THE BENEFITS OF FEEDBACK ON COMPUTER-BASED ALGEBRA HOMEWORK

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Math homework is transforming at a rapid rate with the continuous advances in educational technology. Computer-based homework, in particular, is gaining popularity across a range of schools, with little empirical evidence on how to optimize student learning. In the current study, middle school students (N = 143) solved a set of challenging algebraic problems on a computer-based homework assignment and received (1) no feedback, (2) correct-answer feedback, (3) try-again feedback, or (4) explanation feedback after each problem. Feedback resulted in higher posttest scores than no feedback, and this was true regardless of feedback type. On transfer items, correct-answer feedback has positive effects for low-knowledge students, but neutral effects for higher-knowledge students. Results suggest the provision of basic feedback on computer-based homework can benefit novice students' mathematics problem solving.

Keywords: Problem Solving, Algebra and Algebraic Thinking, Instructional Activities and Practices, Technology

Introduction

Modern advances in educational technology and increasing access to computers gives math teachers a wide range of tools for assigning homework and assessing students' problem-solving skills. Indeed, intelligent tutor systems and computer-based homework are quickly gaining popularity and prevalence in math classrooms across the world, necessitating research that crosses the borders of math education, cognitive science, and educational technology. Many of these computer systems are designed to enhance student learning through the use of targeted problem solving and feedback. However, recent evidence suggests there may be potential consequences of providing feedback during problem solving, especially for learners with higher prior knowledge (e.g., Fyfe & Rittle-Johnson, 2016). The goal of the current research was to evaluate the effects of different types of feedback on computer-based algebra homework for middle-school students. The selection of algebra was motivated by the recognition that algebra is a gatekeeper to future educational and employment opportunities (Adelman, 2006), and by concerns about students' inadequate understanding of and preparation in algebra (NMAP, 2008).

Theoretical Framework

In general, math education research supports the use of feedback during problem solving. Meta-analyses show that, on average, feedback has positive effects on learning outcomes relative to no feedback (Hattie & Timperley, 2007). Indeed, Hattie and Timperley (2007) claim that "feedback is one of the most powerful influences on learning and achievement" (p. 81). They even identify feedback as one of the top ten influences on student achievement, along with direct instruction and reciprocal teaching. However, the effects of feedback vary widely (Kluger & DeNisi, 1996) suggesting that certain types of feedback may be more effective than others. In fact, during mathematics problem solving, students can sometimes learn just as much, if not more, when no feedback is provided (Fyfe & Rittle-Johnson, 2016; Nussbaumer et al., 2008).

A growing body of evidence suggests that some of the variability in feedback effects is due to students' prior knowledge (e.g., Fyfe, Rittle-Johnson & DeCaro, 2012; Krause et al., 2009). Specifically, feedback often has strong, positive effects for students with lower prior knowledge, but neutral or negative effects for students with higher prior knowledge.

Theoretically, there are several reasons why feedback may hinder problem solving relative to no feedback. These reasons are related to the students' perception of the feedback, including their affective and cognitive processes. For example, feedback may draw attention to the self and elicit affective reactions that interfere with learning (Kluger & DeNisi, 1996). For example, feedback on incorrect answers can produce ego-threat (i.e., a threat to one's positive self-image), which may reduce one's confidence or motivation to continue. Students with higher prior knowledge may be especially affected by ego-threat as they likely have some expectation of performing well. Another possibility is that feedback overloads cognitive resources simply by providing additional information that needs integrated with the students' existing knowledge (Sweller, van Merrienboer, & Paas, 1998). For example, feedback can disrupt a student's internal processing of the task and ultimately hinder his ability to learn from it.

One key factor to consider may be the type of feedback provided. Dempsey et al. (1993) outlined a hierarchy of feedback types based on the information provided:

1. *No feedback*: provides no information about the student's response.
2. *Verification feedback*: informs the student if the response if correct or incorrect.
3. Correct-answer feedback: informs the student what the correct answer is.
4. Elaborated feedback: provides some explanation for why a response is correct.
5. Try-again feedback: allows one or more additional attempts to try again.

One possibility is that providing feedback with more information will have positive effects for both low- and high-knowledge students because it provides helpful information for moving forward. However, providing more information may also have consequences because it is more likely to overload cognitive resources. There is some consensus that effective feedback should at least provide the correct answer (Kluger & DeNisi, 1996). But, the benefits of extra information are less clear (see Mory, 2004), particularly in the realm of computer-based homework.

Computer-based math homework has several potential advantages relative to traditional paper-and-pencil homework, including the provision of immediate feedback to students on their performance. The goal of the current study was to evaluate the effects of this feedback using a particular system, ASSISTments.org (Heffernan & Heffernan, 2014). ASSISTments is a computer tutor system that can provide scaffolds and feedback to assist student. The use of computer-based homework offers several advantages for understanding the effects of feedback.

First, computer-based homework provides an ecologically valid context in which to evaluate the role of feedback on problem solving. Many prior feedback studies have been conducted in laboratory contexts in the presence of a researcher. Here, we test the effects of feedback in an authentic learning setting on homework assignments given to students by their teachers. Second, computer-based homework represents a learning setting that may reduce the negative effects of feedback. As mentioned above, one condition under which feedback may hinder learning is when it draws attention to the self as opposed to the task (Kluger & DeNisi, 1996). Attention on the self can invoke evaluations of one's abilities that interfere with the task. Computer-generated feedback is often viewed as a less evaluative source of information than person-generated feedback (Karabenick & Knapp, 1988), and may help decrease attention on the self and increase attention on the task. Computer-based homework may also reduce overload of cognitive resources by giving students control over when and how they process the feedback.

Current Study

The current study tested the effect of different types of feedback for middle school students solving algebra problems on computer-based homework via the ASSISTments system. Teachers and students who were already using ASSISTments were recruited so that the system was familiar to the

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students and part of their regular classroom experience. Students were assigned to receive no feedback, correct-answer feedback, explanation feedback, or try-again feedback. Based on previous research, feedback was predicted to interact with prior knowledge such that feedback would have stronger, positive effects on problem solving for students with lower prior knowledge than for students with higher prior knowledge.

Method

Participants

All students from two sixth-grade teachers' classrooms and two seventh-grade teachers' classrooms were invited to participate. The teachers taught at three different schools and were using ASSISTments.org as part of their regular classroom experience. Of their 160 students, 17 students were not included in the study as they did not complete all required sessions. The final sample contained 143 students (65 in sixth-grade and 78 in seventh-grade).

Materials

All materials were presented using ASSISTments.org. The ***pretest*** included six algebraic equations to solve (see Table 1). There were four different problems types: $ax + b = c$, $b + ax = c$, $a(x + b) = c$, and $a(x + b) + c = d$. The ***homework assignment*** contained two worked examples at the beginning to re-familiarize students with correct problem-solving solutions (see Figure 1 for an example). The remaining problems were equations for students to solve on their own. Students solved 12 or 16 problems (i.e., three or four of each type of problem presented on the pretest). Whether students solved 12 or 16 problems reflected natural variation in teacher preference as two teachers opted for the 16-problem assignment (n = 65 students) and two teachers requested a shorter 12-problem assignment (n = 78 students). Percent correct scores were calculated for each student based on the number of items he or she was assigned. The ***posttest*** included eight equations to solve (see Table 1). Four items were isomorphic to the pretest problems (i.e., learning items) and the remaining four were challenge problems with novel problem structures (i.e., transfer items). Percent correct for each subscale was calculated.

Table 1: Problems Presented on the Pretest and Posttest

	Pretest	Posttest
1.	2x + 3 = 23	5x + 13 = 73
2.	10 + 5x = 30	3 + 6x = 99
3.	3(x + 1) = 9	7(x + 4) = 63
4.	7(x + 3) + 2 = 51	2(x + 3) + 4 = 16
5.	5(x + 3) + 14 = 64	x/2 + 3 = 13
6.	8(x + 2) = 56	4(x + 2) + 3(x + 2) = 35
7.	--	x/9 + 31 = 34
8.	--	6(x + 4) + 2(x +4) = 48

Note. On the posttest, problems 1 through 4 are learning items and 5 through 8 are transfer items.

Design and Procedure

The experiment had a pretest-homework-posttest design. Students completed the pretest during class or at home. Within three school days, students completed the homework assignment on their own. For the homework assignment, students were randomly assigned to one of four conditions: no-feedback (n = 25), correct-answer feedback (n = 44), explanation feedback (n = 41), or try-again

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feedback (n = 33). Finally, students completed the posttest. All teachers assigned the posttest the same day students finished the homework, but three teachers had students complete it in class and one teacher had students complete it at home.

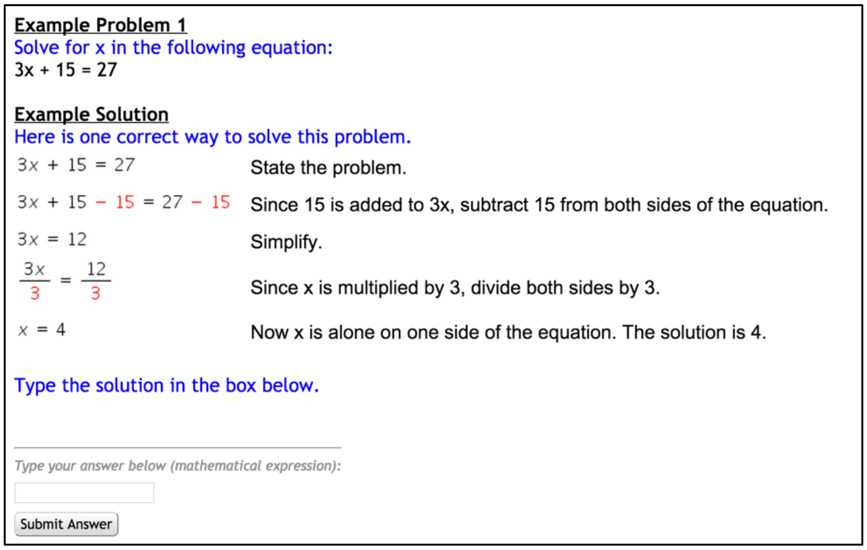

Figure 1. Worked example. An example given at the beginning of the homework assignment.

In the *no-feedback* condition, students did not receive any feedback during the assignment. After submitting each answer, the computer provided an "answer recorded" message and students clicked a button to move on to the next problem. In the *correct-answer feedback* condition, students received immediate, correct-answer feedback after each problem. If students typed the correct answer, a green check mark appeared with the word "Correct!" If students typed an incorrect answer, a red X appeared along with the words, "The correct answer is __" (with the correct answer filled in). In the *explanation feedback* condition, the feedback message included the correct answer, an explanation of why it was correct, and a worked example. For example, for $3x + 12 = 24$, the feedback message stated:

> The correct answer is 4 because when x is 4 both sides of the equal sign have the same amount. Let's plug 4 in for x and simplify to show that both sides have the same amount.
>
> $3x + 12 = 24$
>
> $3*4 + 12 = 24$
>
> $12 + 12 = 24$
>
> $24 = 24$

In the *try-again feedback* condition, the feedback message stated, "Sorry, try again. __ is not correct" (with the student's answer filled in). Students could continue inputting responses until they entered the correct answer or they could obtain the correct answer by clicking on a button.

Data Analysis

To examine the impact of feedback and prior knowledge, regression analyses were used for each outcome measure. Condition was dummy coded with correct-answer feedback, explanation-feedback, and try-again feedback entered into the models (with no-feedback as the reference group). Thus, each regression model included three condition variables, pretest score (mean centered), and three condition by pretest score interactions.

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Results

Pretest

On average, children solved 85% of the pretest problems correctly (SD = 23%). Scores did not differ significantly by condition, $F(3, 139) = 1.09$, $p = .36$. Performance tended to be similar across all six items with percent correct on each item ranging from 80% to 92%.

Homework

The regression predicting homework scores from condition and prior knowledge was significant, $F(7, 135) = 11.64$, $p < .000$, $R^2 = .38$, but the only individual predictor to reach significance was the main effect of prior knowledge, $B = 0.53$, $SE = 0.16$, $p = .002$. Students with higher prior knowledge on the pretest exhibited higher homework scores than students with lower prior knowledge. There were no main effects of feedback or feedback by prior knowledge interactions, ps > .10. Thus, feedback had little effect on performance during the homework.

However, qualitative evidence suggested that children were learning over the course of the assignment. For example, four problems were of the form $a(x + b) = c$. Only 77% of students solved the first of these problems correctly, but 88% solved the fourth of these problems correctly. Similarly, four problems were of the form $a + bx = c$. Only 80% of students solved the first one of these correctly, but 92% solved the final one correctly. Further, on earlier problems, errors tended to reflect common misconceptions about variables, whereas errors on later problems were more diverse, suggesting that students were at least attempting correct strategies later in the assignment. For example, the very first problem was $3x + 12 = 24$. Two of the most common incorrect answers were 1.6 or 9. Students added the coefficient (3) and the isolated number (12) to get 15x on the left side. Then, they either calculated 24 divided by 15 or 24 minus 15 to get their answer. However, one of the last problems was $8 + 2x = 40$, and no student showed evidence of making the mistake of adding the 8 and the 2. Thus, students appeared to improve over the course of the homework assignment, but not differentially by condition.

Posttest Learning Items

Students did well on the posttest learning items, solving nearly 90% correct. Indeed, on 3 of the 4 problems students were near mastery. The fourth problem, $2(x + 3) + 4 = 16$, proved somewhat difficult with only 79% of students solving it correctly. Estimates from the regression predicting percent correct on the learning items are presented in Figure 2. The overall regression was significant, $F(7, 135) = 9.22$, $p < .000$, $R^2 = .32$. There was a significant, positive effect of prior knowledge, $B = 0.36$, $SE = 0.18$, $p = .04$. However, there were also significant main effects of correct-answer feedback, $B = 8.91$, $SE = 4.33$, $p = .04$, and try-again feedback, $B = 13.51$, $SE = 4.58$, $p = .004$, as well as a marginal effect of explanation feedback, $B = 7.40$, $SE = 4.35$, $p = .09$. Although prior knowledge did not significantly interact with any feedback type, ps > .10, an examination of Figure 2 suggests that the effect of explanation feedback was only marginal because it was primarily effective for high-knowledge children, but not low-knowledge children. Descriptively, we also compared the percent of children at mastery by condition. Fewer children in the no-feedback condition (36%) solved all four learning items correctly compared to children who received correct-answer feedback (66%), explanation feedback (66%), and try-again feedback (72%). Thus, feedback boosted learning on the posttest relative to no feedback.

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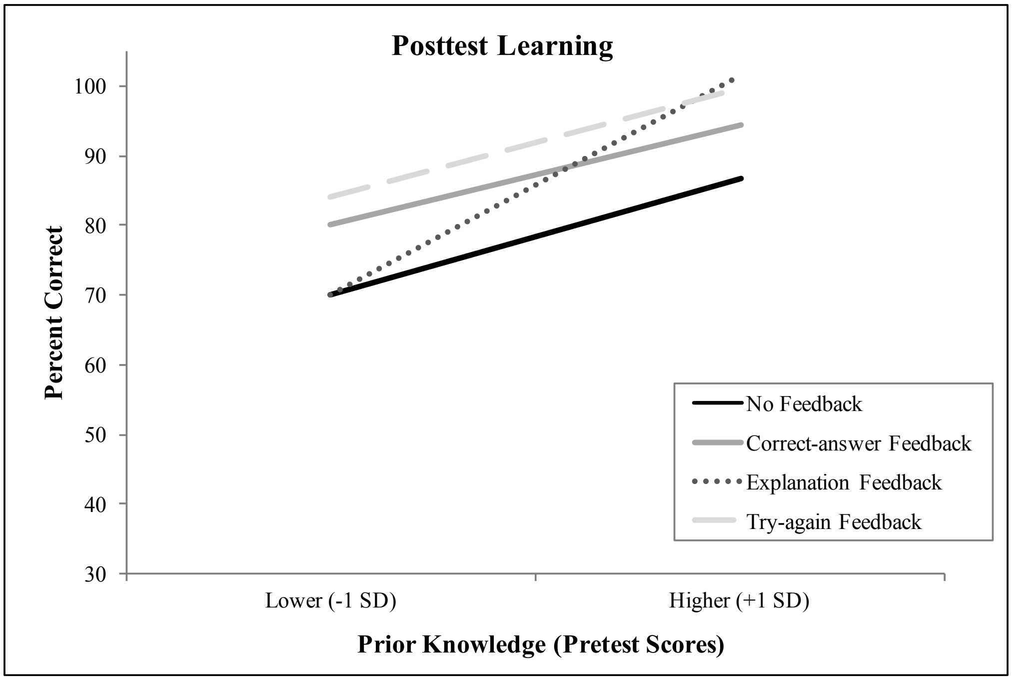

Figure 2. Posttest scores. Scores on the learning items by condition and prior knowledge. Unstandardized regression coefficients are plotted at ±1 standard deviation from the mean.

Posttest Transfer Items

Students struggled on the posttest transfer items, solving only 64% correct on average. This is expected given that these were novel problem types, which students did not practice. Although all four problems were difficult, the problems with two separate terms in parentheses, like 4(x + 2) + 3(x + 2) = 35, were the most challenging. Estimates from the regression predicting percent correct on the transfer items are presented in Figure 3. The overall regression was significant, $F(7, 135) = 5.10$, $p < .000$, $R^2 = .21$. There was a significant positive effect of prior knowledge, $B = 1.12$, $SE = 0.33$, $p = .001$, but no main effects of feedback condition, *p*s > .20. However, there was a significant interaction between prior knowledge and the no-feedback vs. correct-answer feedback contrast, $B = -0.79$, $SE = 0.40$, $p = .05$. To examine the interaction, pretest scores were centered at one standard deviation below the mean in one model (i.e., low prior knowledge) and one standard deviation above the mean in a separate model (i.e., high prior knowledge). For low-knowledge students, there was a significant positive effect of correct-answer feedback, $B = 28.35$, $SE = 12.54$, $p = .03$. The effects of explanation feedback, $p = .14$, and try-again feedback, $p = .35$, were positive, but not statistically significant. However, for high-knowledge students, there were negative, but non-significant effect of correct-answer feedback, $p = .49$, explanation feedback, $p = .40$, and try-again feedback, $p = .40$ (see Figure 3). Thus, the effects of feedback on transfer to novel problems depended on children's prior knowledge on the pretest. Feedback (particularly correct-answer feedback) resulted in better transfer than no feedback for low-knowledge children. But, all three types of feedback resulted in slightly lower transfer scores relative to no feedback for high-knowledge children.

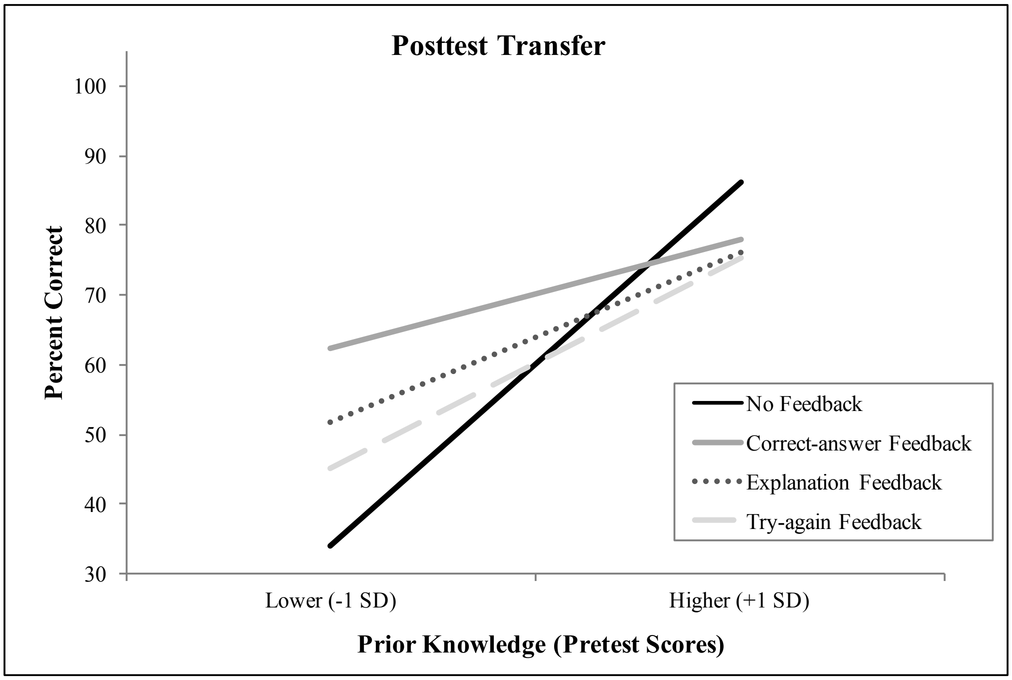

Figure 3: Posttest scores. Scores on the transfer items by condition and prior knowledge. Unstandardized regression coefficients are plotted at ±1 standard deviation from the mean.

Discussion

The goal of this study was to test the effects of feedback and prior knowledge on computer-based algebra homework using the ASSISTments system. Middle school students were assigned to receive no feedback, correct-answer feedback, explanation feedback, or try-again feedback during problem solving. After a single assignment, feedback resulted in higher posttest learning scores than no feedback, and this was true regardless of feedback type. On transfer items, feedback interacted with prior knowledge, such that correct-answer feedback has positive effects for low-knowledge students, but neutral effects for higher-knowledge students.

The findings from the current study make at least three contributions to research in mathematics education. First, the results demonstrate the benefits of three different types of feedback on problem solving. Further, they suggest that providing additional information or attempts does not always increase the efficacy of feedback. In fact, basic correct-answer feedback resulted in the best transfer for low-knowledge students, suggesting a possible threshold account (Phye, 1979). That is, when more information or support is provided beyond what is needed, it does not provide any additional advantage. Second, the results indicate that the benefits of feedback are strong for low-knowledge students, but that high-knowledge students sometimes do just as well without feedback during problem solving. Indeed, on posttest transfer, high-knowledge students tended to exhibit higher scores in the no-feedback condition, which is consistent with recent research (Fyfe & Rittle-Johnson, 2016; Krause et al., 2009). The third contribution is to introduce an exciting new method to conduct experimental research in an ecologically valid classroom. The ASSISTments project is a system bringing researchers and teachers together to better assist and assess student learning (Heffernan & Heffernan, 2014).

Future research is needed to test different types and schedules of feedback that are more dynamic and that adjust based on the student response. Indeed, one of the benefits of computer-based homework is the possibility of adapting the provision of feedback to students' needs. Further, more work is needed to enhance the provision of feedback for high-knowledge students. The high-knowledge students in the current study did not benefit from feedback, but still had room to grow.

One potential solution is to give high-knowledge students more control over the feedback (Hays et al., 2010), allowing them to skip unnecessary feedback and spend more time on challenge problems. Future studies should explore these and other possibilities.

In general, the current research is at the border of mathematics education, cognitive science, and educational technology. The specific results highlight the power and variability of feedback effects on computer-based homework. On the one hand, researchers and educators can marvel that such basic feedback can improve problem solving for novice students. On the other hand, these results challenge the intuition that feedback is always helpful and suggest that some students, under certain circumstances, can do just as well without it.

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