EXTENDING HORIZON: A STORY OF A TEACHER EDUCATOR

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We present a story of a teacher educator's response to a 'disturbance' and describe how her experience enhanced her personal mathematical knowledge and influenced her teaching. In our analysis we attend to different levels of awareness that support a teacher educator's work and illuminate the qualities of a teacher educator's knowledge, in particular, knowledge at the mathematical horizon.

Keywords: Mathematical Knowledge for Teaching, Teacher Knowledge, Advanced Mathematical Thinking

Numerous studies on mathematics teacher development have demonstrated that mathematics teachers learn through their teaching experiences (e.g., Leikin and Zazkis, 2010). This learning is multi-faceted and includes personal pedagogical growth, gaining further insights into students' thinking, learning about the feasibility of various instructional approaches, learning about implementing new curricula or technological tools, as well as enhancing personal mathematics. We extend this research on teachers' "learning through teaching" by focusing on a teacher educator, a teacher of teachers.

FOCUS ON TEACHER EDUCATORS

Research on the work of teachers of mathematics has devoted significant attention to teachers' knowledge and its various facets. Consequently, research on the work teacher educators follows suit (e.g., Jaworski, & Wood, 2008). This is evident, for example, in the working session at the International Group for the Psychology of Mathematics Education Conference on "Mathematics teacher educators' knowledge" (Beswick, Goos & Chapman, 2014.) Within multifaceted discussions on educators' knowledge, our focus is on personal mathematical knowledge in the work of teaching (Watson & Chick, 2013). In addition to knowledge of a teacher, personal mathematical knowledge of a teacher educator includes the ability to mobilize his/her knowledge in supporting teacher development. In particular, this support includes task design aimed at enhancing teachers' mathematical and pedagogical knowledge (Liljedahl, Chernoff & Zazkis, 2007). Particular qualities of teacher educators' knowledge that enable task design in support of teacher development, and how this knowledge develops and manifests, continue to be open areas for investigation.

THEORETICAL CONSTRUCTS

To gain insight into a teacher educator's personal mathematics knowledge, how this can develop through the act of teaching and the subsequent implications for teaching, we rely on an integration of two theoretical perspectives: Mason's *levels of awareness* (1998), and an extension of the notion of *knowledge at the mathematical horizon* (Zazkis & Mamolo, 2011).

Levels of Awareness

According to Mason (1998), awareness in and for teaching has 3 three different forms:

- *Awareness-in-action*: the ability to act in the moment. This level of awareness in teaching is recognized when a teacher poses a certain question, corrects a mistake, suggests an answer or selects a task, but is unable to justify or explain his/her choice.

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- *Awareness-in-discipline*: awareness of awareness-in-action. This awareness is essential in order to articulate awareness-in-action for others. According to Mason, the one important distinction between the two kinds of awareness involves the ability "to do" in contrast with the ability to instruct others. Teachers who possess awareness-in-discipline are able to articulate the choices they make in instructional situations.
- *Awareness-in-counsel*: awareness of awareness-in-discipline. This awareness is essential in order to articulate awareness-in-discipline for others. It describes one's sensitivity to what others require for building or enhancing their awareness.

Each form of awareness may refer to being explicitly or potentially aware, where the former emerges from the latter by noticing change (shifts of attention). Thus, if we wish (and we do) that mathematics teachers are able to articulate the choices they make for pupils in instructional situations (awareness-in-discipline), then a teacher educator must develop sensitivity to what may foster awareness-in-discipline for her students (and so possess awareness-in-counsel).

Knowledge at the Mathematical Horizon (KMH)

As an extension of Ball and Bass' (2009) notion of horizon content knowledge, KMH is identified by Zazkis and Mamolo (2011) as a teacher's use of mathematical subject matter knowledge "beyond" the requirements of school curricula in a secondary or elementary school teaching situation. Ball, Thames, and Phelps (2008) describe horizon knowledge as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (p. 403). This notion was developed by Ball and Bass who focused on teachers' knowledge of "students' mathematical horizons."

Zazkis and Mamolo (2013) extended the notion to teachers' horizons by making a connection to Husserl's philosophical description horizon, which relates to an individual's focus of attention. In particular, when an individual attends to an object (be it conceptual or physical), the focus of attention centers on the object and (some of) its properties, while the 'rest of the world' fades to the periphery and thus exists in the object's horizon (Follesdal, 2003). A teacher's horizon knowledge can therefore be interpreted as an awareness of a mathematical object's periphery, and can be characterized by a flexibility in focus of attention such that relevant properties, generalities, or connections, which embed the object within a greater mathematical structure, are accessed in teaching situations. A teacher's KMH is influenced by her propensity for exploring and studying new (for her) mathematics; teachers of different (school) levels may have vastly different objects of focus, different senses of the mathematical 'landscape', and thus different breadths to their horizons.

We draw a link between a teacher's KMH and Mason's notion of awareness-in-discipline, as knowledge of how mathematical topics are related as per their specific properties or underlying general structure provides a basis for understanding and articulating mathematical choices made in teaching. When considering a teacher educator's KMH, we note that both the objects of focus and the breadth of horizon extend beyond the awarenesses of the mathematics teacher. In what follows, we seek to refine the notion of KMH to suit studying the work of mathematics teacher educators.

Research question: How can the work of a teacher educator be explained in terms of levels of awareness and knowledge at the mathematical horizon?

THE STORY IN TWO ACCOUNTS

In presenting our story of Naomi – an experienced teacher educator, who has taught both content and methods courses in mathematics education – we follow Mason (2002) in distinguishing between *account-of* and *accounting-for*. The term *account-of* refers to a brief but vivid description of the key elements of the story, suspending as much as possible emotion, evaluation, judgment or explanations. This serves as data for *accounting-for*, which provides explanation, interpretation, value judgement

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or theory-based analysis. Naomi, the main character of our story, is an amalgamation of the authors' experience. This is consistent with the narrative inquiry methodology, where "narrative inquiry is aimed at understanding and making meaning of experience" (Clandinin & Connely, 2000, p. 80).

Account of

Naomi's class of prospective elementary school teachers, working on conversion of square units, considered the following "area problem":

An architect is building a model of a park, in which every 10 meters are represented by 3 centimeters. There is a lake in a park. The area of the lake is 7200 square meters. What is the area of the lake in the model?

The problem and its solution scheme paralleled a task that was previously discussed and resolved in class. The approach in class had been to build a proportion to solve the problem, and Naomi expected students to do the same in this case. The solution scheme for the task suggested:

Since 10 m are represented by 3 cm, then 100 m^2 are represented by 9 cm^2.
Possible proportions useful for solving the problem include: $\frac{7200}{x} = \frac{100}{9}$ *or* $\frac{7200}{100} = \frac{x}{9}$,
which yield as a final answer: $x = 648\ cm^2$.

While Naomi expected students to note the relationship that 100 square meters are represented by 9 square centimeters and use this for resolving the problem, not all students took that approach. Mikey, for instance, composed a different solution:

7200 = 80 × 90
if 10 meters = 3 cm, then
80 meters = 24 cm
90 meters = 27 cm
Then, 24 × 27 = 648, so the area of the lake on the model is 648 cm2.

Naomi recalled the following conversation with Mikey:

Naomi: Mikey, please explain how you got your answer.
Mikey: If the lake had length of 90 meters and width of 80…
Naomi: Did you know the shape of the lake?
Mikey: So if it were a rectangle…
Naomi: Have you ever seen a rectangular lake?
Mikey: I pretended it was, and I got the correct answer.
Naomi: And what if it were a different rectangle, would you get the same answer?
Mikey: I have not checked, but this one worked.

Upon additional prompting, Mikey could not explain why his approach was appropriate; "it worked" seemed to be a sufficient reason to accept its correctness. Naomi, unsure at that moment how to explain the situation, diverted the issue, pointing out that Mikey's representations were inappropriate and that the equality sign in the presented solution was repeatedly misused (such as in claiming "*10 meters = 3 cm").*

Upon reflection, Naomi realized why assigning random measures results in a correct answer. One can think of the shape in the model as an image of the shape of the lake under the transformation of dilation/scaling. This transformation preserves area-relations in a way that equal area shapes are always transformed to equal area shapes. This invariance under transformation is why the student's solution resulted in a correct answer: It calculated the area of a particular scaled shape, and this area is invariant across all the shapes that are images under dilation of the shapes with the same area. While this understanding of mathematical structures was accessible to Naomi, it was not immediate

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and it did not seem evident in her teacher candidates, and as such she saw the 'incident' as an opportunity to develop related knowledge among her prospective secondary mathematics teachers.

Naomi developed a task in which she presented a problem and a solution, similar to that of Mikey's, and asked her students to imagine how a conversation between a teacher and student addressing the problem could continue, and to present it in a form of a scripted dialogue. Furthermore, Naomi invited her students to include commentary on how they personally understand the situation and to explain it to a "mathematically mature" colleague. These invitations were aimed at directing possible shifts of attention amongst teacher candidates' that could foster awareness-in-discipline as they attempted to articulate why specific instructional moves were made.

The scripts produced by prospective teachers served as a springboard to class discussion. This included consideration of a variety of pedagogical approaches, but focused on why the student's solution resulted in a correct answer. (A precise mathematical reason was absent in the scripts). Naomi introduced the concept of transformations of dilation/scaling using the dynamic geometry software Geometer's Sketchpad. (The software was familiar to the students, but the particular transformation was not). Her students used the software to explore the features of this transformation, including its invariants.

Accounting for (or Analysis)

Naomi's encounter with Mikey elicited for her a moment of disturbance – it was an unexpected solution and one for which she could not come up with an immediate and satisfactory, at least for herself, rebuttal. In response Naomi revisited the mathematics herself and in doing so, her awareness was broadened on two counts – first with respect to the aforementioned mathematical connection, then with respect to the pedagogical value of addressing such an interpretation as Mikey's.

As a teacher of mathematics, has Naomi learned mathematics as a result of her encounter with the unexpected student solution? After all, she was sufficiently familiar with the transformation of dilation in order to explain the phenomenon mathematically. The situation is similar to that described by Leikin and Zazkis (2010) in which teachers are hesitant to admit learning mathematics through teaching. Often their inability to use relevant mathematics in an instructional situation was explained as "I knew this, but have never thought about it". We agree with Leikin and Zazkis in their claim: "we consider anew "thinking about it" – when an instructional situation presents such an opportunity – as an indication of learning. In this case LTT [learning through teaching] can be thought of as transferring existing knowledge from teachers' passive repertoire to their active one" (p.19).

In Naomi's case she drew a connection between modeling tasks and particular geometric transformations, notions that were not explicitly linked for her before the experience of disturbance. This new connection highlights a change in Naomi's KMH: it broadens the periphery of the modeling task to include connections to the (initially) seemingly disparate topic of invariance under dilation. Knowledge of such connections corresponds to knowledge of major disciplinary ideas and structures and as such instantiates one of the elements of Ball and Bass's (2009) conception of horizon knowledge. Further, it embeds the particular mathematics originally at Naomi's focus of attention (conversion of square units) within a broader mathematical world. It was her awareness of the 'broader world' which allowed her to articulate for herself and her students why the particular features of the geometric transformation of dilation allow the random assignment of convenient measures to lead to a correct result.

This situation illustrates how knowledge beyond the specific curricular requirements is pertinent to mathematics teaching. A teacher's personal mathematical knowledge provided a deeper appreciation of the mathematics at hand, as well as a broader view of the salient features of the content. The technique of applying a transformation or mapping that preserves certain properties allows an individual to shift from working in a potentially unfamiliar space to one in which known properties aide in resolving the problematic situation. Such values and sensibilities, typically

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acquired in advanced mathematics courses at the undergraduate and graduate levels, embed particular topics or concepts within a broader, more general, mathematical world, and as such are included within a mathematical object's outer horizon. These sensibilities correspond to the habits of thoughts and methods of resolutions that Mason attributed to *awareness-in-discipline.* They provide insight into why certain relationships are true and aide in the articulation of said truth.

The goals of the "area problems" were focused on appropriate conversion of square units, building proportions and developing proportional reasoning and computational relationships. The moment of disturbance provoked a shift in attention, redirecting Naomi's line of vision towards alternative mathematical ways of reasoning with the task. This shift enabled a formal resolution and it illustrated not only the facts and techniques of the discipline of mathematics, but also the "habits of thought, forms of fruitful questions, and methods of resolution of those questions" (Mason, 1998, p. 259) characteristic of awareness-in-discipline. However, in developing awareness-in-counsel, as a teacher educator, Naomi's aim is to share this interesting link with prospective teachers and in turn support their own development of awareness-in-discipline.

As a teacher educator, Naomi developed a task for prospective teachers, which has both mathematical and pedagogical value. We suggest that it was Naomi's awareness-in-counsel that invited her to reflect on the contingency of the situation as one which would be of benefit in preparing teacher candidates and in turn she developed an activity that could help broaden her students' perspectives. As Mason put it, awareness of awareness-in-discipline "is what supports effective teaching of that discipline" and allows "for structuring tasks and encounters from which [students] can learn" (1998, p. 260). In what follows, we offer an account-for that attends to Naomi's awarenesses of the discipline of mathematics-teaching.

In particular, clearly, the immediate goal for prospective teachers was to imagine an interaction with a student and present it in the form of a dialogue. In doing this, the prospective teachers considered their pedagogical response to a mathematical dilemma presented by the student's solution. However, Naomi's goal extended beyond involving teachers in playwriting. She considers the task as a vehicle for (1) extending personal mathematical experiences of teachers beyond the prescribed curriculum by drawing stronger connections between various topics in mathematics (e.g., units conversion and transformations); and (2) anticipating possible solutions and evaluating potential reactions to those. Both of these goals connect to fostering teachers' KMH – broadening the peripheral vision of teachers as they focus on particular mathematical objects by connecting the objects both to other mathematical topics, as well as to how students could interpret and reason with those objects based on their prior learning experiences.

The scripting task builds on growing research which attends to various pedagogical motivations for involving students in playwriting (e.g., Zazkis, Sinclair, & Liljedahl, 2013; Zazkis & D. Zazkis, 2014). The benefits of this approach were summarized for prospective teachers, for researchers and for teacher educators (ibid.). Focusing on the latter, the scripts or plays written by prospective teachers can be used as a tool for teacher development. In particular, the scripts can be used to discuss and highlight appropriate pedagogical approaches, to direct further attention to learners and their thinking, and to shift prospective teachers' thinking about preparation to instruction, beyond the traditional "lesson plan". These activities nurture shifts in attention that may raise teachers' awareness of the contingent nature of teaching while providing them with experiences from which they may later draw when responding to unanticipated scenarios.

In an attempt to foster awareness-in-discipline (of mathematics) for her students, Naomi set a task that could provoke a similar disturbance in others and thus trigger a shift in their awareness. Such tasks are intended to "provoke students into rehearsing or exercising skills, but which at the same time attract their attention away from the skill to be automated" (Mason, 1998, p. 259). The skill of responding to student interactions (with appropriate pedagogical sensitivities and strategies)

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was to be rehearsed through the particular task developed by Naomi, but attention was meant to be attracted by the unexpected mathematics.

Naomi's use of scripting tasks exemplifies her awareness-in-counsel of the benefits for preparing teachers in this way – the essential awareness for enhancing the awarenesses of others manifested in the design and development of the task, as well as in the follow up task and discussion. In addition, this particular task developed by Naomi not only invites teachers to examine their approaches to a student solution, but also invites them to revisit their personal mathematics and think mathematically in an unfamiliar situation.

The scripting approach allows the discussion to revolve around the particular approaches presented by the group of prospective teachers. Without singling out a particular student, Naomi initiates a discussion by presenting several different approaches from the scripts and inviting a reaction. The 'scripting-first' tactic provides all students with an opportunity to think and rethink their reaction. This usually does not happen in a whole-class discussion, where the first speakers may influence the ways others consider the task. Further, this opportunity to think and rethink the reaction does not have time constrains. The students may consult any resource or each other, should they choose to do this. This an important difference between script-writing and role play, where the player has to "think on her feet", rather than provide a thoughtful and possibly adjusted response. The time to provide a thoughtful response to a moment of contingency, as a structural aspect of the scripting task, creates space for deeper mathematical engagement – and thus enhancement of teachers' KMH – than is afforded by role play activities. The opportunity to imagine and re-imagine a teaching scenario is geared towards enhancing teachers' noticing, both by completing the task as assigned as well as considering the approaches of classmates in the subsequent classroom discussion. Structuring a task such that students are provided with such an opportunity again shows Naomi's awareness of what are important encounters from which to develop knowledge at the mathematical horizon.

Thus, Naomi's response to a disturbance, coupled with her choice to draw to the attention of her prospective teachers the contingent nature of teaching, illustrates the awareness-in-counsel of a teacher educator.

AWARENESS OF HORIZON OF TEACHER EDUCATORS

When Mason (1998) identified different levels of awareness, the intended discipline in his descriptions was the discipline of mathematics. In our analysis of the work of a mathematics teacher educator we point to yet another discipline – that of mathematics-teaching. With this view, a teacher educator's awareness-in-counsel, which is the awareness that is employed to develop mathematics teachers' awareness-in-discipline (of mathematics), can also be described as the teacher educator's awareness-in-discipline of mathematics-teaching. This awareness results in the development of mathematically salient scenarios that could provoke a moment of disturbance for prospective teachers. We turn our attention now towards the mathematical knowledge required to notice and appreciate such saliency.

We suggest that it is the knowledge at the mathematical horizon of teacher educators (KMHTE) that includes knowledge of what mathematics will catch the attention of prospective or practicing teachers and provoke a disturbance that enables the shifts of attention required to develop their own horizon knowledge (KMH). The disturbance which shifts attention must also provide an invitation to extend the mathematical awareness of teachers in connection to what is applicable for students. It must provide opportunities for teachers to respond without readily available routines and thus incite a shift from awareness-in-action to awareness-in-discipline. At the focus of attention for Naomi is both the specific mathematics content, as well as its contextualization in student/teacher thinking. At the periphery of Naomi's attention is the horizon, accessible via her awareness-in-counsel.

Analogously to a teacher's KMH, we suggest that a teacher educator's KMHTE may be described as having both an inner horizon and an outer horizon. As before, we conceptualize horizon

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as the connections, features, and generalizable properties related to an object of thought and which embed that object within a greater structure. At the inner horizon are features of that object, which are specific to it, yet not at the focus of attention. At the outer horizon are the major disciplinary ideas, practices, and underlying structural components which situate the particular within the general. With respect to KMH, the object of focus is mathematics and as such the features, ideas, practices, and structures of the horizon are also mathematical.

For KMHTE, the object of focus is student/teacher thinking about mathematics, and as such, the features, ideas, practices, and structures of the horizon include a breadth and complexity not already encompassed by KMH. Awareness-in-counsel broadens a teacher educator's ability to direct (and retain) teachers' attention toward mathematical connections and structures, such that they enhance both their mathematical understanding, as well as their abilities to convey relevant aspects of this understanding to (and for) their future students.

CONCLUDING REMARKS

A mathematics teacher educator's disciplinary knowledge is multi-faceted and complex. It combines awareness of pedagogical practice, of mathematical content and processes, of the cognitive, social, affective, and environmental factors associated with thinking and learning. Indeed, an exhaustive list of the "required" knowledge for mathematics teacher education would be near impossible. We focused our attention on the personal mathematical knowledge of an experienced mathematics teacher educator and described how this knowledge was triggered and further developed. Inasmuch as mathematics may not be separated from mathematics pedagogical knowledge, mathematics may not be separated from mathematics education pedagogical knowledge – it is an essential component of the knowledge required to foster the professional growth of the individuals tasked with the responsibility of supporting students in their learning of mathematics.

Various researchers have suggested that teachers' personal mathematical knowledge makes a difference in their ability to plan for instruction (e.g., Watson & Barton, 2011) as well as respond to teaching situations in general, and situations of contingency in particular (e.g., Chick & Stacey, 2013; Rowland, Huckstep, & Thwaites, 2005; Watson & Chick, 2013). We extrapolate that this suggestion is also applicable for teacher educators. The main difference, however, is that the 'students' of a teacher educator are (in our case prospective) teachers, and therefore the teaching situations adhere to the learning of teachers. With this in mind, we illuminated some of the qualities of a teacher educator's mathematical knowledge for teaching – specifically, knowledge at the mathematical horizon – and her extended awarenesses.

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