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Manifestations of Middle School Learners' Problematization Activity using *Flatland* as a Case of Alternative Perspective

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Abstract

In order to address concerns with the underrepresentation of spatial thinking in K-12 curricula, (National Research Council, 2006), Valentine and Kopcha (in press) designed and implemented a learning environment integrating cases as alternative perspective (Jonassen, 2011) in the context of eighth-grade mathematics. The design aimed to provide learners opportunities to investigate multiple representations of space and dimension concepts. Among the seventy cases, *Flatland: The Movie* (Caplan, Wallace, Travis, & Johnson, 2007) and *Flatland: A Romance of Many Dimensions* (Abbott, 1885/1991) operated as sources for learners' mathematization activity. This paper shares phenomenological manifestations of learners' experiences, focusing on their problematization of space and perspective concepts. Learners' problematization was captured as classroom discourse, student blog postings, and phenomenological interviews. Data indicates that both versions of *Flatland* acted as provocation tools, led to impasses, and operated as a source for questions and conjectures.

Keywords: cases as alternative perspective, geometry, mathematics education, perspective, phenomenology, problematization, spatial thinking

Manifestations of Middle School Learners' Problematization Activity using *Flatland* as a Case of Alternative Perspective

This year marks the centennial year for the American Educational Research Association (AERA), a society dedicated to improving education. It also marks a time that the National Council of Teachers of Mathematics (NCTM) released "grand challenges" for mathematics education, including "changing perceptions about what it means to do mathematics" (NCTM Research Committee, 2015, p. 139). The research reported in this paper aims to address this challenge by illustrating the multiple experiences and activities of middle school learners engaged with the mathematics of space, specifically the ways they problematized their world spatially.

The 2011 NCTM Annual Meeting conference theme was, "Geometry: Constructing and Transforming Perspectives." The choice to use the term "perspectives" rather than "figures" or "shape" opens up the way mathematics educators might conceptualize the geometric activity of learners. Rather than foregrounding defining properties of shape and mathematically denoted transformations of figures on a coordinate plane, the council chose to highlight the way perspectives are constructed and transformed (geometrically, epistemologically, and ontologically). The opening keynote included a discussion of the film, *Flatland: The Movie* (Caplan, Wallace, Travis, & Johnson, 2007), based on Abbott's (1884/1991) book, *Flatland: A Romance of Many Dimensions.* The fictional story portrays characters grappling to conceptualize the relationship between dimensions, even dimensions higher than one can embody (e.g., two-dimensional Flatlanders contemplating a three-dimensional space). In this sense, conceptualizing a higher-dimension, off-limits visually and physically, requires conjectures about perspective and the way perspectives are embodied and bounded by dimensions. In this paper, *Flatland* supported conjectures about perspective, acting as a "geometric gift" for learners to construct and transform perspectives, similar to Fröebel's conceptualization of objects young learners might manipulate to learn about their inherently geometric world. According to mathematician Banchoff:

> Fröebel and his colleagues created geometrical gifts from materials available to them, primarily wood, paper, and clay. Today we have the means to improve on the many gifts in many ways – with plastic and Velcro, with tape and magnets, not to mention with the powerful computer graphics. The educator's term "manipulatives" – classroom materials – takes on new meaning when we can put in front of a young student a tool to manipulate not only simple forms but also the very geometry of higher dimensional space. If we care about educating our children toward the perception of space, we should create truly stimulating manipulatives – geometrical gifts for our day. (1990, p. 14)

Purpose

This phenomenological case study seeks to add to the research concerning spatial thinking, in particular, understanding how learners' productive problematizing of space and perspective manifest. Understanding the variant manifestations of learners' experiences may, in turn, change perceptions about what it might mean to do mathematics. The National Research Council (NRC) stresses the need for researchers to increase attention to learners' spatial thinking, operationalized as knowing, representing, and reasoning about space. They argue that not only is spatial thinking "integral to everyday life," but also is "a missing link" across K-12 curriculum (National Research Council, 2006, p. 7). Although there are multiple ways to conceptualize spatial thinking, this paper draws on Hegarty and Waller's (2004) definition of two primary activities, orienting and visualizing. Spatial orientation is synonymous with perspective taking, or "making egocentric spatial transformations," while spatial visualization is synonymous with making "object-based spatial transformations" (p. 176).

Theoretical Framework

Cases as alternative perspective (CAPs) were used in this project as *triggers* to support problematization activity. CAPs is a term used by Jonassen (2011), who advocates their use when the aim of learning seeks to convey the complex and ill-structured nature of phenomena. CAPs are intended to change learners' underlying ways of thinking that Spiro and colleagues saw as central to supporting cognitive flexibility (e.g., Jacobson & Spiro, 1993; Spiro, Coulson, Feltovich, & Anderson, 1988). CAPs problematize being, seeing, and moving by supporting learners to attend to similarities and differences across multiple contexts (e.g., art, photography, video, gaming). They can take the form of video, text, symbols, photos, animations, and simulations and are typically embedded in a hypermedia website to revisit during the course of learning. Two cases in particular will be explicated as they relate to learners' problematization: *Flatland: The Movie* and *Flatland: A Romance of Many Dimensions* – these were the first two CAPs investigated by learners to problematize space and perspective.

Problematization is the act of troubling one's own experiences in order to understand the world (and space) more fully. This is similar to John Dewey's (1929) notion of "reflective inquiry" as interpreted by Hiebert et al. (1996): "Familiar objects, including subject matters in school, are treated as "challenges to thought…They are *to be* known, rather than objects of knowledge…[t]hey are things *to be* understood (Dewey, 1929, p. 103, emphasis in original)" (p. 15). In this sense, learners are iteratively cycling between being triggered by problems and problematizing their world. For this project, the aim was to support learners to construct multiple perspectives, develop flexible concepts of space and dimension, and provoke them to consider their assumptions as part of the learning environment.

Using a hypermedia site (see http://spaceandperspective.com/) with learners allowed them access to seventy CAPs to problematize space and perspective. The design of cases drew on principles from Cognitive Flexibility Theory (CFT) and Realistic Mathematics Education (RME). CFT (see Spiro et al., 1988) is a learning theory suggesting ways to design learning environments to support learners' flexible cognition and avoid reductive biases and misconceptions in later learning. This is primarily done by juxtaposing cases to support learners to represent knowledge in multiple and interconnected ways. RME (see Freudenthal, 1973) is a mathematics educational framework that advocates mathematizing real-world phenomena as starting points in mathematics teaching and learning. The learning environment design, including the design of the hypermedia site and CAPs is described in more detail in Valentine and Kopcha (in press).

Research Method

This post-intentional phenomenological study stems from a four-year design-based research project supporting and investigating learners' shifts in perspective (Valentine, 2014). The research question for this paper asked, "What it is for learners to problematize space using *Flatland* as a case?" Post-intentional phenomenology (Vagle, 2010, 2014) is a recent form of phenomenological research that guides researchers' investigations into questions about living with, in, and through phenomena. This form of living inquiry research emphasizes data as active and generative (see St. Pierre, 2013), recognizing that "findings," or rather tentative manifestations of a phenomenon, resist the traditional notions of data as fixed and static. Rather than claim to describe or interpret in a finalizing way, phenomena, and humans experience living with them, are viewed as manifesting in ways that are dynamic, contextual, non-neutral, embodied, and cultured. There is not a singular method for conducting phenomenological research, however, this research design uses Vagle's (2010, 2014) five component process, including identifying the phenomenon in it's multiple, partial, and varied contexts, devising a data collection plan, making a reflexivity plan, analyzing data using a whole-part-whole reading and writing, and crafting a text that captures the phenomena. Table 1 below shows the research activities related to each of the five components. See Valentine (2014) for more details concerning the philosophical and methodological foundations for post-intentional phenomenology and using it in conjunction with design-based research.

Table 1

Vagle's (2010, 2014) Five Component Process for Conducting Post-Intentional Phenomenological Research and Associated Research Activities

Research Component	**Research Activities**
1. Identify a phenomenon in its multiple, partial, and varied contexts (Vagle, 2010, p. 9; 2014)	a. State the research problem, b. Partial review of the literature, c. Philosophical claim, d. Statement of the phenomenon (research question(s)), including an intentionality statement, e. Contexts, f. Participant selection (Vagle, 2010, p. 10-12; 2014)
2. Devise a clear, yet flexible process for gathering data appropriate for the phenomenon under investigation (Vagle, 2010, p. 9; 2014)	a. Select data sources, b. Align data sources with research questions (Vagle, 2010, p. 15; 2014)
3. Make a reflexivity plan (Vagle, 2014)	a. Create a reflexivity journal, b. Write an initial reflexion statement, c. Continue the reflexivity process as you gather and analyze data (Vagle, 2014)
4. Read and write your way through your data in a systematic, responsive manner (Vagle, 2010, p. 9; 2014)	Whole-part-whole analysis plan described below (Vagle, 2010, p. 18)
5. Craft a text that captures tentative manifestations of the phenomenon in its multiple, partial, and varied contexts (Vagle, 2010, p. 9; 2014)	a. Re-state the multiple and varied contexts, b. Brainstorm potential forms (Vagle, 2010, p. 21; 2014)

Participants and Data Sources

Twenty-one eighth grade learners participated in the study. The two classes with twelve and nine learners met 12 times over the course of 3 months, for an average of 100 minutes each meeting. Table 2 below indicates the data contribution from participants informing this study, broken down by class section.

Table 2

Participants by Class Section and Data Contribution

Class Section (N = number of students)	Audio/ Artifacts	Blog Postings, Photos, Video	Lived-Experience Descriptions	Interviews
8a (N=9)	9/9	3/9	1/9	2/9
8b (N=12)	12/12	10/12	3/12	3/12

Data sources included lived-experience descriptions provided by learners, interviews, follow-up emails, observational data (audio, video, and photo), artifacts from lessons, student blog postings, and the researcher's reflexion journal.

Whole-Part-Whole Analysis

Vagle (2010) describes a process for whole-part-whole data analysis used in this project. First, the entire data set was read as a whole event, but not analyzed. This was followed by multiple line-by-line readings and the insertion of comments/questions on transcripts, artifacts, and texts. Follow-up questions were created for each participant. The second line-by-line reading focused on making meaning and took into consideration any notes, markings, follow-ups, and reflexion up to this point. This ended with a saved document for each participant. The third line-by-line reading focused on "articulating the analytic thoughts for each part for each participant." Subsequent readings "involved reading across individual participants' data, with the goal of looking for beginning tentative manifestations" (Vagle, 2010, p. 18-20). Reflexive journaling occurred throughout the entire research project.

Tentative Manifestations (Findings)

This section shares learners' problematization with *Flatland: The Movie* (Caplan, Wallace, Travis, & Johnson, 2007) and Abbott's (1884/1991) book, *Flatland: A Romance of Many Dimensions.* Findings are organized across three main manifestations of experience: *Flatland* as a source of provocation (both the movie and book versions), impasses articulated by learners, and questioning and conjecturing activity related to *Flatland*.

Flatland as a Source of Provocation

After watching the film, *Flatland: The Movie*, and reading the first few chapters of the book, *Flatland: A Romance of Many Dimensions*, eighth grade learners seemed eager to share their thoughts about perspective, dimension, and the problematic aspects of being a three-dimensional (3D) being watching a film that takes place on a two-dimensional (2D) plane. In an interview with Lynn, she recounts her initial reactions to the film:

> I think it was something that I hadn't really thought about ever. Like just all of these concepts had never crossed through my mind. And so I just remember being so infatuated with it and being like what! This is like a thing that happens!

For Lynn, *Flatland* triggered her awareness of dimensional relationships she never previously considered. Her blog post following the film shows this triggered awareness:

> The movie *Flatland* is an extremely informative and thought provoking movie. The beginning makes it clear what it would be like to live in a 2D world. The way that the government and society is set up was extremely interesting to me and made me personally think about how it is similar to society in our 3D world. As the movie progresses through the story, the concept of what a 0D and a 1D world is made very clear, a topic that I had never personally thought about. Then when the main character discovers a 3D shape, it showed me what 3D is in comparison to 2D. The brief thought at the end of the movie about a 4D world made me feel similar to the 2D shapes. The concept is so abstract it is almost scary. However, it is very much possible that a 4th dimension does exist.

Lynn not only writes about what she considers new dimensional ideas she learned from the movie, but she also ends by reflecting on the possibility of a 4^{th} dimension. Lynn's emotions are tied to ideas of dimension: abstract equals almost scary and the possibility of a 4th dimension. She even writes about the zero dimension (0D) and first dimension (1D) as a new idea to consider. Although Lynn has most likely worked with points and lines in prior mathematical learning, the idea of these as dimensions felt new to her. For Lynn, *Flatland* served as a source for conceptualizing dimensions in a way that is novel for middle school learners.

During book discussions in class, Alistar was especially interested in the parallels between "shapist" constructs (e.g., desired regularity of polygons, more sides equals higher class standing) and his understanding of Victorian society in the late 1800s. In an interview, Alistar describes the book version of *Flatland*: "With that book, there's a lot to be discovered in between the lines as kind of a metaphor." He wrote more about this "in between" space in his lived-experience description, articulating connections between phenomena he deemed impossible to explain on a 2D plane and how this motivated him to read the book:

> During the time we were reading *Flatland*, I started to realize that parts of the book that were not explained might not have been possible. If everything was flat, would laws like gravity and density actually apply? The comprehension of questions like these compelled me to read the book.

In this sense, what was elusive, "compelled" him:

> Well, I think that, the idea of why I had those questions was just thinking about the world. Since it was flat, like, if it rained, where would the rain go? Or like, if there was gravity, why aren't they like – are they stuck to the world or are they just always falling or something – I don't know. Is the world really flat or like propped up like this? [Gestures vertically with hands] That's what I could never really think about - because that wasn't really ever explained too much. So that's what I had questions about. And, I felt like I kept reading because I wanted to know if it ever explained that. And while they might not of, I still thought it was a good book, for sure. Even though I feel like there are – some things that I would have liked to know more about it, for sure.

Seeking to understand the world of Flatland, especially as it relates to common 3D phenomena, like rain and gravity, was Alistar's provocation for further reading and questioning. Later in the interview, Alistar continues to engage with the story of *Flatland*, connecting it to his identity as a gamer and budding game designer. Interestingly, a year after this investigation, Alistar signed up for high school courses in programming and film-making to carry out his goal of designing a dimensional-switching video game based on the story of *Flatland*. See Valentine and Jensen (in press) for more information about his video game design idea.

Jack and Albus were eager to talk about the book as well. In a joint interview, they expressed feeling similarly provoked by what they considered impossible phenomena described in *Flatland:*

Jack: I was trying to figure out how the problems would work with paint. I tried to figure out how paint would work and how water would work and gravity would work.
Albus: That's interesting
Jack: I kind of explained them I think.
Albus: I mean paint wouldn't make any sense, but you'd be able to – I mean because you can't hold anything, so in a 2-dimensional space
Jack: Yeah, well I said that if this was like a flat surface, paint would just be around it and that would be it and then because they only see the sides of things so that would look like it was paint.
I: I remember you actually talking about that. I think what you were saying is that it just wouldn't have any area – maybe?
Jack: The shape – I don't know. I said that it would have area, just that area wouldn't have paint on it. Things couldn't be on top of each other – it would just be around it.

As a result of reading *Flatland*, Jack is oriented towards thinking through the phenomenon of paint in planar *Flatland*. To give the reader a bit of context, in the story of *Flatland*, polygons started painting themselves during the color revolution in order to help (and sometimes trick) other polygons to recognize their shape. In Jack's discussion, he is distinguishing between the impossibility of paint "on" the surface of Flatland (which he sees as a layer and thus not part of the same plane as Flatland), recognizing "things can't be on top of each other" but rather "around it," which is the notion of perimeter. It is in identifying and working through these problems that *Flatland* shows itself to be a source of problematization.

Jack and Albus continue to problematize *Flatland,* especially the movie version. In the discourse that follows, they point out the problems with portraying a 2D world for 3D humans in movie form. The problems in translation most likely benefited the discourse, allowing for further problematization:

Albus: The movie was less realistic I think.
Jack: It is, the movie had to show it, but the book could only talk about it theoretically, but the movie had to show it or else it wouldn't actually – it would just be a book.
Albus: Yeah, and if things were constantly falling through space, I feel like the movie would have been less entertaining to watch. It would have been like nauseating. So I think that the book had the freedom to explain it without having to tweak anything.
Jack: Or without actually having to show it. It did use diagrams though but it didn't really have to show it all.
Albus: Yeah, it could just explain things that people – like it's easier to understand something when you're shown it, but I feel like it's harder to show something that's unshowable.
I: That's true, yeah. And you had talked about that. Like *Flatland: The Movie* was almost 3-dimensional in the sense that you have this top-down perspective.
Jack: Yeah, also there was different layers and stuff.
I: Oh right, with the background.
Albus: Stuff was moving on top of the background and that doesn't make any sense. Like that would have to be moving too with them, like they'd have to be on the same plane because they can't have multiple
Jack: Yeah.

Later in the interview, they were asked a follow-up question concerning the differences between the movie and book:

I: You said that the reason that the movie was different than the book was because they had to sort of – I mean you had to be able of see it [Jack: Yeah], so they had to make these choices in developing the film. But, did you think it changed the concepts? You think the concepts were represented differently?
Albus: I think slightly [Jack: yeah] but only, only in order to show it in the movie – I think because I mean they probably wanted to keep it the same concept but they wanted it to have characters and they wanted it to be simpler for probably younger kids to understand – to like give them a broader explanation of it for people who haven't really talked about perspective that much so like it was a good intro movie to watch – which is, I mean, that's how we used it in our class. And then we kind of used it to talk about the book and I think that the book is like kind of a secondary thing to go on to after *Flatland*, like to enhance your understanding of what would actually be going on in Flatland.

I: Okay, so do you think that the order was helpful?
Jack: Yeah definitely [Albus: Yeah] because in the movie there were a lot of little things that were wrong but you didn't really notice them until you actually dissected the movie. But the main thing was accurate and that led you into thinking about 2-dimensional space and 4-dimensional space. And the book actually helped you understand it.

The way they perceived *Flatland* differently across mediums is indicative of cognitive flexibility theory and the associated CAPs that seek to support learners in this type of differentiated noticing. Jack and Albus are able to see the complexities of a 2D plane and the misrepresentation of the plane because of a second variation of the case, *Flatland: A Romance of Many Dimensions*.

In all of the examples discussed so far, learners express *Flatland* primarily as a thought-provoking phenomenon, sometimes awakening their noticing about dimension in new ways as was the case for Lynn. For Alistar, Jack, and Albus, *Flatland* triggered a questioning stance about planar embodiment (gravity, rain, paint) and even the difficulties of representing 2D relationships in 3D space. The next section discusses another manifestation of learners' experience with *Flatland*, that of impasse. Although a separate section, this activity of being provoked is related to the experience of impasse.

Impasses Articulated by Learners

For Aristotle, an impasse provides the entry point into inquiry. He writes, "it is profitable for those who want to get through something well to do a good job of going over the impasses…it is not possible to untie a knot one is ignorant of" (Aristotle, 2002, p. 35). As indicated above by Jack and Albus, *Flatland* provoked them to consider impasses, or unresolved dilemmas related to problems inherent in viewing a 2D film. In class discourse following the film, learners expressed additional impasses related to the film, while also considering their embodiment as 3D beings and the complexity of ideas like higher dimensions and animal seeing.

Table 3 below presents ten main impasses that arose after viewing *Flatland: The Movie*. These impasses emerged from a 20-minute conversation with both classes jointly.

Table 3

Ten Main Impasses That Arose After Viewing Flatland: The Movie

Impasse	**Description**	**Example**
1. Implied thickness of Flatland	Flat objects moving over a planar background cannot happen if both are part of the same plane. Problematic that we can view a film taking place on a 2D plane.	The cloudy background moves independent of the polygon Hex. Something with height would be invisible.
2. 2D characters performing 3D activities	Eating and dragging objects, 3D activities, don't make sense in *Flatland*.	2D characters in film dragging objects and eating food
3. A. Square and his wife seeing and speaking through prison wall (line)	Is a line in 2D like a wall in 3D – can sound travel through it?	Class argues about the nature of a line in Flatland
4. 2D just as elusive as 4D space	Limited understanding of other dimensions not 3D	If the direction up describes 2D to 3D, what is the direction from 3D to 4D? How can you see 2D without height?
5. Gravity in 2D plane	Standing in 2D is hard to conceptualize	Gravity not the same in 2D
6. Are negative/partial dimensions possible?	If there are infinite positive dimensions, can it work in a negative direction	One student asked this question and the class tried to find examples, like the zero gravity fair ride.
7. Motion of 4D animations	Is motion part of the structure of a hypercube or 4D object?	Maybe time and space are made up constructs – more about perspective
8. 4D as time	Conjecturing that 4D is time	Learners have hear this concept in the past
9. What is it to see in 4D	Not able to see 4D	4D seeing may be like seeing all sides of a 3D object at once – maybe we see 2D and synthesize 3D
10. Animal seeing	Can we really know what animals see?	Class discussing animal seeing and human's seeing color

**Impasse 7 and 9 start to combine towards end of discourse*

Of the ten impasses, the first three point to problems inherent in a 3D rendition of a 2D world. As discussed by Jack and Albus in the previous section, the movie version of *Flatland* breaks the rules of a 2D plane, especially if one considers the embodied perspective, or point-of-view, of a Flatlander occupying the plane of Flatland. These inconsistencies in the film comprised the first three impasses shared in the discussion and include the implied thickness of

Flatland, 2D characters performing 3D activities, and a part of the film where Arthur Square and his wife seem to be able to see and speak to each other through the prison wall (line in Flatland). The discussion concerning problems representing 2D space in the film was vibrant, with many students building on each others' ideas and conjectures. For example, the first impasse, the implied thickness of Flatland, included examples about the clouds moving behind the character Hex. For students, these were two different Flatlands – an impossibility. Building on this, students started to question the very nature of even seeing 2D objects at all, leading to the fourth impasse of comparing the elusivity of the 2nd dimension to that of the 4th dimension.

The remaining impasses represent phenomena that learners were unable to access. This started with a conversation about the 2nd dimension as compared to the 4th dimension. It was agreed upon in the discussion that the 4th dimension is out of reach in our 3D world, although many students conjectured that the possibility of a fourth dimension, even an infinite number of higher dimensions, most likely exist. However, once the 2nd dimension was brought into question as being "off-limits," because of the height requirements to view objects in 3D space, students started bringing more phenomena into the discussion. These included considering 2D gravity, negative and partial dimensions, and animal seeing. Motion was a particularly salient impasse that learners continued to discuss throughout the project. The last scene of Flatland ends with an animated hypercube rotating in and out of itself. For learners, the idea of motion as part of the structure of the fourth dimension became an impasse and questions arose such as, "is the motion shown so that 3D beings can access and make sense of the fourth dimension or is it part of the structure of four dimensions?" Cases such as *Flatland* created these moments of impasse and were preceded by learners problematization activity. Rather than view these impasses as negative, they acted to propel learners to continue posing questions and conjectures, leading to many debates among learners. In the section that follows, more detail about these questions and conjectures are explicated.

Questioning and Conjecturing Activity Related to *Flatland*

A map was generated that compiled discussion points and questions from all 14 class sessions. The map revealed 83 ideas and questions put forward by learners. Sixteen of these related to content from both *Flatland* cases, while 44 ideas and questions addressed broader issues about perspective, motion, dimensionality, etc. Table 4 below displays example questions and conjectures posed by learners.

Table 4

Example Questions and Conjectures Posed by Learners

Questions Posed	• Do we see in 2D or 3D? • From the 2nd to the 3rd dimension, you go up or down. What's the direction we would discover about the 4th dimension? • Since numbers are infinite, are there an infinite number of dimensions? • Where does light come from in Flatland?
Conjectures Posed	• If dimension = N, then N-1 is what we see. • Maybe 4D is in and out, but experience in 4D would be seeing 1, 2, 3 pieces of this spatial existence and experience in 5D would be seeing 4 at a time. • You can't have partial dimensions because as soon as 2D shapes have any height at all, its 3D • Maybe 4D is the exception to moving. If 4D can see all sides of something, they don't need to move. (Followed by question: But won't 4D have to move to see/perceive 4D?)

Although the questions and conjectures posed are not all specific to the *Flatland* story, it is *Flatland* that spurred them to consider complex notions of dimension (e.g., higher dimensions, seeing and perceiving among and across dimensions, the direction of dimensions). For example, learners used *Flatland* to problematize their own 3D space and conjecture what it might be like to see and perceive higher dimensions (as well as lower dimensions). Even the last scene of the movie, with the rotating 4D hypercube, gave rise to ideas about motion, especially its role in mediating sight and perception. One conjecture that students posed continuously was: if dimension = N, then N-1 is what we see. In the case of 3D beings, we see two dimensions and perceive or piece together a three-dimensional space. For Flatlanders, 2D beings, they are able to see 1D lines and perceive two dimensions. This analogous reasoning supported learners to access concepts about four-dimensional (4D) space, namely, a 4D being would *see* 3D and *perceive* four dimensions. This analogy was extended to the 5^{th} and eventually an infinite number of dimensions.

Discussion

Flatland as a case garnered reaction from all learners and acted as a provocation tool, leading them to problematize space and perspective. The case has qualities of a discrepant event (Posner, Strike, Hewson, & Gertzog, 1982) meant to cause a perturbation, or in this case create impasses for questioning and conjecturing activity. *Flatland*, both the movie and book versions, provided the reader or viewer with an alternative perspective, each in a variant way. It is in this sense that *Flatland* operated as a case of alternative perspective meant to support productive problematization.

Hiebert et al.'s (1996) conception of problematization emphasizes the notion of problems as triggers for "reflective inquiry" (see Dewey, 1929, p. 189) and the iterative problematizing of one's experiences "in order to understand them more fully" (p. 15), and in this way, a seeker of problems. Hiebert et al. (1996) write, "When we treat an object as a problem to be solved and examine it carefully, said Dewey (1929), we begin to understand it, to gain more control over it, and to use it more effectively for our advantage" (p. 15). Mathematics education is oriented towards supporting learners to solve problems, invent strategies, and construct understandings about relationships in the world. Problematization is visible in the residue of learners' discourse as shown here. An important consideration when developing and implementing the cases with learners that Hiebert et al. (1996) make clear, is selecting "tasks" that encourage cognitive demand and opportunities for learners to struggle with meaningful concepts and relationships. This is an important consideration because "[t]asks are inherently neither problematic nor routine" (p. 16).

Scholarly Significance of the Study

This paper opened by highlighting the NCTM Research Committee's (2015) "grand challenge" concerning "[c]hanging perceptions about what it means to do mathematics" (p. 139). The aims of this project are oriented to exploring how the mathematics education community might address this challenge in the area of spatial thinking. *Doing* mathematics for this project is oriented towards problematizing, supporting conjecturing activity, and engaging learners as investigators of mathematics, rather than prioritizing activities of procedure implementation. The committee's second challenge includes "[c]hanging the public's perception about the role of mathematics in society," where we "see mathematics as something that human beings normally do and that has relevance and beauty" (p. 139). In this project, learners' questions and conjectures show that mathematical ideas can be embodied as they consider seeing, perceiving, and moving among dimensions. The last grand challenge concerns equity, specifically "achieving equity in mathematics education" (p. 139). Flores (2007) view of access and equity concerning "opportunity gaps" helps articulate the importance of this project: "talking about achievement gaps without mentioning opportunity gaps that cause them invites a focus on deficit models to "explain" low performance in terms of factors such as cultural differences, poverty, low levels of parental education, and so on" (p. 40). For this project, opportunities for learners to engage in the mathematics of space was central, specifically an *opportunity* to conjecture in mathematics class, an *opportunity* to ask questions about our world, an *opportunity* to consider the ill-structured aspects of a typically routinized domain, an *opportunity* to talk and reflect with each other, and mostly, an *opportunity* to be mathematicians.

By designing and researching learning experiences of the type described in this paper, it is possible that future policies and initiatives can point to the value added of exploring learners' experiences, adding an underrepresented voice to the conversation. The author acknowledges that mathematics is foremost a "human practice" and "teaching and learning mathematics are not politically neutral activities" (Gutiérrez, 2010, p. 4). A first step may be to shift our discourse about success from one of "proficiency on standardized exams" to one about becoming "better people by our own definitions" (Gutiérrez, 2010, p. 7). The exploratory results in this paper suggest a shift in how success might manifest for learners. Opportunities to "grasp space" (Freudenthal, 1973), to visualize, to conjecture, and to consider the ill-structured, complex nature of mathematics seems to capture a non-standardized type of success. As Jörg, Davis, & Nickmans (2007) eloquently wrote, "Rather than being framed in ends-oriented terms, education might become possibility-oriented" (p. 152). It is the possibility of what could be that drove the exploratory research in this project.

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