NEGOTIATING MEANING: A CASE OF TEACHERS DISCUSSING MATHEMATICAL ABSTRACTION IN THE BLOGOSPHERE

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Many mathematics teachers engage in the practice of blogging. Although they are separated geographically, they are able to discuss teaching-related issues. In an effort to better understand the nature of these discussions, this paper presents an analysis of one particular episode of such a discussion. Wenger's theoretical framework of communities of practice informs the analysis by providing a tool to explain the negotiation of meaning in the episode. Results indicate that the blogging medium supports continuity of discussions and can allow for the negotiation of meaning, but that a more nuanced treatment of the construct is necessary.

Keywords: Teacher Education-Inservice/Professional Development, Technology, Learning Theory, Informal Education

Introduction

> I've been throwing around the term "pyramid of abstraction" recently, and there was some great pushback on Twitter this evening. This post is my attempt to clarify what I mean, and why I think it is a useful perspective to building students' knowledge. (Kane July 31, 2015)

Mathematics teacher Dylan Kane wrote the passage quoted above in a blog post after a series of Twitter interactions with mathematics teacher bloggers Dan Meyer and Michael Pershan. The series of interactions were prompted by Dylan's original blog post in which he explained ideas about a metaphor for mathematical abstraction as pertaining to his teaching practice. The discussion that resulted from this formulation presents an instance of a negotiation of meaning around a metaphor related to the teaching and learning of mathematics. Most importantly, it provides an example of how mathematics teachers are engaging in the practice of blogging, which is the focus of this paper. It should be noted, however, that this paper is not concerned with teacher knowledge (Ball, Thames, & Phelps, 2008; Shulman, 1986), but rather with the social interaction among mathematics teachers that the blogging practice affords.

Mathematics Teacher Blogging

Mathematics teachers around the world are choosing out of their own will to create, organize, and manage their own personal blog pages. On these publicly visible virtual pages, they make relatively regular posts, which are most often related to their work as mathematics teachers. These posts can include written and/or media enhanced recounts from their teaching experiences, links to interesting resources, and responses to posts that other bloggers have made.

Since blog pages are individually managed and no universal blogging platform exists, finding like-minded bloggers may at times be difficult. To mitigate this issue, many bloggers extend their practice through Twitter, a universal micro-blogging platform that allows users to create searchable profiles. There are currently 431 profile listings on the Math Twitter Blogosphere directory, most of whom have both a Twitter handle and a blog page listed on their profile. Although micro-blogging on Twitter is slightly different in nature than blogging due to the 140 character limit on each post, which forces users to communicate ideas more concisely, it is still considered a form of blogging (Ebner, 2013). Blog pages allow users to explore ideas deeply, but Twitter makes it easier for like-minded bloggers to connect. Mathematics teacher bloggers often use both mediums to express ideas, linking between the two when necessary. For instance, some bloggers include snips of Twitter conversations

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in their blog posts, and other bloggers link to their blog posts within their Twitter posts. In this way, teachers are communicating with peers across the world.

This sort of collegial interaction is unusual for teachers because teaching is generally an isolated profession (Flinders, 1988; Lortie, 1975). Mathematics teachers' typical contact with other mathematics teachers is limited to that of department meetings and teacher lunchrooms, where teachers usually plan lessons and grade papers individually (Arbaugh, 2003). Sparse one-time professional development initiatives are not generally found to be effective in stimulating teacher collaboration (Ball, 2002), and they don't often extend into more regular professional development opportunities because they require time, funding, and facilitation.

However, there are hundreds of mathematics teacher bloggers who seem to be overcoming these constraints. It is evident that they spend hours writing publicly about their daily practice, posting resources, and sharing their dilemmas with no compensation and no mandate. This unprompted, unfunded, and unevaluated teacher activity is a rich phenomenon of interest that deserves attention. Ironically, this phenomenon is largely unstudied. Empirical investigations related to blogs in mathematics education are limited to studies on the utility of blogging as a pedagogical tool within either a mathematics course (Nehme, 2011) or a mathematics education course (Silverman & Clay, 2010; Stein, 2009). These studies do not account for the autonomous and self-driven nature of the blogosphere and there is no clear work in mathematics education exploring the activities of these teachers.

Research Question

As such, this study is guided by the overall research question of what the mathematics teacher blogosphere affords for teachers who engage in blogging in relation to their practice. To this end, in this paper I pursue an investigation of one episode that exemplifies the type of interaction that is possible within the mathematics teacher blogosphere.

Theoretical Framework

Since blogging is an individual practice that is made public (Efimova, 2009), a mid-level theory that accounts for situated participation is desirable. C*ommunities of practice* (Wenger, 1998) is one such theory: it is a social theory of learning where learning is considered as increasing *participation* in the pursuit of valued enterprises that are meaningful in a particular social context. *Practice* is at the heart of Wenger's (1998) *communities of practice*, and a key aspect of *practice* is the ability to motivate the social production of *meaning*. The continuous production of *meaning* is termed as the *negotiation of meaning*, and is further defined by the duality between *participation* and *reification*. For Wenger (1998), *participation* is "a process of taking part [as well as] the relations with others that reflect this process" (p. 55), and *reification* is "the process of giving form to our experience by producing objects that congeal this experience into 'thingness'" (p. 58). Both *participation* and *reification* shape the participant and the community in which they participate in an ongoing manner.

According to Wenger (1998), *participation* and *reification* imply each other, require and enable each other, and interact with each other. Wenger (1998) notes that the benefit to viewing the *negotiation of meaning* as a dual process is that it allows one to question how the production of meaning is distributed. He notes that there is "a unity in their duality, [because] to understand one, it is necessary to understand the other, [and] to enable one, it is necessary to enable the other" (Wenger, 1998, p. 62). Various combinations of the two will produce different experiences of *meaning*, and together they can create dynamism and richness in meaning if a particular balance is struck.

Wenger (1998) notes that "when too much reliance is placed on one at the expense of the other, the continuity of meaning is likely to become problematic in practice" (p. 65). If *participation* dominates, and "most of what matters is left unreified, then there may not be enough material to anchor the specificities of coordination and to uncover diverging assumptions" (p. 65). However, if

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reification dominates, and "everything is reified, but with little opportunity for shared experience and interactive negotiation, then there may not be enough overlap in participation to recover a coordinated, relevant, or generative meaning" (p. 65). In essence, *participation* allows for re-negotiation of *meaning*, and *reification* creates the conditions for new *meanings*. In this interwoven relationship, the two aspects work together to drive the process of *negotiation of meaning*.

The interplay between *participation* and *reification* is different in each unique social situation, contributing to a different experience of *meaning* for participants of that *practice*. What is of interest in this paper is how mathematics teacher bloggers experience the social production of *meaning* within the blogging *practice*.

Method

In order to be able to view mathematics teacher blogger conversations on Twitter, I have spent over a year collecting subscriptions. Every user sees different posts based on who they subscribe to, and it is important to note that my pervasively subjective position influences what I can notice in this ultra-personalized and dense virtual environment.

Initially, my position was predominantly that of a 'lurker' in that I had not made significant contributions to the blogosphere. This position changed slightly after my attendance to the MTBoS conference 'Twitter Math Camp 2015' (TMC15). Physically meeting many of these bloggers connected me to them more than before. As my subscription list grew, I was also able to view more of their conversations. It was during this time after my return from TMC15 that I encountered a particular conversation between Dylan, Michael, and Dan that I flagged as interesting based on my theoretical framework and for its power to illustrate a possible mode of interaction in the blogosphere.

After the conversation took place, I used *storify.com*[1] to identify all tweets related to the conversation from the feeds of each of the participants (Dylan, Michael, and Dan), and rearranged them chronologically. I copied the written content of each post, the name of the participant, and the time stamp, and pasted it into an offline spreadsheet document. I also included the written content of any blog post that was linked to in the Twitter posts within this document. This reconstructed conversation made of Twitter and blog posts comprises the data set for this paper.

This data set was then coded according to Wenger's (1998) *negotiation of meaning* so that conclusions could be drawn about participant experiences of the social production of *meaning,* as embedded within the *practice* of teachers in the blogosphere. As part of the analysis, moments of *participation* and *reification* were coded and labelled at each instance. In general, *participation* was considered to be any action that a blogger took as part of the blogging *practice*, and *reification* was considered to be any trace that was left from a *participation*. The interplay between these aspects was then considered in relation to the data, and conclusions were drawn about the nature of the *negotiation of meaning* as exemplified in this case of mathematics teacher blogging.

In what follows, a reduced version of the reconstructed conversation is presented in the results section, and is then reviewed in terms of Wenger's (1998) *negotiation of meaning* construct in the analysis and discussion section.

Results

On June 25, 2015, Dylan writes a blog post about his ideas regarding how teachers can help students deal with solving difficult mathematical problems by helping them become better equipped to transfer prior knowledge to new contexts in mathematics. He refers to the popular 'ladder of abstraction' metaphor, and claims that it is incomplete.

> I often hear references to the "ladder of abstraction" — the idea that students' understanding begins with the concrete, and climbs a metaphorical ladder as it becomes more and more abstract. I think this is a useful metaphor, but is also incomplete. (Kane June 25, 2015)

Dylan then suggests an alternative to the metaphor that would make it more useful in classroom practice and more reflective of how he has experienced students learning mathematics.

> I think the metaphor of a ladder of abstraction would be better replaced by a pyramid of abstraction . . . I worry that the ladder of abstraction metaphor leads me to believe that, once a student understands one concrete example of a function, they are ready for a more abstract example. While some students may be, I want to focus on building a broad base first, and then moving up the pyramid after we have spent time analyzing the connections between the examples and the underlying structure. (Kane June 25, 2015)

A month later, on July 17th, Dylan writes a reflection about his experiences at PCMI, a three-week summer mathematics institute. In this post, he discusses the importance of letting students engage in productive struggle within mathematical problem solving, without oversimplifying. Just after making this point, he uses a hyperlink (italicized below) to refer back to the June 25th blog post in which he had introduced the idea of a 'pyramid of abstraction.'

> This gets at something I wrote about recently that I called the *pyramid of abstraction* – that students build abstract ideas from looking at connections between a wide variety of examples, rather than simply jumping from concrete to abstract. (Kane July 17, 2015)

Just as for his June 25th post, Dylan publishes a link to his July 17th post on Twitter (Figure 1).

Figure 1: Dylan's Twitter post linking to his blog post.

Shortly after publishing the link to his July 17th blog post on Twitter, Michael Pershan responds in agreement with Dylan's reference to the 'pyramid of abstraction' metaphor (Figure 2).

Figure 2: Michael's response to Dylan's post.

Fifteen days later on July 31st, Dan Meyer challenges Dylan and Michael on Twitter by asking about the meaning of the apex of the 'pyramid of abstraction' (Figure 3).

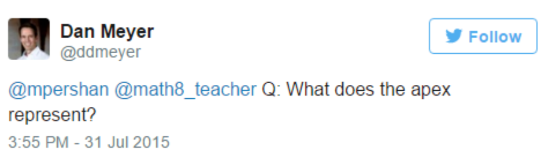

Figure 3: Dan's question to Dylan and Michael.

This is followed by a series of interactions on Twitter between Dan, Michael, and Dylan that occurs over the course of a few hours. This series of interactions is presented in transcript form below. Some comments have been removed for brevity.

> *3:55 Dan:* Q: What does the apex represent?

4:13 Michael: Like, the uber-apex? Or the apex for a skill family?
4:23 Dylan: an abstract principle that can be transferred among multiple contexts-think of all math as nested pyramids of abstraction
4:24 Michael: And at the very tippy tippy top something like "do math," I guess.
4:28 Dan: I'm asking about the significance of the pyramid's tippy-top in this newfangled metaphor.
4:29 Dan: I don't think Dylan's explanation works. I can always abstract the "abstract principle" he puts at the top.
4:30 Dan: Abstraction has no end. Seems to me you guys are in a jam.
4:31 Dylan: maybe abstract isn't the best word. Heart of this for me is knowledge that will transfer to a new context
4:35 Dan: Pyramid of something-other-than-abstraction then? Curious what it is you're trying to describe.
4:38 Dylan: I'm defining abstract as knowledge that transfers. I want to stick with that, but it's worth defining more carefully
4:39 Michael: I'm staying away from such a tough problem as trying to define "abstract"!
4:40 Michael: When I am interested in pyramids of abstraction, it's an attempt to describe mathematical thinking.
4:40 Michael: There are strategies that we use that often represent bundles of strategies, and so on.
4:55 Dylan: my premise is that students need many representations to abstract from, hence a pyramid.
5:07 Dan: But abstraction requires multiple instances /by definition/.
5:08 Dan: Just saying this pyramid thing complicates an already complicated concept.

A few hours later, at 9:28PM on July 31st, Dylan publishes a blog post in response to this conversation, starting with the post initially quoted in the introduction of this paper, and proceeding to explain how he has defined 'abstraction.'

> I'm defining abstraction very specifically. A student abstracts a concept, or builds abstract knowledge, if they can apply that knowledge in multiple contexts. (Kane July 31, 2015)

He then discusses why he thinks a 'pyramid of abstraction' is useful in teaching mathematics and why this metaphor matters to him as a teacher.

> There are a set of teacher actions we can take to facilitate *transfer* — that moment when a student applies a concept they understand to a context they haven't seen before. That's what I'm chasing. (Kane July 31, 2015)

In the last section of his blog post, he lists issues he still has with the metaphor, acknowledging that it is possible the metaphor obfuscates the concept he wants it to represent, and asking the reader if it makes sense to them. He also concedes that there may be no end to abstraction, but that he personally doesn't believe this is true.

> If points in the coordinate plane are an abstract concept, and linear relationships are another abstract concept, and functions are another abstract concept, do we end up in an infinite pyramid of abstraction? I don't think so. I think each of those ideas can then be a building block for broader mathematical concepts. (Kane July 31, 2015)

Finally, he admits he is integrating knowledge transfer and abstraction into one idea, and implicates that this may be a result of his own understanding of 'abstraction.'

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> Do I Actually Understand Abstraction? I'm redefining abstraction a bit, and I'm also lumping knowledge transfer as one giant idea, which it might not be. (Kane July 31, 2015)

Analysis and Discussion

Initially, Dylan *participates* in thinking about his past formulations about problem solving and writes these ideas in the blog post that he publishes on June 25th. Within this process of *participation,* Dylan engages in a process of *reification* when formulating his 'pyramid of abstraction' metaphor. He also produces a public trace of this process, which can be seen as the product of his *reification*. This trace is made even more public than a blog post because Dylan also links to it on Twitter.

On July 27th, Dylan *participates* again in this same manner by posting a blog post and linking to it on Twitter, but this time with a focus on recapturing his experiences from a professional development encounter. In this process of *participation*, he uses a hyperlink to reference a *reification* he made on June 25th regarding his formulation of the 'pyramid of abstraction' metaphor. In this way, the June 25th *reification* has prompted further *participation* around this topic on July 27th, resulting in a new *reification*, and a *re-negotiation* of meaning. The concept now has a rich history, and is traceable through the use of hyperlinks.

When Michael publicly agrees on Twitter with Dylan's metaphor, a *participation* and *reification* in itself, it makes Dylan's 'pyramid of abstraction' *reification* even more public, and catches the attention of Dan fifteen days later, who then challenges the metaphor. The power of Dan's *reification* (3:55) is that it is pervasively public. Dan currently has 46,324 followers, Michael has 4,761, and Dylan has 1190. The nature of Twitter is that if one member tags another in a post, only those who follow both members see the post in their feed. Since Dan, Michael, and Dylan share followers as mathematics teacher bloggers, the number of people who see such a post is large.

This now very visible *reification* prompts a series of *participations* and *reifications* from all three of these members as well as from any 'lurkers' who may be reading. In particular, Dan's question about the apex of the pyramid (3:55) is a *reification* that prompts *participation* from both Dylan and Michael, who *reify* their interpretations of the 'pyramid of abstraction' metaphor. Dylan *reifies* his focus on knowledge transfer as the implication of the pyramid metaphor (4:23), and Michael *reifies* his vision of the pyramid metaphor as a skill family (4:13) that ultimately comprises mathematics (4:24). In this way, the *meaning* of the metaphor is being *negotiated*.

Dan subsequently *participates* by posting that he does not think Dylan's explanation works (4:29) and that there can be no 'top' to abstraction (4:30). This *reification* stimulates a *re-negotiation* of the term 'abstraction.' Dylan *participates* further by *reifying* his focus on knowledge transfer (4:38) while Michael *participates* by *reifying* that the focus should not be on defining abstraction, but rather on how students bundle strategies in mathematics (4:39-4:40). This shifts the *negotiation of meaning* to a focus on the applicability of this metaphor to mathematical learning, and Dylan *reifies* that his intent with the metaphor is to explain that students need multiple representations of a concept in order to be able to abstract mathematical meaning (4:55). Dylan then restates his impressions of the conversation and refines his concept of the 'pyramid of abstraction' metaphor in the blog post he publicizes a few hours later. This post may be seen as a reflective form of *participation* in which Dylan *reifies* a heightened level of awareness regarding the 'pyramid of abstraction' metaphor. Perhaps the most powerful *reification* Dylan makes in this blog post is that 'abstraction' is not ubiquitously defined. This ultimately reflects Dylan's experience of *meaning* in this context.

Wenger (1998) states that "having a tool to perform an activity changes the nature of that activity" (p. 60). It is clear that the blogging tool has done just that for these mathematics teachers because unlike in a face to face discussion, the practice of blogging results in *participations* that directly produce permanent, public, and traceable *reifications*, making them prime contenders for prompting *participation* and holding participants accountable for their statements. As such, the

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continuity of *meaning* within this medium clearly does not suffer as Wenger (1998) warns can happen if too much emphasis is placed on either *participation* or *reification*.

Further, the asynchronicity of the medium allows participants to take time between responses, which may imply various degrees of *participation* in the practice, and in turn, various degrees of *reification*. Unfortunately, Wenger's (1998) construct does not provide a mechanism for such differentiation. He defines each component broadly, including a wide variety of interactions, most of which would occur in a face to face setting. However, in the case of blogging, *participations* include not only writing posts, but also restating, questioning, and devising examples, while *reifications* include not only publicized posts, but also ideas that have now attained a state of 'thingness' in the community, which here is most prominently the 'pyramid of abstraction' metaphor.

Conclusions

I have illustrated that the blogging medium changes the nature of mathematics teacher discussions because its design allows for *participation* and *reification* to be closely intertwined in a way that their co-evolving and co-implicated relationship drives the *negotiation of meaning* among participants, ensuring continuity of *negotiation*. Such continuity is an important affordance of the mathematics teacher blogosphere experience, and it can be attributed to three important features: asynchronicity, permanency, and publicity. Asynchronicity allows users to *participate* and *reify* to different degrees depending on how long they take to respond, permanency allows *reifications* to prompt further *participation* even days or months later, and publicity allows *reifications* of *participations* to be visible by many 'lurkers,' whose presence makes *participants* accountable for their *reifications*.

I have also revealed that a more nuanced treatment of *participation* and *reification* is needed for mathematics teacher blogging in particular. Some *participations* are quick and spontaneous tweets, while others are prolonged and accompanied by reflective activity. Some *reifications* are mere traces, while others are concepts such as a 'pyramid of abstraction.' Heightened levels of awareness resulting from a prolonged *negotiation of meaning* can also be considered *reifications*. There is currently no terminology within Wenger's (1998) construct to refer to such higher order *participations* and *reifications.*

Finally, the construct of *negotiation of meaning* also has the power to expose *what* is being negotiated. In this case, teachers found it valuable to invest time into engaging in *negotiating the meaning* of 'abstraction' and the metaphor of a 'pyramid of abstraction' as opposed to a 'ladder of abstraction.' Looking for more of these instances may help identify teaching-related issues of interest to mathematics teachers. Most evidently, the development of ideas can be traced within the blogging medium. Each idea has a history and a future. The history is traceable, and the future is quickly unfolding.

Endnotes

[1] *Storify.com* is an online service that allows users to curate content from various social media sources, and arrange it in any order, with narration if desired. It also makes it possible to automatically arrange content chronologically.

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