FROM THE UNIVERSITY TO THE CLASSROOM: PROSPECTIVE ELEMENTARY MATHEMATICS SPECIALISTS' PEDAGOGICAL SHIFTS

Kayla D. Myers
Georgia State University
kmyers@gsu.edu

Susan L. Swars
Georgia State University
sswars@gsu.edu

Stephanie Z. Smith
Georgia State University
szsmith@gsu.edu

This project focuses on the development of prospective Elementary Mathematics Specialists (EMSs) in a K-5 Mathematics Endorsement Program. Program courses emphasized elementary mathematics content and pedagogy while providing opportunities for participants to evidence their learning through classroom teaching practice, all in an attempt to facilitate pedagogical shifts toward more standards-based instruction and dialogic discourse. Data presented include scored observation rubrics of classroom practices, professional portfolio reflections on implementation of new teaching practices, and semi-structured, individual interviews.

Keywords: Teacher Knowledge, Teacher Education-Inservice/Professional Development, Problem Solving

Purpose

Many in the field of mathematics education are proponents of Elementary Mathematics Specialists (EMSs) and the critical role they have in elementary schools of supporting effective mathematics instruction. The recent unified position of several prominent mathematics education organizations, including the Association of Mathematics Teacher Educators (AMTE, 2013), asserts every elementary school in the U.S. should have access to an EMS and that advanced specialist certification should be offered via intensive preparation programs. One salient challenge associated with EMSs' preparation is their development of new ways of teaching in elementary classrooms.

Adopted by most states in the U.S., the Standards for Mathematical Practice in the *Common Core State Standards for Mathematics* (CCSS-M; National Governors Association Center for Best Practices [NGACBP] & Council of Chief State School Officers [CCSSO], 2010) and the National Council of Teachers of Mathematics' *Principles to Actions* (NCTM, 2014) both strongly depict *standards-based learning environments* (SBLE) that foster dialogic discourse and conceptual understandings of mathematics. In such learning environments, students make conjectures about their mathematical ideas and explain their thinking and reasoning while teachers value students' multiple perspectives and carefully craft mathematical discussions by using their ideas to bring the classroom to shared mathematical understandings. This pedagogical approach requires thoughtful planning and questioning on the part of the teacher and is a key shift away from traditional classroom instruction. An important goal of EMS preparation is the development of these pedagogical competencies.

Our university is located in one of the limited number of states in the U.S. offering EMS certification (i.e., a K-5 Mathematics Endorsement [K-5 ME]). Key goals for participants in the K-5 ME program are pedagogical shifts toward alignment with a SBLE, development of the *specialized content knowledge* (SCK) necessary for teaching mathematics, and changes in mathematical beliefs. Research in general on EMS preparation is limited, and our previous research on this particular program shows teachers had significant: increases in SCK, changes in pedagogical beliefs toward a cognitive orientation, and increases in mathematics teaching efficacy (Swars, Smith, Smith, Carothers, & Myers, 2016). This study in particular extends the existing inquiry on the program by focusing more so on the translation of learning in the K-5 ME courses into instructional practices. It was guided by the following research questions: What pedagogical shifts do prospective EMSs experience as they complete a K-5 ME program? How do prospective EMSs describe their pedagogical shifts during a K-5 ME program?

Literature Review

EMSs are generally considered to be teachers, teacher leaders, or coaches with the expertise to support effective elementary mathematics instruction and student learning in the classroom, school, or other levels (AMTE, 2013). Over time, the roles of EMSs have mostly been considered in two ways, including EMSs who work with students and EMSs who work with teachers (AMTE, 2013; Reys & Fennell, 2003). In general, the specific roles and responsibilities of EMSs vary according to the contextual needs and plans of schools and school systems, with an increasing number of schools utilizing EMSs in some manner (Gerretson, Bosnick, & Schofield, 2008). The extant studies on EMSs have focused on improving instructional practices, designing coaching programs, and improving student achievement, with overall results showing positive impacts of EMSs on teacher development and student learning (Campbell & Malkus, 2011; McGee, Polly, & Wang, 2013).

A body of research shows that classroom pedagogy has more influence on improving student learning than the use of particular curriculum materials (Brown, Pitvorec, Ditto, & Kelso, 2009; Remillard, 2005; Tarr et al., 2008). The *Principles and Standards for School Mathematics* (NCTM, 2000) and the CCSS-M recommend the intersection of mathematical content and process standards requiring a pedagogical approach different from the traditional direct instruction in computational skills still found in many U.S. classrooms. Many of these suggestions are grounded in constructivist compatible instruction, where teachers: engage students in real-life contexts; provide students with original, non-routine problems; and develop a classroom community grounded in dialogic discourse intended to develop students' individual and shared understandings of mathematical concepts and practices in ways that nurture their abilities to problem solve, reason, and communicate mathematically (Charalambous & Hill, 2012; Cobb & Jackson, 2011; Tarr et al., 2008). Teachers are creating SBLEs and ensuring their instructional tasks hold high levels of cognitive demand (Porter, 2002; Stein, Smith, Henningsen, & Silver, 2009). According to the Tarr et al. (2008) study, improved student learning and achievement was connected to the extent of enactment of such a SBLE.

Methodology

This inquiry used a descriptive, holistic case study design. The case was the described experience of implementation by a group of elementary teachers in an EMS preparation program. We focused on their efforts to enact instructional practices in their classrooms drawn from their learning in the program. Collected data were both qualitative and quantitative in nature.

Participants and Setting

Participants were 13 elementary teachers enrolled in a graduate level K-5 ME program at a large, urban university in the southeastern U.S. The teachers worked at one urban, elementary school recently converted to a charter school. They held varying teaching positions at the school, including those of classroom teacher, small group resource teacher, and instructional coach, but all worked with the primary grades.

A foremost goal of the program was the development of a deep and broad understanding of elementary mathematical content, including the *specialized content knowledge* (SCK) for teaching elementary mathematics (i.e., the "mathematical knowledge needed to perform the recurrent tasks of teaching mathematics to students" [Ball, Thames, & Phelps, 2008, p. 399]). The program also focused on high-leverage teaching capabilities in the elementary classroom, including: (a) selection and implementation of mathematical tasks with high levels of cognitive demand, (b) use of multiple mathematical representations, (c) use of mathematical tools, (d) promotion of mathematical dialogic discourse, explanation and justification, problem solving, and connections and applications typical of a SBLE, and (e) use of children's thinking and understandings to guide instruction. Learning during course sessions occurred via: (a) active inquiry and analysis of the mathematics in the elementary curriculum, specifically the CCSS-M, (b) study of children's thinking and learning by way of video

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clips and teaching cases, (c) examination of examples of classroom practice through video clips and teaching cases, and (d) scrutiny of the research base related to mathematics teaching and learning in the elementary grades.

The program was two semesters in duration and included four 3-semester-hour mathematics content courses for elementary teachers that integrate pedagogy, plus one 3-semester-hour practicum course providing an authentic residency. The four courses were: Number & Operations, Algebra, Data Analysis & Probability, and Geometry & Measurement. Each course was offered for 7 weeks, meeting 1 evening per week for 5.5 hours at the elementary school. The Number & Operations course had a significant focus on Cognitively Guided Instruction (CGI). Key program assignments included: clinical-style interviews of children's mathematical understandings; selection, adaptation, or generation and analyses of worthwhile mathematical tasks; an in-depth data design, collection, and analysis project; and critical examination and presentation of extant research on elementary mathematics education via synthesis papers. The participants also completed a 3-semester hour practicum course during the second semester of the program that provides an authentic residency enacting the synthesis of content knowledge and problem-based pedagogy emphasized in the program. Practicum assignments included the creation of a portfolio demonstrating expertise in teaching elementary mathematics, analyzing impact on diverse student learning, and technology integration. Successful completion of the practicum and all four of the content/pedagogy courses led to recommendation for the K-5 ME.

Data Collection and Analysis

Quantitative data were collected using scored observations of classroom teaching practices with what we have called the Standards-based Learning Environment Observation Protocol (SBLEOP), which documents the degree to which the teacher facilitates and the students experience a SBLE (Tarr et al., 2008). This observation protocol was adapted from an observation tool in the Wisconsin Longitudinal Study (Romberg & Shafer, 2003) and slightly modified for the K-5 ME program. It consists of a rubric assessing the extent to which specific mathematics classroom learning events are apparent during an observed lesson, using a scale of 1 to 3, with a higher score indicating more alignment with a SBLE. Five of the classroom events were included as data in this study.

Qualitative data include professional portfolios with teacher reflections, as well as six individual interviews. The professional portfolios with teacher reflections and two classroom observations, conducted by the university supervisor, were completed as part of the authentic residency course in the second semester of the program, and provided the opportunity to document: the degree to which classroom instruction included worthwhile mathematical tasks and evidence of a SBLE; analysis of diverse student work and achievement using formative assessments and remediation; technology integration; and personal reflections on teaching practices. This documentation experience served as a cumulative reflection for the participants and a summative evaluation for the university supervisor. Portfolios were required to have at least ten enacted lesson plans with at least one lesson from each of the four mathematical domains of the program's courses. Given the participants' consistent focus on the Number and Operations domain for the classroom observations, as well as the majority of the additional six chosen lessons included in the portfolios, teacher reflections were drawn from the Number and Operations section of the professional portfolio for analysis. The semi-structured, individual interviews were conducted with six randomly selected participants within two weeks of completion of the program, exploring their experiences with implementation and documentation (i.e., the portfolio, classroom observations, reflections, etc.) during the program.

The qualitative and quantitative data were intended to reciprocally illuminate and extend the findings, particularly in the drawing of conclusions. The interview and teacher reflection data were analyzed using line-by-line open coding that generated numerous meaning units (i.e., embedded coherent and distinct meanings), which were then documented in a coding manual. These meaning units were then compared across cases and as consensus was reached between the researchers, coded meaning units were collapsed and renamed until final shared themes were determined. Descriptive statistics were used for analysis of the items on the SBLEOP.

Results

Quantitative Findings

The SBLEOP from the authentic residency course in the final semester of the program provided categorical data for five classroom events from two classroom observations. We used the same approach as Tarr et al. (2008) to convert these data to numerical data. We summed the individual scores (1, 2, 3) from the two observations, then determined whether each event for each participant should be rated as high (5-6), medium (3-4), or low (2). These categorical codes were then assigned numerical values of 2 (high), 1 (medium), or 0 (low). These numerical values were then summed across the five classroom events to find a composite score for each participant ranging from 0-10, which indicate teachers' enactment of a SBLE. Table 1 shows the percentages of participants who were rated as low, medium, or high by classroom event and the percentages of participants in each range of composite scores. The composite scores show that 85% of these teachers enacted a SBLE at a high level (7-10), and the remaining 15% enacted a SBLE at a medium level with a composite score of 6 out of 10.

Table 1: Percentage of Participants by Score for SBLEOP and Composite Score

	Scores with Numerical Codes of 0, 1, 2		
Classroom Event	*Low (0)*	*Med (1)*	*High (2)*
CE1 Making Conjectures	31%	62%	8%
CE2 Conceptual Understanding	0%	8%	92%
CE5 Explaining Strategies	0%	15%	85%
CE6 Multiple Perspectives	0%	0%	100%
CE7 Using Student Statements	0%	54%	46%
	Sum of Event Scores (0-10)		
	Low (0-2)	*Med (3-6)*	*High (7-10)*
Composite Scores	0%	15%	85%

Qualitative Findings

The analysis of the interview and reflection data revealed several commonalities across participants' experiences with classroom implementation. These themes tell a story of movement through shifting beliefs and practice across the program. Participants began in a place of *skepticism*, troubling this new way of teaching and learning mathematics, with resistance that came from doubt and uncertainty. It was not until participants *tried it on*, by putting pieces into practice and experimenting in the classroom, that they began to see things differently. This move to the classroom was often credited to the implementation assignments and documentation requirements (i.e., classroom observations and creation of the portfolio) from their authentic residency course, which served as *shifters,* or impetuses for change in their practices and beliefs about teaching and learning mathematics. Ultimately, though, we found that participants *need more support* in their classroom endeavors. The changes they were making were challenging and significant in scale, and the feedback and support needed to maintain these new practices were acknowledged.

Skepticism. When these participants started the endorsement courses, they were coming from a place of skepticism. This community of teachers was immersed into a very rigorous program of study, and the initial reaction was resistance. Participants were holding on to well-entrenched habits and customary practices, which provided a level of comfort and safety. These long standing teaching

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practices were troubled in the endorsement courses. Their first course, Number & Operations, introduced them to CGI, including the problem types, solution strategies, and the power of a problem-based pedagogical approach when implemented with rich dialogic discourse in the classroom. Participants watched videos in the course of CGI implementation and voiced skepticism because they thought: (a) it took too much time to develop such a pedagogical routine, (b) their own students were not capable like those from the videos, and (c) the expectations for classrooms were unrealistic.

Participants initially believed that incorporating CGI would be too difficult because it would take a long time to develop an aligned pedagogical routine with students. This assumption came from a place of skepticism about the capabilities of students' thinking and learning in their own classrooms. When reflecting on watching the videos in class, one interviewee said, "I was still wondering, ok, how long had they been exposed? It just was really scary especially um with me teaching the kids who are struggling in math… will they be able, you know, to do this?" However, that same interviewee came to think about her students in a new and promising way: "I'm thinking, like, these kids aren't, you know, as LOW as people think…"

The biggest place of skepticism was in this disbelief about what children can do mathematically and how they can learn: "I was a non-believer at first… I'm like, word problems are the hardest things that kids have to do in math. They struggle with that the most, and to just see it is unbelievable." This skepticism about their own students quickly diminished as they began to witness firsthand their learning in the classroom, as one participant asserted, "I think for me the biggest a-ha was my mindset about what I thought kids could do… so for me, it's just planning it out and knowing that kids can." Participants started to see their students "can do more than we give them credit for", and "they can surprise you with what they know".

Trying it on. In the authentic residency course experience, participants were required to complete two passing classroom observations of their mathematics instruction. The observation rubric was highly conducive for a CGI lesson, so nearly everyone chose to try that on. In this trying on, this implementation of CGI, participants felt anxious about their execution, wondering if they were "doing it right". They also felt uncomfortable in not knowing if it would work. In trying it on, though, and engaging in it for themselves, they grew in confidence about the effects of CGI and also their impact as mathematics teachers. They were valuing the mathematics, the thinking involved, the conceptual understanding and problem solving skills, the multiplicity of strategies, and the explanations their students were using. Said one participant, "Kids that can't even read word problems necessarily work them out and have the strategies and the conceptual understanding… it's just that powerful."

The mathematics that participants noticed was another focus in their stories of implementation. They spoke of "witnessing a shift in their [students'] mathematical thinking," and their students "feeling successful in their ability to solve problems" and "going from procedural to conceptual understanding". The discourse that participants facilitated in their classrooms also led to important shifts in their thinking about how children learn mathematics. Finding (for themselves and from their students) that there are multiple ways to solve problems, many ways to think about the same mathematics, and numerous perspectives on the same story problem. Most importantly, participants began to value discourse, this opportunity for the class to hear and learn from each other, as a way for students to explain and justify their thinking. One participant asserted, "I think [CGI] has been the best thing I have learned in math as a teacher thus far, hands down. Nothing compares to it… I think that's the most important thing I have learned in math thus far." Additionally, they found it built a stronger classroom community.

Having 13 prospective EMSs as participants in the same school, working on the same coursework and implementing the same practices, built a strong community of teachers as learners as well. This support system provided a push to ask questions, to try new things, and to persevere. This

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parallel, between what CGI did for their mathematics classroom community and what a collective implementation did for the school community, is noteworthy.

Shifters. In connecting their learning of theory, methods, and content from the courses to enactment in the classroom setting, there are many factors that contributed to shifts in instructional practices and beliefs. These factors, or "shifters", include: receiving and using critical feedback, program coursework (e.g., clinical interviews and worthwhile mathematical task collections), implementation of and reflection on classroom observations, documentation in the professional portfolio, listening to children's mathematical thinking, trying on new and different instructional practices, and deepening their own mathematical understanding, to name a few. In particular, interviewees spoke specifically about the value of the implementation (classroom observations) and the documentation (portfolio), in their shifts: "I think I would've continued to do it the way I was doing it and probably would've kept getting frustrated with not getting the results that I was looking for. So I think that really helped me, having [the university supervisor] come in here."

Facilitating this buy-in and meaningfully shifting the prospective EMSs' instructional practices and beliefs was not easy. These salient experiences, these "shifters", generated a change that pushes beyond imagination into reality: "There's no way that I would have ever bought in to this had I not had the practicum experience." By taking their course learning and having a space as a community to implement and document over time as they tried on these new teaching practices in their own elementary classrooms, participants began to notice their beliefs shifting, their practices shifting, and their attitudes toward mathematics shifting. Said one participant, "The mantra that I have lived by every year, I want to reinvent myself, and I will say that every year. I say it and then [in] the summer I'm like, did I reinvent myself? I don't think I did. But I think if I reflect THIS time I can say I reinvented myself."

Need more support. Across the data, there was an emphatic call for further support and feedback. Each participant made note of their appreciation for the feedback they received; however, two evaluative classroom visits provided limited time and support from the university supervisor. One participant in particular recommended some sort of follow-up as they tried on new practices: "just come back and check on us". Another participant, one of several, mentioned the need for non-evaluative spaces for feedback and support. For their two observations, participants would choose lessons that they felt confident implementing; but, in their struggles and changing instructional practices and beliefs, they often had questions and a need for support apart from their evaluation: "It would be great for us to kind of still have mentors, somebody to come and check in on us."

Another consistent desire voiced by the teachers was sustainability. Participants recognized that they learned a lot in one year, and they voiced an aspiration to make sure they continue to implement these new teaching strategies. Some ideas for this sustainability were to implement ongoing professional development, to have more classes over the summer, and to have more opportunities to bring university supervisors or mentors into their classrooms.

Discussion

EMSs provide crucial and needed mathematics expertise in elementary schools. However, there is limited research on how to best prepare EMSs and in particular support the difficult process of pedagogical shifts in classrooms. This case study explored the experiences of one group of prospective EMSs as they sought to connect their learning of theory, teaching methods, and content knowledge in program courses with their classroom instructional practices.

The quantitative and qualitative data revealed the prospective EMSs were facilitating instruction mostly aligned with a SBLE. The SBLEOP showed the largest gain across the two observed lessons was the classroom event of enacting lessons that foster development of conceptual understanding. Prospective EMSs were encouraging and valuing students' multiple strategies and perspectives and creating learning opportunities grounded in dialogic discourse as they valued students' mathematical statements and used them to build discussion or develop shared understanding, and students were explaining their mental reasoning and problem-solving strategies. However, they struggled to provide frequent opportunities for their students to make conjectures (i.e., informal generalizations) about mathematical concepts and processes in the context of problem-based learning. From our own experiences, overcoming this difficulty requires more time for classroom discourse, deeper understandings of the mathematical connections to and within the content by both teachers and students, and higher expectations that young children can make important connections and generalizations.

These teaching practices evidenced on the SBLEOP did not come easily for the participants. Skepticism and disbelief marked their perceptions of their own students' mathematical capabilities. When viewing videos of children's mathematical reasoning and explanation during the course sessions, the teachers were highly dubious that such could occur in their own classroom realities, due to students' inabilities and time constraints of a pedagogical routine grounded in problems. However, experimentation with what they were learning in the courses, particularly a problem-based pedagogy aligned with CGI, made them believers. Trying it on and seeing the benefits for and capabilities of their own students were instrumental in the prospective EMSs changing their practices and beliefs about teaching and learning mathematics.

The qualitative data revealed the key supports for trying on new instructional practices, including the authentic residency course assignments. Shifters, like supervisor feedback, clinical interviews and worthwhile mathematical tasks assignments, classroom observations, and the creation of the professional portfolio, pushed participants to make these pedagogical changes. Many participants began the program from a place of skepticism, but after implementation (trying on new teaching practices) and documentation (portfolio assignments and reflections), changed their practices and beliefs. Participants identified the importance and profound impact of implementing a SBLE in their classrooms, notably the use of CGI as a guide for instruction. However, the prospective EMSs voiced the need for more support. They wanted help and feedback beyond the two enacted lessons in the program and also after program completion. Participants wanted opportunities for non-evaluative classroom observations with reflection and feedback.

In sum, it is vital that the efficacy of EMS preparation programs, such as the one in this study, be carefully studied in order to determine their impact. Based on our findings, the participants in this program connected their learning in the courses with their classroom teaching practices. Even more importantly, the participants found immense value in that connection and desire continued support in maintaining these new practices. It seems the implementation and documentation expectations during the authentic residency course supported this very difficult translation, facilitating the ultimate goal of the program of pedagogical shifts.

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