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Dialogic Teaching Model For Ninth Class Students To Conceptualize Inequalities

Mustafa Çağrı Gürbüz[1*] Murat Ağsu[2]

1. Faculty of Education, Uludag University, Bursa, Turkey
2. Faculty of Education, Uludag University, Bursa, Turkey

* E-mail of the corresponding author: mcgurbuz@uludag.edu.tr

Abstract

It is known that difficulties are often experienced in conceptual learning of mathematics, which is an abstract lesson. For this reason, it is difficult for students to conceptually learn inequalities, one of the difficult subjects of mathematics. The aim of this study is to investigate the effect of dialogic teaching to overcome the general mistakes and difficulties of 9th grade students in deepening the conceptual teaching of inequalities. This study was designed as an action research. The answers and solutions given to 7 open-ended questions prepared to determine students' misconceptions and mistakes were scored between 0 and 2 points. When a detailed analysis of solutions written by the students was done, it was determined that the students had difficulty in establishing the concept of numbers, that they ignored the real numbers in a defined range and only focused on integers, that they ignored zero when finding the square of the inequality in a defined range, and that they had difficulty in understanding the principle of reversing when the inequality was multiplied by a negative number and also had difficulty in the solution of inequalities when two inequalities were combined into a single inequality. According to the results of the research, dialogic teaching played a supporting role for the students to reach the conceptual learning of inequalities. It was also seen that high school students were able to reconstruct the concept of inequality conceptually in the learning process.

Keywords: dialogic teaching, inequalities, conceptual teaching, reconstructing

Introduction

This study emphasizes that algebra teaching, which is an abstract language of mathematics, does not just consist of a procedural teaching, it also emphasizes the necessity to bring to the fore the conceptual teaching process. Common understanding in mathematics education in Turkey is that students try to learn the mathematical knowledge presented to them by memorizing instead of constructing the knowledge in their minds (Pesen, 2006). Tall and Razali (1993) revealed that the difficulties confronting the procedural adapters of mathematics learning were more than the difficulties confronting the conceptual adapters. In addition to procedural teaching studies, studies that emphasize the process of doing mathematics for conceptual teaching are increasingly taking place in algebra teaching (Bennett, Burton & Nelson, 2010; Brizuela et al., 2015; Nathan & Koellner, 2007; Pedersen, 1993). Yetkin (2003) stated that conceptual learning in mathematics was a difficult phenomenon besides being important, and that actions including the identification of learning difficulties and their sources in mathematics as well as the determination of teaching methods to overcome these difficulties were some of the important steps in achieving this goal. Moreover, equations and inequalities are known to be one of the subjects that students generally make mistakes (Ersoy & Erbaş, 2005; Şandır, 2007; OECD, 2016).

1. Inequalities and Challenges of the Concept of Inequalities

Definition: An open statement that includes the "$\leqslant$" or "$<$", "$\geqslant$" or "$>$" relations is called an inequality (Argun et al., 2014, 176). In other words, the inequality is the numerical expression created by using the symbols $>$, $<$, $\leqslant$ and $\geqslant$, when comparing the amounts of quantities with each other. In algebra teaching, there are two basic algebraic axioms constructed in the mind of the individual, balance and change. While the variable remains as is in the teaching of inequality, the balance is upset with bias. What is expected of the person is to explain the mathematical relations of the upset balance. The variable, which is the other axiom, maintains its parameters in inequalities, too, as in the teaching of algebra. Inequality in algebra refers, at the same time, to the definition range of numbers. For example, $x<3$ means that x is the numbers less than 3. Moreover, the set of numbers in which the inequality is defined is important. If x is a natural number, it can be 0, 1, and 2. If x is an integer, it can be ... -2, -1, 0, 1, and 2. If x is a real number, it can be expressed by (-∞, 3). For this reason, it is necessary for the students to have a command of the number sets. It has been shown in many studies that students at different school and grade levels have difficulties, common mistakes and misconceptions about understanding basic algebraic concepts (the inequality concept, equation, algebraic expressions, problem solving, variable, etc.) (Booth, 1984; Dede, 2004-2005; Ersoy & Erbaş, 2003; Gürbüz & Akkan, 2008; Herscovics, 1989; Kieran, 1992; Mac Gregor & Stacey, 1993; Oktaç, 2009). The errors that students often make about the issue of inequality in the literature can be summarized as follows:

1. It is possible to examine student errors in four groups, skill errors, discretization errors, typographical errors and incidental/random errors (Sleeman, 1984).

2. It is seen that students make mistakes in changing the sign of a number when transferring it to the other side of the equality and in changing the direction of inequality when they multiply and divide the inequality by a negative number (Cortes & Ptaff, 2000).

3. It has been found that students fall into errors when transferring their knowledge of algebra to inequalities and that their ability to process is poor (Dede et al., 2002).

4. Verikios and Farmaki (2006) categorized the problems students encounter with inequalities into three: (i) the problem of the change of the direction of the inequality sign when the inequality is divided or multiplied by a negative number, or the change of it without interpretation, (ii) the consideration of inequality as an equality and the attribution of only a single value rather than finding the range of the solution, and (iii) the misconceptions of students about 0 (zero).

2. Dialogic Teaching in Mathematics

The aim of dialogic teaching in mathematics is not only to teach concepts but also to teach mathematical dialogue in which concepts are questioned and developed (see Kazak et al., 2015). Students' construction of mathematical concepts cannot be considered as separate from the linguistic processes (Lemke, 1990). Students are constantly in interaction with activities, gestures, conversations, and mathematical symbols while learning mathematical concepts (Airey & Linder, 2008). Language acts as a tool in meaning making mathematical processes. Students use language to think about their own ideas and their peers' ideas and to talk about and discuss mathematical concepts. In other words, students construct mathematical knowledge using various forms of language. Hence, some researchers have offered suggestions about use of language in the mathematical knowledge construction process. Language use can take on the form of either a monolog or a dialog. In a monolog, the teacher is dominant, and knowledge is transmitted from the teacher to students, resulting in rote memorization. Each student in a dialogue takes the perspective of the other into account when they speak. Therefore, there is no boundary between the students, rather a shared area is developed. According to Wegerif (2007), dialogue is the source of creativity. Although we can describe how we teach the ability to use mathematical concepts correctly, new things can be learned without explaining how to encourage children to think for themselves. Previously unknown, this means thinking creatively. Teaching for creative thinking implies drawing students into genuine open-ended dialogue. One goal of education is to move students away from rote learning to meaningful learning. Meaningful learning requires making connections between newly introduced concepts and prior knowledge (Novak, 1993). In Mathematics Education there is a tradition of using discourse to analyze how mathematical concepts and connections are being understood by students (Edwards, 1993; Greeno, 1997). Sfard (2002) especially offers "communication" and uses the metaphor of "thinking" as a form of communication. He thinks that "thinking is almost equal to communication, but not the other way around" (Sfard 2002, p.13). Sfard uses an instrument to analyze how students enter into dialogue among themselves and how they support their discourse to explain or justify their answers. Sfard's study is also consistent with other research, such as Kieran (2002) and Wertsch (1998), which recommend learning through participation. According to Kieran and Dreyfus (1998), when students solve a problem collectively, it is possible to have a few moments of "universes of thought" in which participants get to understand mathematical concepts. Dialogic discussion occurs when participants participate in discussions based on valid assertions. Participants who demonstrate this approach try to justify their answers by participating in discussions and using assertions that may have been verified by their peers. In this sense, participants need to use mathematical objects (and their representations) to support their claims. Such an interaction may have the potential to encourage learning among participants in the group.

3. Criteria of Conceptual Learning

Knowledge of operations consists of two parts. The first part is the symbol and language of mathematics. Mathematical symbols draw general lines of a subject but do not give its meaning (Hiebert & Lefever, 1986). Ideas not only specify the meaning of the symbol, but also provide visualization of it in the mind. The symbol that is not associated with thought does not have a meaning. However, the same concept can be represented by different symbols. For example, for the concept of "five", the symbols "5" and "V" can be used. The second part of rules consists of the mathematical relations, concrete processes and diagrams (Hiebert & Lefever, 1986). Since knowledge of operations is acquired as a rule by memorization without its reasons being investigated, more emphasis is placed on processes than on concepts in instruction. In learning by memorization, the student acts like a mirror. He does not interfere with the information conveyed to him or does not question it. He reflects it as is. Because Operational learning accepts mathematics as information that is directly transmitted to the student without questioning, it recognizes the teacher as an authority who knows and transfers rules and knowledge to students (Cobb, 1986). Conceptual knowledge is not only to know the definition and name of a concept but also to see the relationships and networks of relationships between concepts. A concept alone does not make sense. In order for the meaning of the concept to emerge, it must be associated with the group whose meaning it carries. It is a difficult situation for learners to organize their knowledge on their own, without the discussion environment of this network of relationships. People build newly learned knowledge on top of the previously learned knowledge. Mathematical knowledge is, too, added to prior knowledge. In order to understand the concept, the new knowledge must be associated and reconciled with the prior knowledge in an appropriate way (Skemp, 1971).

A characterization scale was developed by using the definitions and classifications of operational and conceptual information in the related literature (Skemp, 1971; Schoenfeld, 1989; Hiebert and Lefever, 1986; Ernest, 1991) and by utilizing Baki and Kartal's (2002) criteria that characterize the knowledge of Operations and Concepts.

Criteria Characterizing the Knowledge of Operations and Concepts Together

1. Understand, use, write, abbreviate and simplify the symbols and expressions that constitute the language of mathematics.

2. Converting a problem into an equation, solving the equation and checking the logic of the solutions.

3. Associating given relations with each other to convert them into another relation.

Since concepts are organized in a conceptual ecology that controls and modifies the process of conceptual change, it is extremely difficult or even impossible to understand concepts without establishing meaningful relationships between them (Strike & Posner, 1992). This study has emerged as a result of seeking an answer to problems similar to the ones outlined above that became chronic in the class where the researcher taught. It was observed that students could not learn the concepts of inequality from the subjects of algebra in class throughout the teaching period. As a result of informal observations, which is a way of finding conceptual difficulties (Tanner & Jones, 2000), it was seen that the students had fallen into mistakes commonly cited in the literature. In order to produce a solution, researchers have sought a different way of teaching. Several methods have been applied in the literature for the purpose of conceptual learning. The researchers of this study blame teacher-centered and inactive mathematical processes for the reasons for the students' inability to attain conceptual learning. They gravitated towards dialogic teaching, presuming that an inquiry-based environment for students to conceptually construct mathematic concepts would be the starting point. On the other hand, in a dialogic environment, authority is shared between the teacher and students, and both teacher–student and student–student interactions result in the construction of meaning for conceptual learning (Alexander, 2008; Reznitskaya, 2012). Dialogic teaching is not a one-way interaction, it is a two-way interaction (Kazak, et. Al., 2015). The goal is not just reaching the correct solution. It is also to see things from multiple perspectives. Dialogic teaching is concerned with putting students into open-ended learning dialogues using examples from such dialogues. Including learning how to ask good questions and how to show respect for other views.

Purpose and research questions

The purpose of this study is to promote high school students' conceptual understanding of the inequality concept by using dialogic teaching. The study aims to answer the following research questions:

1. What are high school students' conceptual difficulties about inequalities?

2. How do dialogic interventions influence high school students' conceptual understanding of inequalities?

Method

A qualitative approach using the action research methodology was utilized in this study (the effect of dialogic teaching methods on High school students' conceptual understanding of inequalities. Mertler (2009) states that action research provides teachers with the opportunity to work in their own classroom, thereby improving their qualifications and effectiveness by better understanding their teaching methods, students and assessment systems. Action research involves people in a change process in an explicit and purposeful way by including them in programs and communities to solve their own problems (Whyte, 1989). Action research aims to solve problems in a program, organization and community (Patton, 2015). One of the reasons for choosing action research in the study was to estimate that the difference between research and action would decline at an insensible rate during the resolution of the identified problem (Patton, 2015). Another reason was the growing belief that the realization of conceptual learning would be observed in the process. The formative assessment inherent in action research coincides with this situation. Formative evaluations in action research look for ways to increase the effectiveness of a program or product (Patton, 2015). In this regard, it is seen that action research is appropriate for the assessment of students' level of conceptual learning in the dialogic teaching process.

Methods of dialogic teaching

This study was conducted using dialogic teaching methods, including the group dialogue technique.

1. Group discussion: Discussion groups were formed on the basis of the solutions of the students on the evaluation forms so that discussions could be carried out in a healthy way and richer ideas emerged in the discussions. In the discussion groups created, the students targeted in different solution categories were brought together, thereby forming three groups, and dialogic teaching was carried out in these groups. The discussions were not done involving all the students in the class (N = 38). The reason for that was the thought that the students had not encountered such an instructional method before in the classroom environment, that the students would not be able to express their thoughts in a comfortable way because their discussion experiences were weak, and that the students' ideas in a study to be done in large groups could not be deeply penetrated into. When the solutions of the students in the evaluation forms were examined, dialogic teaching was deemed appropriate through questions 4, 5 and 6. When the solutions given by the students to these questions were examined, it was determined that they often made mistakes that include reaching the solution through value-substitution or guessing, the problem of changing the direction of the sign of the inequality, and ignoring the definition range. When these three groups were formed, students were selected according to the errors they made.

Table 1. Groups created according to the error type

Group	Type of the Error Made
1st group	Value-substitution - Guessing
2nd group	The problem of changing the direction of the sign of the inequality (-/+)
3rd group	Ignoring the definition interval

For example, it was seen that the solutions and therefore the errors of the students selected for the first group were close to each other. The reason why the groups were formed according to student solutions was to increase the diversity of ideas that would emerge during the debate and ensure that healthy ideas emerged.

Because of these reasons, three different groups of three students were created. It was thought that the students' discussion skills, which were weak, would improve and that they would be able to reach a shorter and more effective solution by analyzing their own ideas in depth.

2. Participants

The purposeful sampling method was used to determine the participants of this study. The participants included 38 first-grade high school students (25 males and 13 females, aged 15 to 16 years) attending a public high school in the western part of Turkey and taking the General Mathematics course. To protect confidentiality, the participants were given name codes from P1 to P38. In addition, ethical approval documents were signed by all participants prior to the study.

During the course of the study, a total of nine students from the three groups selected out of 38 students and participating in dialogic teaching were selected. Three students were present in each of the three selected groups in the study. The selected students were determined by first examining the solutions they made on the 7-question evaluation form. 38 students were divided into three groups: very successful, moderately successful, and less successful. Three students each were then randomly selected from each of these three student groups to form three groups. Dialogic teaching studies were conducted with 9 students in these three groups. A list and abbreviations of the aliases belonging to the students in the three groups are shown in Table 2 below.

Table 2. A list and abbreviations of the aliases belonging to the students in the groups

Group 1		**Group 2**		**Group 3**	
Alias	Abbreviation	Alias	Abbreviation	Alias	Abbreviation
Galip	G1	Meryem	M2	Selen	S3
Türkan	T1	Yusuf	Y2	Ali	A3
Alp	A1	Dila	D2	Hilal	H3
*Teacher-T					

3. Procedure

This study was conducted during regular classroom hours over a 2-week period. The classroom instruction for both groups included two 45-min periods per week. The exam papers of the students were examined before the classroom instruction to determine the errors they made insistently on the subject of inequalities. Subsequently, open-ended questions were prepared by identifying the mistakes that students made regarding the subject of inequalities found in the literature. The questions prepared were presented to three experts' opinions to sort out the similar questions and to clean out the mistakes in the questions. 7 open-ended questions were prepared considering also the opinions of the experts. These 7 open-ended questions were administered to 38 9th grade students. Afterwards, rubrics for the questions were prepared by taking the opinions of the experts, and the solutions made by the students were evaluated. As a result of the evaluations made, the success of the students in 3 questions was very low in the evaluation form of the 7-question, open-ended questionnaire. When the solutions of these three questions were examined in detail, it was seen that the mistakes made in these three questions covered the mistakes made in the other questions. Thus, dialogic teaching was carried out through these three questions. The studies were carried out using the dialogic teaching methods on the three open-ended questions determined by selecting the three groups of three students out of 38 students.

4. Data Collection

The data consisted of both written texts and video recordings. The study employed a purposefully designed questionnaire consisting of seven open-ended questions. The questions were determined based on the difficulties experienced by students with regard to inequalities as presented in the literature. In addition, during the written examinations administered by the teacher, the mistakes frequently made by the students were detected and appended to the questions of the questionnaire. Furthermore, three experts checked the questionnaire for content validity. They also checked the validity of the responses in terms of scientificness. These texts were used to determin conceptual difficulties and implement the possible changes following the interventions. In addition, video recordings of the dialogic interventions were used to better understand the influence of dialogic interventions on students.

5. Data Analysis

The collected data were analyzed by "interpretational analysis". The purpose of this method is to identify the constructs by classifying the content and to define concepts based on the researcher's interpretive understanding (Borg,& Gall, & Gall, 2006). For this purpose, the content was formulated according to "dialogic teaching", and so it was interpreted based on characteristics of Dialogic Teaching.

Findings and Discussion

The study was designed to answer two research questions. Within the scope of the research questions, student solutions for the evaluation form consisting of 7 open-ended questions were examined in order to determine the answer to the first question. The academic standing of the students is shown in Table 3.

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Table 3. General analysis of students' solutions

		Failure to change the sign	Definition range	Value-substitution/Guessing	Ignoring 0	Inequality-equality confusion	Operation error	$\bar{x}$	S
Q1	38		2	11		1	3	1.29	.565
Q2	38	2				4	3	1.13	.414
Q3	38	1	5				9	1.21	.577
Q4	38				26	1	8	.87	.475
Q5	38	12		19		2	9	.87	.475
Q6	38	10		19		2	11	.55	.504
Q7	38	7		3		5	4	1.08	.632

When these questions were examined, it was seen that the students had very low success in three questions. Errors made in questions with low success often had the same tendency.

The error often made in the fourth question emerged from the fact that the memorized knowledge as a mathematical notation could not be expressed correctly. In other words, it is seen in Fig 1 that the students ignored the number "0" when they extended a range by squaring it.

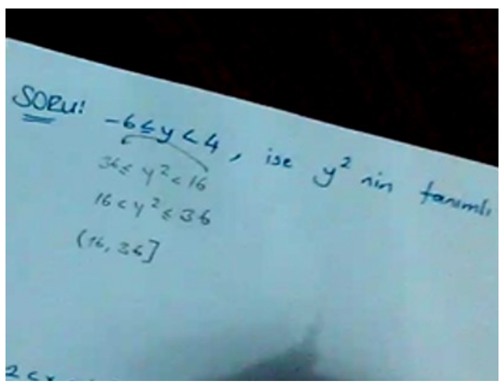


Figure 1. The error of ignoring "0" in the fourth question

For the fifth question, the students could not correctly express the ranges where the "x" and "y" variables were defined. In other words, it was seen that they did not pay attention to the <,>, ≤, ≥ signs in the definition set. The range in which the newly formed "3y-2x" variable was defined as the result of sequential operations was misinterpreted by most of the students. Another situation in this question was that, most of the students tried to solve the question by using the value-substitution method, but they found that the solution was wrong due to the special condition of the range where the variable was defined.

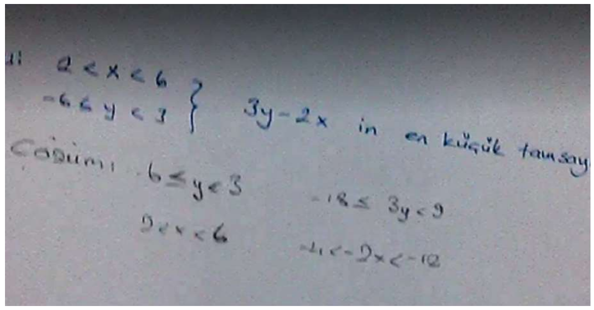


Figure 2. Example of sign error in the fifth question

As shown in Table 1, the lowest success was observed in the 6th question. In order to identify this situation, student solutions were examined in detail, and it was seen that majority of the students used value-substitution as a method in the process of question solving.

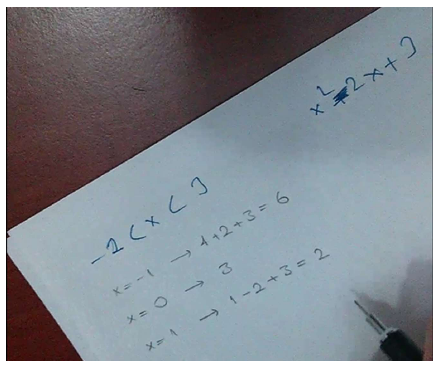


Figure 3. An example solution with value-substitution frequently used in the sixth question

Among these 7 questions, the lowest success was observed in the 4th, 5th and 6th questions. When student solutions were examined, it was expected that if solutions were delivered by addressing the mistakes made in the solution of these three problems, the mistakes made in other questions could also be eliminated. In recent years, researchers have paid increasing attention to the role of language and social interaction in the learning and pursuit of mathematics (e.g. Barwell, 2005; Forman & van Oers, 1998; Hoyles & Forman, 1995; Monaghan, 1999; Sfard, 2000; Sfard & Kieran, 2001). This interest relates to the function of language both in teacher–student encounters and in peer group activities. It is often claimed that working and talking with peers while carrying out maths activities are beneficial to student learning and the development of their mathematical understanding. Placing the responsibility in the learners' hands changes the nature of learning by requiring the learners to negotiate their own criteria of relevance and truth.

Since the mathematical processes that were expected to be performed in the solution of these three problems were also inclusive of the other questions, it was decided that the instruction through dialogic teaching would be taught on the basis of these questions. Discussions within and between groups were carried out on these questions.

Let's first look at how dialogic teaching is structured by the researchers in teaching.

Table 4. Process of dialogic teaching is structured by the researchers

Characteristics of Dialogic Teaching	Dialogic Inquiry Tools		
	Indicator	**Monologic**	**Dialogic**
Flexibility (the content of the discussion form)	Authority	The teacher controls the process and content	The student controls the process and content
Expander, horizontal questions	Questions	The teacher directly asks target-specific questions	They are aimed at revealing the thoughts of the students
Promoting	Feedback	The teacher reminds the formula	The student is helped to discover the process
Communication and Reflection	Connecting student ideas	The teacher evaluates the answers given (True-false)	A question-and-answer environment is formed among students
Questioning	Explanation	Rules are applied directly (Why and how questions are ignored)	Students support their ideas with their evidences
Structuring of knowledge	Collaboration	Inexplicable results	Information is restructured

Adapted from (Reznitskaya, 2012)

In a dialogic classroom, the comparison of monologic and dialogic teaching was made under the heading Dialogic Inquiry Tools. Things to be aware of during the instruction were revealed. In this study, the key behaviors and characteristic applications of dialogic teaching described and explained above were clarified in order to create a dialogic discussion environment. The researchers identified an instructional plan that drew on these characteristic applications. The characteristics of the Dialogic Inquiry Tools, which were described by Reznitskaya (2012) and were a significant measure of discrimination from monologic teaching, and dialogic teaching were compared. As a result of this comparison, discussions were opened and deepened on the topic to be taught, and reference points leading students to deep thinking were determined. Notes were taken to use these reference points during the implementation, and they were introduced according to the progress of the discussions. The characteristics of the key behaviors and practices that were defined in studies (Alexander, 2008; Billings & Fitzgerald, 2002; Mercer & Littleton, 2007; Nystrand et al., 2003; Soter et al., 2008) to organize the teaching-learning environment and establish the dialogic discussion in a dialogic classroom and that were to be followed were as follows:

1. What is expected of dialogic teaching in the first step, the “Authority” step, is not the determination of the instructional content by the teacher during the discussion process. On the contrary, how the content will form will be shaped by the way the discussions lead in the process. the relationship between the lesson control is more flexible. The authority of form and content is shared among the discussion groups. The answers generated by the students guide the discussion. The role of the teacher in this study was not to tell the students their mistakes and information about how to solve their mistakes. The teacher just acted like a moderator.

2. In the “Questions” step, the teacher directed the students’ thinking skills with deepening questions. Dialogic teaching is conducted mainly on questions that are open and different (Burbules, 1993, p. 97). What is implied by an open question is that it will be an adequate answer and it will not be ambiguous. The purpose of different questions is not to test students and ask them to give short answers, but rather to show them a meaningful and new way. Guiding is a responsibility of the teacher. During the discussion of the fourth question, the thoughts of the students were generally in the direction of squaring the number, but they did not pay attention to the sign of the number when squaring it. In this case, the teacher asked the question “What does a number squared mean?” in order to deepen the students’ thinking. With this question, the teacher provided the students with the opportunity to deepen their thoughts about what kind of changes would happen to the number if it was squared.

3. “Feedback” is in the third place. In this step, it is expected that the teacher helps the student discover the mathematical process that has been established. It is expected of the teacher practicing dialogic teaching to make meaningful and specific feedback to improve the group’s questioning. Questions to be asked should be directed to the students to create evidence and to advocate. Teachers’ feedback should encourage students to negotiate and construct new understandings. If there is a negative number in a range, the lower value of the range must be zero when the range is squared. To enable students to discover this situation, the dialogue between the teacher and the student was as follows:

T: What was the defined range of x?

S3: from -6 to 4.

T: Now, how about I tell you a value from that range, and you find its square.

S3: -5 squared became 25. If I take the 3. 3 squared is 9 ... Hmm, it did not work.

T: Very nice. He took 3. Its square was 9. It was not a value in the range. 9 is not in that range then. Then, what do we need to do?

M2: Do we need to add? I’m very confused right now. Why was not it in the range?

M2: Hmm, 2 squared is 4 and is not in this range.

M2: Yes, 1 squared is 1. 0 squared is 0. Wait a minute, it is ZEROOOO. We must start the range from zeroooo. It is from 0 to 36 then.

As understood from this dialogue, the question “Now, how about I tell you a value from that range, and shall we square it?” asked by the teacher is suitable for an example feedback. Later on, it was understood from the sentences they expressed in the dialogue that the students completed the process of discovery.

4. “Connecting student ideas” is in the fourth place. At this stage, the student is expected to explain and defend his ideas in a discussion environment encouraged by the teacher. The critical task of the teacher at this step is to provide hints to help students build their ideas on each other. Participants in Dialogic discussions should always be inclined to enter meta-level reflections. Therefore, in order to defend his own right idea, the student should cherry-pick the debate processes. According to Gregory (2007), the primary task of a teacher is to ask students additional questions to improve the quality of their responses. An example situation showing that this occurs is for students to connect their ideas by debating among themselves. For example, in the solution of the fifth problem, students were expected to express the set of definition of the newly formed inequality (i.e., they were expected to pay attention to the signs of $<, >, \leq, \geq$ in the definition set) when two inequalities were added top to bottom. Firstly, discussion of the answers given by the groups to the question was provided. In order for this to happen, the teacher asked a question like this: “Guys, there is a difference between the result that the first group found and the result that 2nd group found. The first group, please explain your solution (Because the first group was the group that gave an incorrect answer, it was chosen). To the second group: Please find out the difference between the first group’s solution and your solution.” The discussions of the two groups triggered the students to think deeply on the question and review what they did. As a result, a situation misstated by both groups was identified: The examination of the inclusion and non-inclusion conditions (the use of $<, >, \leq, \geq$ signs in the definition set) in inequalities. As a result of students’ hesitations, the teacher asked the following question in order to prompt for deeper thought:

T: Well, when writing the inequality in the two groups, there was an equality in the inequality y but not in the inequality x. But they did not use an equality in the two groups when summing top to bottom. Do you think the operation you are doing is right?

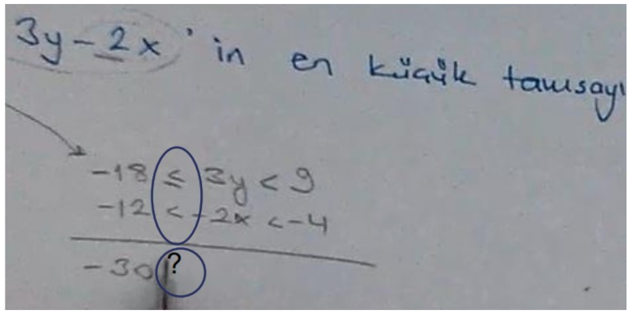


Figure 4. Appropriate marking

Y2: Hmm. Yes, sir.

G1: Sir, we made it wrong.

T: Why is it wrong?

G1: I think if there is an equality, there must be an equality in the resultant sum.

Y2: No, I do not agree. I think we did it right.

T: Could you explain why it is right?

Y2:

(-18≤3y<9)

(-12<-2x<-4)

-30<3y-2x<5,

A1: Yes, sir. I agree with my friends. Because -18 is included but -12 is not included, -30, which is their total, should not be included.

T: Why?

A1: Because for -30 to be included, both -18 and -12 must be included.

T: Yes, it's true. Your friends explained it beautifully. In an inequality, if there is an equality in one and there is no equality in the other, then there should be no equality in the value, which is the sum.

The teacher completed the process of revealing the relationship between the ideas of the students (connecting the student ideas) in two stages. As a first step, he had the groups explain their solutions to each other in order to create depth between student ideas. First of all, he gave the group having the incorrect solution the right to explain. He provided an opportunity for all students to identify the errors in the solution. In the second stage, the problem state was the focus (how to use the <,>, ≤, ≥ signs in the definition set in the new inequality). Here, the grounds for constructing a high-level knowledge based on the ideas put forward by the students were established.

5. Fifthly, there is the "Explanation" or "Inquiry-Explanation". It is expected that students will be able to explain their ideas through evidence during this Inquiry phase. In this phase, the teacher gets students to question why such a rule is formed in reaching the solution different than the practice of directly applying the rules in reaching the solution. Students are asked to describe their solutions and explain in detail how they reached that result. While these processes are happening, in the background students need to be aware of what they are doing, and it is a prerequisite for them to convince their friends that their solutions are correct. In the process of convincing their friends to the correctness of their solutions, students are enabled to think intensively on the question and the solution path. It is inevitable for students to make new discoveries during this intensive thinking. Students' ability to elaborate on their ideas here reveals conceptual learning, which is ahead of learning by memorization. In a Dialogic class, students should explain their ways of thinking in detail. Students continuously proceed in a way to respond to why and how questions. What is important in the transition to the "Explanation" phase is that the discussion of the incorrectness of the incorrect ideas is encouraged. At this stage, what is expected of the teacher is to be able to prompt students' existing knowledge. As a result of this prompting, students should be able to establish communications between the existing knowledge and superordinate knowledge. The process of

creating and reflecting new knowledge is started by providing communication with the knowledge that the students have acquired in the previous questions. For example, in the solution of the sixth question, the students used discriminant to compare the results they found and to prove their correctness (see Fig. 7). They presented explanations by developing a set of mathematical proofs to determine the correct answer.

6. They engage in collaborative reconstruction of knowledge during Dialogic discussions. Once students have listened to each group, they create reactions that broaden and support their ideas. During dialogue discussions, students join and listen to a collaborative knowledge society and react to each other's position and justifications for reconstruction of knowledge. Thus, they benefit from the constructs they have established in the fourth and fifth steps to further develop the thoughts of the group (Reznitskaya, 2012, p.448). The following dialogues summarize the students' reconstruction of the information in the group;

The discussions were continued over the question "If -2<x<3, find the smallest integer value of the expression x2-2x + 3."

The first group explained everyone in the class environment how they reached the solution.

First, let's square x.

Then, the expression would be -4<x2<9. But let's write 0≤x2<9.

T: My question to the second group is that why did your friend write 0<x2<9 instead of -4<x2<9?

Y2: In the previous question, I fell for that but the other group did not fall. Our friend wrote so, because the square of each number is equal to or greater than zero. This happens that way, because when a negative number is squared, the result is positive.

Y2: -2<x<3

Let's multiply x by -2.

The result became 4<-2x<-6. But, now, I will not fall for this trap. Actually it will be like this: -6<-2x<4

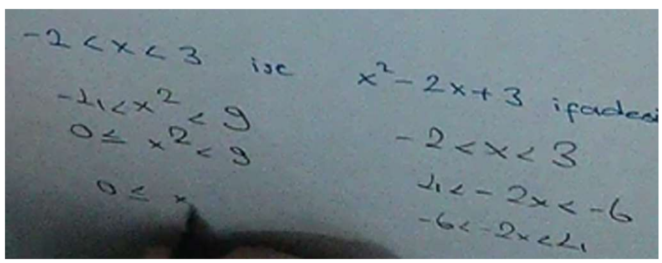


Figure 5. The first step students make in this question

Now, I shall add top to bottom

Now, let's add 3 to both sides

From there, the answer becomes -2.

T: Now, let's see the answer of the third group.

A3: We tried to do it by value-substitution here.

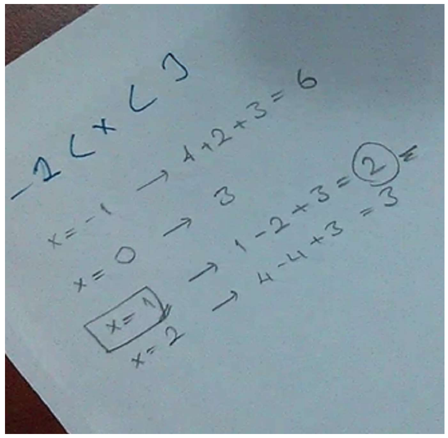


Figure 6. Solution that most students find

The smallest of these values was marked. They said their answer would be 2.

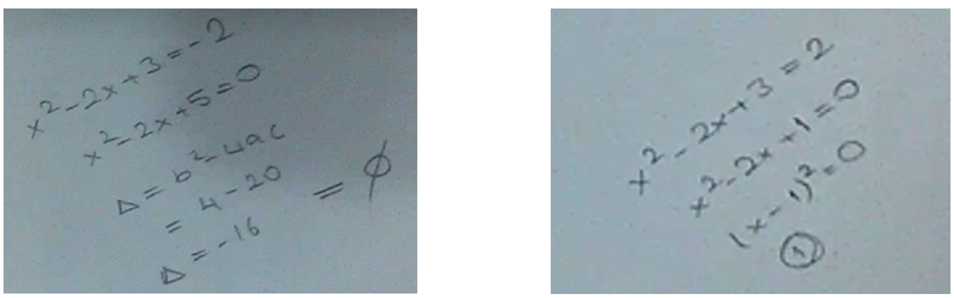
H3: Sir, how do we know which is right?

T: Then, shall we make a comparison?

D2: Sir, shall we substitute the x values we found in the equation?

D2: Sir, thus we will have found out which one is right, won't we?

T: Let's do it and see.



Because ⌂<0, there is no real root. The answer is incorrect.

Because ⌂>0, there is a real root. The answer is correct.

Figure 7. Finding root deltas

The students were able to compare the answers they developed with the mathematical proof. They were able to prove the correct solution using discriminant. As stated in the upper sections, it was seen that the students were influenced by the steps of Connecting student ideas and Explanation.

Thus, the students compared the previous questions with the procedures they performed on the 6th question based on their teachers' guidance. To bring these comparisons to a conclusion, the teacher made use of all the ideas that had emerged to encourage the students.

The discussions brought the ideas of the students to the point of examining the interventions applied to the variable.

H3: Sir, something like this came to my mind: We did an operation on a variable in the 5th question, but we did two operations on a variable in the 6th question.

T: How many variables were there in the 6th question?

H3: There is only one variable.

T: There was one variable, and did we do two operations?

H3: Yes.

D2: What did we do in question 5?

H3: There were two variables. One operation was done on one of them and it was summed top to bottom.

T: What was done on the 6th question?

H3: Each variable was added top to bottom by doing two operations, and the error was found. Hmm, then we will do one operation in this question.

D2: Why do you want to do a single operation?

H3: Because there is one variable here, I have the right to do only one operation. I do not have the luxury to do a second operation.

T: Then, you are saying that I have to solve this question using one operation.

H3: Yes, sir.

T1: How will you do that?

H3: When I saw the quadratic form, I told my friend that could I turn this expression into an expression with full square?

T: Why do you want to turn it into a full square? Why did you want to solve it by doing a full square?

H3: Because, sir, if I do a full square here, I will just add or subtract a constant. Adding or subtracting a constant is not an operation.

T: That is, adding or subtracting a constant does not intervene with the variable?

H3: Yes, I will already do it right now.

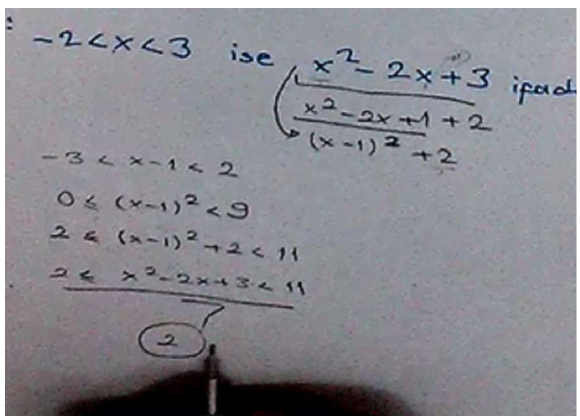


Figure 8. Final solution

T: I have listened to your discussions so far. I can summarize from your conversations that if we ever encounter such a question again, we will perform only one operation in the range where one variable is defined. If we execute two operations, it means that we are forming two different definition ranges for that variable. This was so according to your result, too. That is, we are going to perform one operation with one variable, and two operations with two variables. With the preceding sentences, the discussions were completed and concluded by the teacher.

These comparisons forced them to establish a concept. The students constructed all the steps for the 6th question on the knowledge that they gained as a result of the discussion in the previous questions. For them to clarify this situation provides both the establishment of the concept and the solution of the problem that has arisen.

5. Conclusion

The errors the students made and the difficulties they experienced were given in detail in the Findings section. It was seen that the difficulties of students about inequalities were parallel to the results found in the literature (Payne & Squibb, 1990; Cortes & Ptaff, 2000; Verikios & Farmaki, 2006). It can be said that the main reason for the difficulties the students experience in inequalities is the fact that the procedural instruction is too dominant in education. Because procedural learning accepts mathematics as knowledge directly transmitted to students without questioning, it accepts the teacher as an authority that communicates knowledge and know-how to students (Cobb, 1986). Contrary to this situation, a teaching environment where students were active was established through dialogic teaching. Dialogic teaching is an effective tool for raising student participation to a decent level and increasing the quality of classroom interaction (Lyle, 2008). For students to actively participate in the process played an important role in achieving the goal of the study. Dialogic teaching offers a pedagogical approach that allows teachers to assess their students and encourage reflective learning. Dialogic teaching interventions were tried to overcome the difficulties of students about the concept of inequalities. Consequently, it was expected that dialogic teaching would have a positive effect in achieving conceptual teaching. Dialogic teaching was used as a method to provide conceptual learning of students. Co-construction, the last step of dialogic teaching, was targeted to support the conceptual knowledge of inequalities and to achieve conceptual teaching.

When we examined the conceptual learning of students about inequalities, it was seen that the students could reflect the knowledge they constructed in the fourth question to other questions. For example, although the teacher asked "Why did you write 0≤x2<9 instead of -4<x2<9?" the students said, "I had fallen for it in the previous question but the ones in the other groups did not. Since the square of each number is equal to or greater than zero, our friend wrote so." The students in the first group transferred and used their knowledge learned in the previous question. A situation emerged indicating that the students reconstructed the previous knowledge. A condition that was consistent with the first of the criteria expressed in the study of Baki and Kartal (2002) emerged. Students showed that they could use expressions and symbols that made up the mathematical language, which was beyond understanding. It can be said that dialogic teaching had a positive contribution in eliminating the difficulty of "ignoring the number zero" found in the study. According to Kieran and Dreyfus (1998), when students solve a problem collectively, it becomes easier for participants to arrive at mathematical concepts. The current study revealed results consistent with Kieran and Dreyfus (1998).

The students were able to make a logical check by comparing the mathematical answers they developed. The relation giving the roots of the equation was questioned in order to verify the two different answers, which the students thought were correct during the discussion of the different answers given by the groups to the 6th question. As a result of discussions, the students decided to use discriminant to determine which answer was correct. Here, it can be said that the students performed the processes according to the second criterion in terms of characterization of conceptual information. It was also determined that they used the connection with student ideas and explanation steps of dialogic teaching (see the Findings section). Here, the students received support from the steps of dialogic teaching when creating a conceptual structure of inequalities. The fact that they developed and explained their own ideas influenced by their friends' ideas is the proof that dialogic teaching extends the ideas of students. This is similar to the studies of Palomar and Olivé (2015) and Kazak et al. (2015).

Dialogical debate forms when the students participate in discussions to prove their ideas. Students participating in these discussions attempted to justify their answers using the assertions verified by their peers. In this sense, as in the case of Discriminant, participants needed to use mathematical objects (and their representations) to support their claims. Such an interaction encouraged learning among participants in the group.

Discussions based on concept difficulties are aimed at expanding students' ideas. The aim of this study is for students to conceptually develop ideas with regard to inequalities. The students raised their conceptual ideas of inequality, like a step, with the two previous questions discussed. It gave them a repertoire that would allow them to make mathematical discoveries. According to Reznitskaya, et. al., (2009), students acquire generalizable knowledge of argumentation, or an argument schema, through participating in dialogic discussions with their peers. We have come to the conclusion that Dialogic Teaching, despite the complexity of classroom dialectics, can serve as a useful mechanism for promoting the development of individual discussions. Based on the study design, in the sixth question, students were asked to argue on a problem (Hızarcı & Elmas, 2004) that would lead to difficulties if the conceptual learning of inequalities did not occur. The observed situation shows that the students reconstructed the concept of variable and the knowledge of the range in which the variable was defined by using the knowledge of identities in the solution of the sixth question at this stage. The development of students' ideas can be seen as the best way to ensure they perform co-construction. Discussions by students about the correct solution of the inequalities question led them to the conclusion that at most one operation could be done on a variable in operations on inequalities. The rule found here is not a situation that is much encountered in school mathematics. It is not even presented as a rule or as a mathematical situation. This confirms the third characterization of the conceptual information that Baki and Kartal (2002) expressed, "reaching another relation by associating relations". It can be said that the students completely met the criteria that characterize both conceptual and operational knowledge.

In conclusion, it was seen that the students were able to reach the outcome targeted in the study. Specifically, it was understood that in reconstructing the subject of inequalities, dialogic teaching had positive contributions. Similar to the literature (Palomar & Olivé, 2015; Hajhosseiny, 2012; Reznitskaya, et al., 2009; Lyle, 2008), in this study, it was found that dialogic teaching supports the conceptual development of students. Dialogic teaching supports students' conceptual development of mathematics learning. Dialogic teaching played an encouraging role in students' conceptual learning of inequalities, developing a mathematical repertoire and producing reflective ideas. It can also be said that it also positively affects class dynamism and communication with the teacher. Finally, it can be said that dialogic teaching is a valuable method of increasing student participation at a profound level and improving the quality of classroom interaction.

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