggesting that she wanted to argue about the placement of the problem or about Elsa's edict, but she then acquiesced so they could move on. This tiny slice of interaction mirrors the broader episode, in which Morgan puts forth an idea and temporarily resists Elsa's opposing idea before giving in and adopting Elsa's strategy.

From a mathematical perspective, Elsa's actions strongly suggest that she did not perceive Problem 1 as an arithmetic problem at all. She argued that the problem was actually a numerical "pattern" (i.e., a sequence of numbers governed by an unknown rule that determines each entry), thus she ignored the plus signs and the equal sign as well. Morgan momentarily bought into the idea of a "pattern" (Line 9) but then quickly moved to a strategy based on a different conception of the problem—one that recognizes the plus signs and the equal sign (Lines 11–17).

In our analysis below, we argue that Morgan's strategy contained the seeds of an emerging relational strategy that both children could have profited from exploring further; however, these seeds ultimately went untilled due to Elsa's influence on the collaboration (Lines 18–21). Thus, an asymmetric power dynamic was co-constructed by Morgan and Elsa and an opportunity to enter into dialogue and productively explore a new (relational) strategy was missed.

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Morgan: [moves problem closer, orients it so that it's visible to them both]
Elsa: Can—can you not put that [Problem 1] on the paper? [moves Problem 1 to the center of the table]
Morgan: Yeah.
Elsa: Or how about there on the paper? That's good, on the paper.
Morgan: Wait! [glances at Elsa then looks back down at the problem; indicating each number on the left side in the problem with her marker as she speaks] eight plus five plus four equals—and that's the middle [indicates the equal sign], but [indicating the blank space "__"] four plus five plus eight wont FIT.
Elsa: No, what would be after that?
Morgan: A five?
Elsa: [briefly glances in the interviewer's direction, then back to the worksheet] Wai—ooh yeah! Would—it's a pattern!
Morgan: Ay, yeah!
Elsa: And we have to figure it out. It's eight, five, four [writes "8 5 4" on the big sheet]. Ooh three! I think.
Morgan: Wait, no! //Eight [writes "8" on the big sheet]..
Elsa: //I think it's three.//
Morgan: Five [continues writing, "8 5"]..wait!
Elsa: Four.
Morgan: Plus [writes a plus sign in her inscription, "8+5"], //plus four [continues writing, "8+5+4"]..
Elsa: //It's counting down.// [adjusts in her seat, leans in farther] It's counting down!
Morgan: Equals four plus five plus eight. [completes inscribing her equation as she talks, "8+5+4=4+5+8"]
Elsa: No we have to, we have to—the answer to the problem is [writes "3" at end of her string of numbers, as "8 5 4 3"] three.
Morgan: [gazes at Elsa's string of numbers "8 5 4 3"] Oh! [scribbles over and completely blacks out her equation]
Elsa: *Because* it's counting down. [writes "3" on her answer sheet]
Morgan: Oh! Eight, five, four, three. [writes "8 5 4 3" above her scribble, then puts down marker, picks up pencil, writes "3" on her answer sheet]

Morgan begins by reading the problem aloud, and pauses to note that the equal sign is in the middle of the problem, suggesting that the equal sign's location struck her as unexpected (Line 5). In this same turn, she expresses confusion, because she wants to balance the equation by repeating the numbers from the left side on the right, but there was only room for one number in the answer space (Line 5). This reveals her nascent understanding of the equal sign as a relational symbol; this idea could have led them to a correct strategy had they pursued it.

While Morgan is contemplating her emerging strategy, Elsa offers an alternative: that the problem is a "pattern" and they need to find the next item in the sequence (Lines 8, 10). Thus, Elsa does not view the string of characters as an arithmetic problem. At first, Morgan accepts this idea (Line 9), but then temporarily returns to her own strategy (Lines 11–17). Morgan's actions (Lines 15, 17) indicate a strong commitment to her emerging idea and getting it down on paper; she does not acquiesce to Elsa's attempts to get her attention and completes writing her equation as "8+5+4=4+5+8." However, this brief moment of agency is broken by Elsa's statement that the answer is three (Line 18). Morgan quickly agrees to Elsa's suggested answer, using her marker to cross out all the written work she had produced (Lines 19, 21). Morgan appears to be on the cusp of

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understanding the equal sign as a relational symbol, but because Elsa guides the interaction through her comments and actions—and because Morgan allowed her to do so—Morgan was persuaded to abandon her emerging understanding in favor of Elsa's pattern strategy which, in this context, was less effective.

Problem 2 (9 + 7 + 5 = ___ + 9). On the second problem, Morgan made statements that assert her status as an equal collaborator (Line 23), yet her contribution still went unrecognized by Elsa. Thus, the emerging social relation that was being co-constructed remained asymmetric. From a mathematical perspective, they proposed a final solution that was based on the same "pattern" game as for Problem 1 and again ignored both the plus signs and the equal sign.

Elsa: Nine... uh. [giggling] I have no idea.. What's nine plus seven plus—
Morgan: Wait! This is question number two. Since the other one was going *down*, this one might be going UP.
Elsa: No, because see [indicating numerals on the problem sheet] nine, seven—oh counting by twos? No counting by *odd* numbers.
Morgan: Yeah!

Interestingly, this time it is Elsa who initially suggests a summation strategy (Line 22), and Morgan reminds her that they are looking for a pattern. Once reminded, Elsa quickly returns to the pattern approach, but also quickly rejects Morgan's suggestion that the pattern "might be going up" (Lines 23 & 24). Both have fully adopted Elsa's idea that the goal is to find a pattern within the sequence of numbers, and neither child references addition or the equal sign. Elsa continues to drive the interaction, suggesting a series of possible patterns, and proposes a solution of "3"—the next odd number, counting down—that Morgan immediately takes up.

After the collaboration episode, when the children solve equivalence problems individually on the posttest, neither child used a correct, relational strategy. Thus, neither seems to have benefited from the collaboration, despite the fact Morgan appeared to have the seeds of a relational strategy. We argue that the nature of their collaborative interaction made it impossible for this nascent correct strategy to fully emerge and be beneficial for either child.

Pair 2: The Case of Dylan and Shawn

Problem 1 (8 + 5 + 4 = 4 + ___). Dylan and Shawn represent another case of an asymmetric power dynamic. Just prior to the excerpt below, both children made statements about potential solutions, however the conversation flowed mostly in one direction. Dylan acknowledged Shawn's propositions but Shawn did not give any indication that he considered Dylan's ideas. Shawn kept his gaze on the problem, whereas Dylan made several attempts to make eye contact, suggesting that he wished to check in with Shawn as they went along, but this never occurred. Shawn and Dylan essentially worked separately (as indicated by overlapping speech and unacknowledged turns of talk), each developing his own understanding of the problem. Shawn eventually articulated a solution based on a relational understanding of the equal sign (Line 26), placing Dylan in a position to "buy in" to Shawn's strategy and abandon his own approach (Line 27). The only time Shawn looked away from the problem was to assert his proposed solutions.

Shawn: [referring to the left side of the equation] So that's 17. [leaves tip of pencil on left "8"; glances in Dylan's direction] Four plus *what* equals 17?
Dylan: Four plus what? Four plus... well// [looks at Shawn] it's four so of course there's ten.
Shawn: //Four.// [quickly glances up in Dylan's direction] Four plus three!
Dylan: Wait! What?! [lifts gaze up, staring out at nothing in particular]

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Shawn: [glances up at Dylan] Four plus 13! [reaches for problem, orients it so that he can write "13" on it]
Dylan: Four plus—oh yeah! Yeah! Plus 13.

At the outset of their collaboration, Shawn immediately begins adding the numbers on the left side of the equation, while Dylan's attention is drawn to the middle of the equation, suggesting that the structure of the problem is unfamiliar to him. When Shawn offers a strategy based on a relational understanding of the equal sign (Line 26), Dylan leaves his train of thought and attempts to follow Shawn's (Line 27). Dylan's final utterances (Lines 31) suggest that he agrees with Shawn's relational strategy; however, he may simply be appropriating Shawn's method without the relational understanding of the equal sign that undergirds it (Line 31). As Dylan records his final answer, his facial expressions, posture, and tone suggest that he is still uncertain about their final solution, despite the fact that he acquiesces to Shawn.

Problem 2 (9 + 7 + 5 = ___ + 9). The opening moments of Dylan and Shawn's interaction with Problem 2 showed great promise of authentic and equitable collaboration. They worked in tandem for several turns of talk, and successfully determined that the left side of the equation sum to 21. However, in approaching the right side, this emerging intersubjectivity broke down, and they began to work separately again. Shawn used the relational strategy they had used on Problem 1, whereas Dylan offered a strategy, based on an operational conception, of adding all the numbers: "Oh! That would be.. that would be 21 [referring to the left side] and then plus nine [referring to the right "9"]. So.. hmm." This cognitive–conceptual divergence resulted in a communication break-down that was not successfully repaired. For the remainder of their interaction, Dylan attempted to make sense of what Shawn had proposed, but to no avail. In the end, the pair offered the final answer of "12" that was proposed by Shawn. Despite his agreement, Dylan's speech and nonverbal behavior suggest that he was not convinced; he appeared dejected when he gave in to Shawn. Even as they wrote down "12" on their worksheets, Dylan quietly says that he thinks the answer might be something different.

In both excerpts involving Shawn and Dylan, they both reached correct solutions using a relational strategy. However, the social power dynamic that was simultaneously co-constructed, in and through their discursive productions, was asymmetric and, moreover, was not beneficial for both children. When Dylan and Shawn solved additional problems individually on the posttest, Shawn solved all four problems correctly, while Dylan solved only one correctly. This outcome suggests that, although Shawn gained insight into the underlying structure of the problems and how to approach them effectively, the *quality* of his engagement with a peer left much to be desired from a *relational equity* perspective (Boaler, 2008) (see below).

Pair 3: The Case of Marie and Jenny

Problem 1 (8 + 5 + 4 = 4 + ___). Marie and Jenny both immediately interpreted the first problem as involving arithmetic, and together they added the left side to arrive at 17. Moving beyond this point proved to be challenging; they worked in tandem to add the left side but reached an impasse when they arrived at the equal sign. They could not easily reconcile how the left side, which they both agreed was 17, was equal to "4", and the extra "blank" on the right side was also lost on them. They seemed on the cusp of giving up, but instead reexamined the problem. This process led Marie to see the equation in a new way and she proposed a relational solution strategy. The transcription below begins as Marie launched excitedly into an elaborate explanation in which she used both speech and gesture to communicate her newfound conceptualization to Jenny. Jenny, in turn, attended to Marie's speech and gestures and took up what Marie was attempting to explain (Lines 32–34). Together they arrived at the correct final solution based on a *shared* relational understanding. They worked mostly in tandem, sometimes interrupting one another and in some cases finishing one another's sentences.

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Marie: Four. No.. Maybe it's like [leans back], so that's 17 [indicates left "8"; at this point, her speech speeds up and she speaks excitedly] so four [indicates the "4" on the right side] plus [indicates plus sign] what [indicates "blank"] would make it seventeen [both hands open, palms facing up and slightly towards each other, gesturing to left side of gesture space] also [shifts her hands to right side of gesture space], which makes it equal together. [Brings hands together, palms toward her chest, tips of fingers of the two hands touch]

Jenny: I don't get what you're saying.

Marie: So like.. that's 17 together [gesture underlines left side with pencil] so four [points with her whole hand to the RS "4"] plus what [point to the blank] would make that 17 [fingers and thumb bunched together in a "wide pinch" position, gestured at the right side] so that these two [index points to the left "4" then the right "4"] are equal together [hands open, fingers spread, palms toward each other as if holding something]? So since this is 17 together [drags hand across entire left side of equation, her hand in a pinch shape, with her thumb underlining the equation and her other fingers tracing above the equation]..

Jenny: Oh.. [releases tension she was holding in her posture then leans back]

Marie: Four [indicates right "4"] plus what [uses same dragging gesture as before, on right side] makes that equal together? Like, 17 [indicates left side with pencil] plus four [indicates right "4"] makes 17 [indicates blank],// makes it equal [waves hands, both open and facing down, back and forth above the whole equation]?

Jenny: //So should we// write 13? [Poises pencil above the blank, looks at Marie]

When Jenny expresses confusion about Marie's emerging relational strategy (Line 33), Marie offers an in-depth explanation. Jenny understands the strategy after this explanation and even offers the answer before Marie does (Line 37).

Problem 2 (9 + 7 + 5 = ___ + 9). Unlike Shawn and Dylan, the strategy that was articulated by Marie and Jenny during Problem 1 was robust enough that they were able to maintain it as a shared strategy and use it to solve Problem 2. Marie and Jenny were now acting in concert, sharing the discursive space, and this resulted in an efficient, authentic, and equitable interaction centered on a shared task. They approached Problem 2 enthusiastically and expeditiously, both stating at the same time, "Like we did it last time." Marie again used complex gestures and speech to articulate a relational strategy that was now shared with Jenny (Line 38).

Marie: Well, let's count it up. So seven plus five [covers up left "9" with her left thumb, and points to left "7" and left "5" with her right hand], what does that [cup shape under the left "7 + 5," grouping the numbers together], cause the nine is already used [indicates left "9" then the right "9," then she covers both nines, one with each hand]. So [indicates left "7 + 5"] seven plus five equals?

Both Marie and Jenny went on to solve all four individual posttest problems correctly after the collaboration episode.

Summative Comments Across All Three Pairs

All three dyads notice the location of the equal sign in the problems, and one child in each pair shows at least partial knowledge of the equal sign as a relational symbol. Despite these similarities, the three dyads differ greatly. Elsa does not consider Morgan's relational strategy, but rather asserts her own pattern strategy—one that lives outside of the realm of arithmetic and mathematical equivalence. She ultimately persuades Morgan to abandon her line of thinking in favor of the pattern strategy. In contrast, Shawn and Dylan eventually agree to use a relational strategy to solve the problems during the collaboration episode. However, both of these dyads experience a similar type of asymmetry in their interactions. While Elsa persuades Morgan to abandon a correct line of thinking, Shawn forges ahead with his relational strategy and Dylan eventually accepts Shawn's answer despite not understanding it. Finally, although Marie and Jenny have a brief moment of asymmetry when Marie first offers her relational strategy and Jenny expresses confusion, the way these children navigate the asymmetry leads to both children learning. Marie works to help Jenny join her in her understanding so they can move forward together. This is in contrast to Shawn, who seems to drag Dylan along despite his protests. Marie and Jenny both make an effort to get on—and *stay on*—the same page, something that was missing in the other dyads, in which one of the two children was always leading the way.

Discussion and Implications for "Relational" Equity

In the spirit of the conference theme, "without borders" (sin fronteras), the analysis presented here is a first step toward bridging educational psychology and mathematics education research, with a focus on both the conceptual challenges that elementary students face when dealing with a certain genre of mathematical activity (equivalence problems), and the social aspects of collaboration with a peer. Our main finding, in broad strokes, is that students navigated an emergent power dynamic, and this dynamic in turn affected the quality of engagement and consequently, their learning. From a psychological perspective, this finding bears on classic questions about where new ideas come from and why they are (or are not) taken up. Our findings support the hypothesis that noticing the atypical location of the equal sign in mathematical equivalence problems can lead to the development of relational strategies, but our findings also suggest that the social context in which the act of noticing occurs may partially determine whether students develop and use a relational strategy.

From a mathematics education perspective, this finding bears on issues of equity and inclusion. Equity in mathematics education can be conceptualized in terms of issues related to status hierarchies, participation, and identity. Most relevant to this report is Boaler's (2008) notion of *relational equity* (where "relational" refers to "social relationships" not "relational understandings of the equal sign"—but we appreciate the coincidence in terminology). Boaler defines relational equity as "equitable [social] relations in classrooms; relations that include students treating each other with respect and considering different viewpoints fairly" (pg. 168). Boaler proposes that we focus on the *quality* of students' interactions during collaboration. We find her notion of relational equity useful for articulating the implications of our findings.

Boaler (2008) argues that when students come together to collaborate on mathematical tasks, they are not only learning content and concepts, but they are also learning and reifying values such as respect and responsibility. Specifically, Boaler assumes "that the ways students learn to treat each other and the respect they learn to form for each other will impact on the opportunities they extend to others in their lives in and beyond school" (pg. 168). We agree.

It is important to note that Boaler's notion of relational equity was developed in the context of a diverse, urban high school and refers to the ways in which students interacted with others from different social classes, cultural groups, and ability levels. Our data come from an educational psychology study involving clinical interviews, and our participants were demographically homogeneous. That said, we nevertheless propose a corollary to Boaler's definition: we argue that relational equity is contingent on an emergent social-mathematical power dynamic that is co-constructed, in situ, via students' actions and discourse (Gutiérrez, in press). Boaler, too, sees that (asymmetric) power dynamics play a role in mathematics learning:

> A common problem in the enactment of group work is an uneven distribution of work and responsibility among students, with some students doing more of the work and others choosing to opt out or being forced out of discussions." (Boaler, 2008, pg. 171)

This definition applies to the ways the children in our study responded to one another's mathematical perceptions and actions. Morgan's and Dylan's discursive contributions, for example,

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were not taken up and discussed with their partners. In a sense, their personal meanings of the equal sign were “forced out” of the semiotic space. The lack of relational equity in these two cases is striking when compared to the case of Marie and Jenny. In this pair, Marie sensitively responded to fact that Jenny was falling behind. Moreover, when Marie viewed the equal sign as a relational symbol before Jenny, there was a possibility of another social-mathematical hierarchy emerging, yet Marie demonstrated a strong commitment to Jenny’s learning, as indicated by her use of an elaborate array of communicative means. This kind of complex communication, commitment, and respect was not found with Dylan and Shawn, nor with Morgan and Elsa, which leaves us with two very different versions of “relational” equity.

Acknowledgments

This research was supported by the U.S. Dept. of Education, through I.E.S. awards #R305B090009 and #R305B130007 to the Univ. of Wisconsin-Madison. The opinions expressed are those of the authors and do not represent the views of the U.S. Department of Education.

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