"A BANK ON EVERY CORNER": STUDENTS' SENSE OF PLACE IN ANALYZING SPATIAL DATA

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This paper describes the role of sense of place in students' analysis of spatial data toward understanding the distribution of financial services in their city. High school students participated in a 10-session module about their city's two-tiered financial system of banks and alternative financial institutions. The paper analyzes two class sessions organized around the use of ratios, or intensive variables, to understand the distribution: an embodied distribution activity atop a large floor map and individual exploration of scalable, data-rich digital maps. Analysis investigates the role of students' sense of place as they grappled with these ratios. Students drew on their senses of place to interpret data and generate their own sets to associate in ratios. Abstract measures and data visualizations contained in the digital maps were less accessible to students in this iteration of the module.

Keywords: Equity and Diversity

The concept of ratio cross cuts school mathematics at all levels and is an important tool for making sense of the world, in relating pairs of quantities and as building blocks of rates. Reasoning with ratios "plays such a critical role in a student's mathematical development that it has been called a watershed concept, a cornerstone of higher mathematics, and the capstone of elementary concepts" (Lesh, Post, & Behr, 1988 as cited by Lamon, 1993, p. 41). Ratios are integral to investigations of issues of social (and spatial) justice, as they are useful for analyzing equitable or fair-share distributions of resources.

This paper examines student work with associated sets ratios in the context of a spatial justice investigation around the theme of local access to financial institutions. Banks and alternative financial institutions (AFIs, such as pawnshops, wire transfer, and check-cashing stores) form a two-tiered financial system. Banks offer credit-building opportunities as well as lower rates for services than their AFI counterparts, but they tend to cater their services to people with better financial credentials and more flexible income (Servon, 2013). As part of a larger project, a curricular module was designed for high school students to use mathematics to contrast the interest rates of banks and AFIs and to build on that contrast in analyzing the local spatial distribution of these institutions. This paper focuses on the latter component and pursues the following research question: How did students' sense of place support and complicate their use of ratios in evaluating the spatial distribution of financial institutions in their city?

Guiding Frameworks

Place, Space, and Social Justice

Teaching mathematics for social justice entails reading and writing the world with mathematics; "reading" with a critical perspective on the social and political circumstances that structure the world, and "writing" with agency to respond to or change inequities or injustices (Gutstein, 2006). This paper presents findings from a mathematics module oriented around an extension of teaching mathematics for social justice: teaching mathematics for spatial justice (Rubel, Lim, Hall-Wieckert, & Sullivan, 2016).

Traditional notions of "space" in mathematics refer to geometric abstractions in which the concept of "justice" would seem to lack pertinence. This paper draws on social rather than geometric

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definitions of “space” (Soja, 1996; Tuan, 1977). With such an understanding, space encompasses social, historical, and geographical dimensions, each contributing to the world’s ongoing state of affairs. Abstract “space” comes to be bounded by human experience and imbued with historical meaning, taking on character as a particular, locatable “place” (Tuan, 1977). The associations that people have with a particular place, or “sense of place,” can color their perceptions of the world around them (Lim & Barton, 2006; Tuan, 1979). Maps and GIS-enabled digital presentations of data can present authoritative, bird’s-eye-view perspectives about place (Wood, Fels, & Krygier 2010) but can misrepresent the reality of being on foot in a particular location, and can conflict with people’s sense of place (Monmonier, 1991; Pickles, 2004).

The extent to which the multi-dimensional relations that structure space and place are fair or unfair constitutes spatial justice (Soja, 2010). Spatial justice provides a context for mathematical questions and tools focused on uncovering and examining hidden biases and agendas which structure the spatial relations people inhabit. An example of this can be seen in a critical mathematical examination of a state’s lottery, which can analyze lottery ticket sales by location to investigate relationships between lottery spending and income level (Rubel et al., 2016).

Intensive and Extensive Variables

Variable quantities of space can be categorized as extensive (i.e., ‘quantity of space,’ such as area) or intensive (i.e., ratios of extensive variables, such as population density) (Lawvere, 1992, p.18). For example, banks per square mile is an intensive variable that relates two extensive variables (number of banks and area in square miles) and coordinates a single measurement to formulate a property description of a place. Extensive and intensive variables can be used to frame data for mapping (Goodchild & Lam, 1980), as in choropleth maps, which correspond color shades to ranges in numerical data (Buckley, 2013). Since geographic areas vary in shape and size (and other properties), comparing spatial data typically necessitates normalization of data using intensive variables. In the case of this project’s digital maps, extensive variables that quantify numbers of banks and AFIs by neighborhood demand normalization, in relation to other properties like area (e.g., pawnshops per square mile) or number of households (e.g., households per bank), or by comparing them against each other (e.g., banks per AFI).

As associated sets, intensive variables like these are known to be accessible to students in the way that the relationship between the two extensive measures can be imagined more concretely than in other categories of ratios (Lamon, 1993). Students’ sense of place could serve as a resource for concretization of the intensive variables necessary to analyze the spatial justice of a distribution.

Methods

This paper presents findings from the first iteration of a 10-session mathematics curricular module. The module was piloted in a 10th-grade advisory class taught by a mathematics teacher at a high school located in Harwood, a low-income, predominantly Latin@ neighborhood.

Researchers audio recorded and collected detailed field notes for each class session, and recordings were used to clarify and enrich fieldnotes. The focus here is on sessions five and seven because of their emphasis on using intensive variables to normalize data. The fifth session (“Big Map Day”) featured an embodied distribution activity to highlight the conceptual underpinnings about normalizing data, atop a big floor map of the city (Edelson, 2011). The seventh session (“Ratio Map Day”) engaged students with custom-made digital maps that were produced as part of a suite of digital tools for the larger project.

Whole group discussions from these two sessions were transcribed from audio. Focal students’ work (3 groups) with digital maps was captured using Camtasia (Techsmith, 2010), which video-records students’ screens, including the actions of their cursor, in sync with audio/video of students at work though the computer’s built-in camera. The Camtasia recordings were used to produce

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narrative descriptions of students' actions with the digital maps with corresponding transcription of spoken utterances. Two main sets of codes were used to analyze the data: 1) the various ways to read and compare ratios (i.e. comparing numerators additively), and 2) the variables students identified to explain distribution as well as how students used the idea of proportionality to make an argument about fairness. A collaborative process was used to generate and apply the codes to the data.

Results

Big Map Day (Session 5)

Students began with a free-form exploration of a walking-scale, 140-square-foot map of the city temporarily installed on the classroom floor. Next, standing in and representing respective counties, students received various scaled props to demonstrate relative numbers of pawnshops and banks in these spaces and were prompted to discuss their reactions to these various distributions (Figure 1). As we have noted elsewhere (Hall-Wieckert, Rubel, & Lim, 2016), the activity engaged students' sense of place (Lim & Barton, 2006) about the city. This place-based knowledge served to support students in understanding the need to normalize data and to begin to coordinate such measures with demographic characteristics.

Figure 1. Students participate in embodied distribution on large map.

The activity started with a hypothetical equal distribution of pawnshops by county. A student, Sheeda, almost immediately refuted this as not making sense, saying that "some places are bigger than others, and some places are poorer than others," which suggested that the normalization should compensate for these distinctions. The teacher highlighted the former part of Sheeda's observation and led an activity distributing the institutions by area, then progressed to a distribution normalized according to households by county. Finally, the teacher revealed the actual distribution, which corresponded to none of these hypothetical and seemingly fair arrangements. When the actual distribution was shown, students confirmed the distribution using their sense of place. Sheeda explained the large number of banks in one particular county by pointing out, "...he got a bank on every corner though." This idea of normalizing by "corner" seems to suggest that she recruited her sense of the density of banks there to confirm the skewed distribution and add further nuance to the distribution of banks at a finer level of spatial scale.

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Students also drew on their sense of place to generate their own variables associated with the distribution of banks and pawnshops. Some students conjectured that some areas have more banks because they have a higher concentration of stores or more expensive stores. Several students associated variables related to their sense of place in terms of income inequalities across their city. For example, Rebecca stated, “I think that the ones that have the most pawnshops is where people have less money.” Rebecca later reacted to the idea that pawnshops and banks might not be distributed fairly, and pointedly asked the teacher, “Why they never fix that, Miss?” In this case, “fixing” or redistributing institutions for equitable access and services would involve a transformative “rewriting” of the world, a line of thought that is encouraged by teaching mathematics for social (or spatial) justice, but beyond Rebecca’s query is not further picked up by the teacher or other students in this discussion.

Ratio Map Day (Session 7): Whole-class Discussion

A suite of digital maps enabled students to explore the distribution of financial services in the context of various socioeconomic variables. The suite included three types of map layers: 1) points representing the locations of banks, AFIs (pawnshops, wire transfer, and check-cashing stores), and McDonald’s, which was included as a service likely more familiar to youth than financial institutions; 2) demographic variables (e.g., median household income); and 3) ratio maps, which used intensive variables to normalize distribution of institutions by relating the quantity of institutions to area, number of households, or to another category of financial institution (e.g., banks per AFI). The demographic and ratio maps were choropleth layers that could be displayed concurrently with the location points. In other words, the intensive variables were displayed using a color scale by intensity and could be layered underneath the view of the extensive variables, showing the count and location of each category of institution (see Figure 2). Session 7 was the second day that students engaged with the digital maps and the first day that the ratio maps were introduced.

At the start of this session, the teacher conducted a whole-group discussion orienting students to the McDonald’s maps as a way of priming them to conduct similar explorations about financial institutions later in the session. Of course, the notion of a fair distribution of McDonald’s is not equivalent in significance to a fair distribution of financial services, and some might consider the presence of McDonald’s as a disservice, but the example was intended to serve as a tangible, familiar variable for students to situate their reasoning about distribution in relation to area and households.

The teacher-led exploration began with a choropleth visualization of the number of McDonald’s per square mile. Where on the huge map, students’ sense of place about locations in the city served to prompt their association of variables toward normalizing the distribution, in this session, they recruited their sense of place as a way of interrogating the accuracy of the digital maps being presented as authoritative and official (Monmonier, 1991; Pickles, 2004). Immediately, Lina questioned this normalization and wanted to know how many blocks were equivalent to a square mile. Lina’s understanding of units of measurement in city distance understandably did not conform to the traditional measure of square miles used in the maps. In this instance, Lina recruited her sense of place about the city’s spatial arrangement to interrogate the decision to normalize density with square miles. Lina’s skepticism was a shift from her participation on the Big Map Day when she generated associated variables with other students, like the density of stores or malls, to interrogating the variables presented on the digital maps.

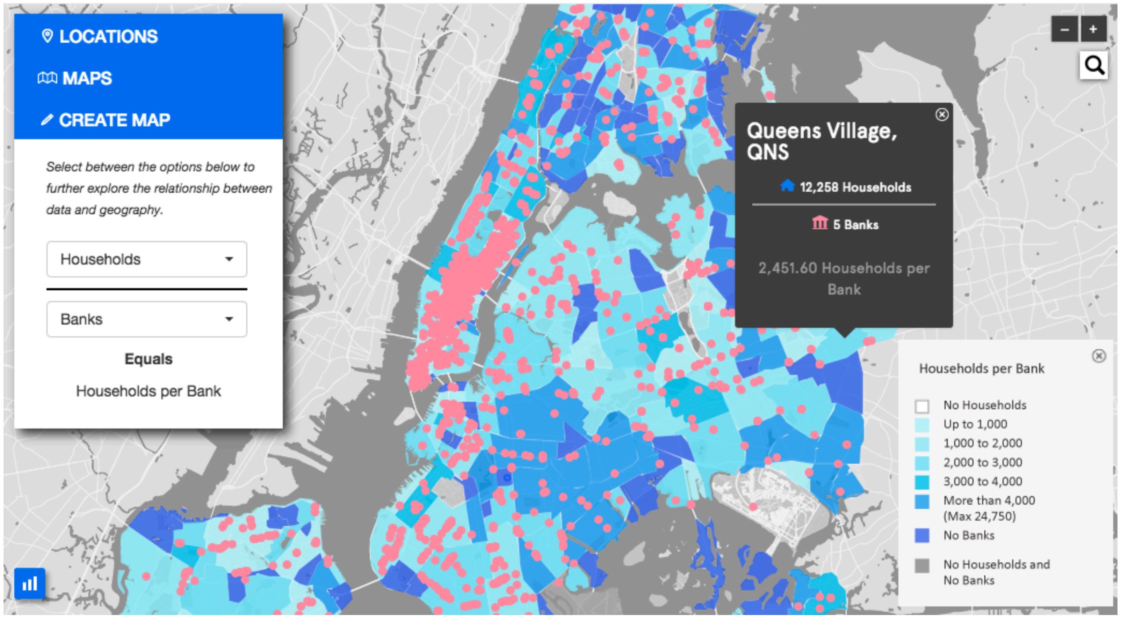

Figure 2. Location and distribution of banks in relation to number of households.

Lina's skepticism of the digital maps carried over to a questioning of the basic counts data embedded in the intensive variables like households per McDonald's. When presented with data showing that Harwood had 35,000 households for only one McDonald's, Lina exclaimed, "And one McDonald's? They lyin'!" She then proceeded to enumerate additional McDonald's located in what she considered to be Harwood; however, her sense of place in terms of the neighborhood's boundaries did not correspond with the boundaries encoded in the map.

Students' sense of place drew them to focus primarily on the map's extensive data, comparing the absolute number of McDonald's by neighborhood rather than intensive relationships. Which neighborhoods had "more" or "less," in an absolute sense, was more tangible and important. An exception to this was when Rebecca noted the crux of the relationship between the school neighborhood's McDonald's and another's: "So we only have one we share with more, and then they have way more and they share with less." This observation enabled Lina to reduce the fraction 23,000/23 to its unit ratio and follow up with, "So a thousand people go to one McDonald's?" These combined observations fed into Rebecca's expression that the distribution was unfair, and she pointedly asked the teacher again, "So why doesn't anyone fix that?" Rebecca's realization suggests that by understanding the intensive variables at work in the maps, she was able to question fairness using spatial data.

Ratio Map Day (Session 7): Map Exploration

After an introduction to the ratio maps included in the mapping tool, students explored digital ratio maps of their choice. Students, in pairs, were prompted by a worksheet to choose one map layer (i.e., one intensive variable), interpret the map's data for Harwood, notice and examine patterns across the map, and compare the data for any two neighborhoods. The different ways in which students engaged their sense of place contributed to differences in their sense making about the predetermined intensive variables to evaluate the distribution of financial institutions across the city.

Green pair. Miguel spent most of the session silently clicking through and across all of the different ratio maps as Rafaela watched, and engagement with their senses of place was not apparent. His clicks focused on each map's dark-shaded neighborhoods; that is, the neighborhoods with the highest ratios for each intensive variable. This does not necessarily mean he was looking at the most-serviced neighborhoods. For example, darker shading in the households per bank map layer would

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mean a lower rate of services. In his explanation for choosing the banks per square mile variable, he stated, “Because we wanted to see how many banks were in each square mile”; and he reported that in Harwood, “for each 1.81 square miles, there’s only 3 banks.” The use of the word “each” in both of these statements indicates an understanding of the multiplicative relationship between the extensive quantities. Miguel’s interpretation of the map shading did not coordinate between a neighborhood’s number of banks and number of square miles and referred only to the extensive variable of the counts of banks. He wrote, “The darker the color becomes, the more banks you will find in that location. And the more lighter the color becomes, the less banks you will find.”

Orange pair. Sheeda and Miriam engaged their sense of place by focusing on comparing Harwood with Montgomery, another familiar and adjacent neighborhood. These students ignored the worksheet, and instead spent most of their time exploring the absolute numbers of financial institutions by neighborhood. As a typical instance, upon examining the pawnshops per square miles map, Sheeda clicked on Montgomery and noted, “There’s 3 pawnshops,” by reading only the numerator from the fraction shown in the pop-up box. She ignored the square miles in the denominator and the resulting ratio of pawnshops per square mile. Again, examining the ratio of AFIs per bank for Harwood, Miriam compared the quantities in the numerator and the denominator additively: “We got more pawnshops than banks.” At no point did Sheeda or Miriam demonstrate thinking about the intensive quantities, and they persisted in engaging only with the extensive components, even with adult intervention. A researcher guided the students to observe that Harwood was smaller in area but had more AFIs than Montgomery. When the researcher asked the students to explain what this means, Sheeda focused only on the extensive variables and responded, “That means Montgomery is bigger than Harwood.”

Purple pair. Rebecca worked alongside an assistant teacher and focused on the households per bank map. Rebecca wrote an analysis of Harwood’s data that went beyond Miguel’s by not only interpreting the ratio terms but also how it related to the unit ratio: “The data says that there are 35,521 households for each 3 banks. So 11,840 people share each bank. It says that Harwood shares each bank with a lot of people.” After reading the number of households and institutions in her chosen neighborhoods, she stated, “So this one [Portmore] has less households and it has more banks. And this one [Easington] has more people and just one bank.” She concluded, “They [Easington] should have put … had more [banks].”

Rebecca did not generalize her interpretation beyond making sense of specific data points. When prompted to explain what a higher ratio meant for her chosen variable, Rebecca said, “When it says the ratios are higher, I think it means where the banks are more at.” She did not recognize that in the case of households per bank, a higher unit ratio would indicate a lower proportion of banks per household. Rebecca’s sense of place confounded her analysis in that her hypothesis that the number of banks was related to a neighborhood’s income level was a distraction from interpreting the given legend. Rebecca stated that she “just wanted to know where was the more banks for houses with less money,” and she expected to be able to answer this question through a single map layer. This expectation led her to try to inject income into her analysis of the households per bank map and read the categories in the legend as referring to ranges in household income rather than ranges in number of households.

Discussion

Sense of place contributed to students’ understanding of the rationale for launching a mathematical exploration of the distribution of financial institutions. The idea of dividing up the number of institutions equally by county evoked a voicing of ideas about variables, related to their sense of place, that they considered important in analyzing this distribution. Students’ consideration of these variables provided a launching point for the teacher to introduce data normalization.

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In contrast, students' senses of place challenged the representations in the digital maps. Some student-generated associated variables, like relative size of different parts of the city, connected well to ratio measures of distribution like density per square mile, which were included on the digital maps. However, many of the suggested variables by students, like the density of stores in different parts of the city, were unanticipated and not available as intensive ratio maps. The map tool did not allow for students to follow through with their own ideas about what variables might be salient and thereby engage in more authentic data explorations.

In addition to a fixed set of variables to examine, the way that the maps were designed did not allow for flexibility in approach. The mathematization of the selected variables in these maps were imposed; for instance, the choice of square miles made sense to the designers as a unit of areal measure, but blocks or "corners" could have been a more accessible unit. Similarly, neighborhood delineations on the map did not always correspond with students' senses of boundaries. For Rebecca, her desire to examine a direct relationship between income and number of banks on the map was constrained by the designers' decision to include demographic maps as separate static maps to be compared, but not combined, with the chosen ratio maps.

Sense of place served as an anchor for students to grasp the tangible aspects of the given variables; at the same time, it distracted them from engaging with more abstract, intensive variables. During the whole-class discussion and in their investigations with digital maps, students tended to compare extensive variables–in this case, the discrete number of institutions. Scaffolding concrete representations of abstract intensive measures, such as households per institution as the number of people sharing one institution, aided in students' interpretation of these variables and supported their mathematical arguments about fairness. Miguel and Rebecca, for example, were successful at interpreting intensive variables with respect to specific neighborhoods but struggled to interpret the more abstract choropleth layers.

Conclusion

Students' sense of place engaged them in considering and analyzing spatial data. The open-endedness of the huge floor map enabled students to understand the conceptual basis of and suggest strategies for normalization. Students drew on their sense of place to imagine concrete representations of intensive variables, which allowed them to interpret data points, make comparisons, and even formulate arguments about fairness. These concrete representations could serve as building blocks toward making sense of the data represented more abstractly in the map's choropleth layers, which were more elusive for students to interpret.

The closed nature of the larger project's digital maps did not enable students to use the maps to fully explore variables that connected to or built on their sense of place. Rather, students' sense of place became a tool to critique the point of view presented by the seemingly authoritative digital maps. More support towards activities that combine the open-endedness of the big map activity with the abstraction and mathematization of the digital maps is needed. A future goal is the design and implementation of digital mapping tools that enable students to generate their own intensive variables or ratios, based on their sense of place, which can then be visualized in authoritative map layers. The authors are currently investigating new designs for activities to engage students in map-making of this kind.

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