A FRAMEWORK TO GUIDE THE DEVELOPMENT OF A TEACHING MATHEMATICS WITH TECHNOLOGY MASSIVE OPEN ONLINE COURSE FOR EDUCATORS (MOOC-ED)

Karen F. Hollebrands
North Carolina State University
kfholleb@ncsu.edu

Mathematics teacher educators face a challenge of preparing teachers to use technology that is rapidly changing and easily available. Teachers have access to thousands of digital tools to use with students and need guidance about how to critically choose and use tools to support students' mathematics learning. Research provides guidance to teachers about what features to look for in a technology tool and suggestions are offered about how mathematics teacher educators and researchers can support teachers in using technology to teach mathematics.

Keywords: Technology, Teacher Education-Preservice, Teacher Education-Inservice/Professional Development

Introduction

Technology is an essential component of today's workplace and a ubiquitous part of our society. 82% of high school students and 68% of middle school students have access to smart phones. 75% of these students would like to use their devices to support learning (Speak Up, 2014). Students report that using technology better engages them in learning. Parents state that the use of technology will better prepare their children for the workforce of tomorrow. While researchers have evidence to show the positive impacts the use of technology can have in classrooms, and while there are increasing numbers of freely accessible digital tools available to use, teachers' incorporation of technology has been slow.

We know technology, like any tool, must be selected and used carefully. Mathematics teachers have access to more open-access digital resources than ever before. While ten or twenty years ago, teachers were creators of activities, today, teachers search, find, select, and often modify activities they find using Google, Pinterest, or the Blogosphere. In addition, many teachers have access to digital resources that accompany their curricula. With so many activities easily available, many teachers have become curators rather than creators of digital resources. As teachers gather materials together to present to students, they need guidance to assure that what they have selected will meet the needs of their students and will achieve their learning goals. Teacher educators face a tremendous challenge in preparing teachers to use digital technology that is rapidly changing to support students' mathematics learning. For example, the resources available today may be different tomorrow. Research can provide guidance and advice to assist teachers in using technology in the mathematics classroom.

In this paper, a framework developed to guide the design of a Teaching Mathematics with Technology massive open online course for educators (MOOC-Ed) will be shared along with questions that teachers can consider when making decisions about using technology to teach mathematics.

Guiding Framework

The Didactic Triangle.

A framework was created to guide the development of a MOOC to support teachers in using technology to teach mathematics. The foundation of this framework is the didactic triangle. The didactic triangle is a representation that has been used by several researchers (e.g., Brousseau, 1997;

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Articles published in the Proceedings are copyrighted by the authors.

Freudenthal, 1991; Steinbring, 2005) to describe interactions that occur among a teacher, his or her students, and the content that is being taught. These interactions can be described in terms of pedagogical activities the teacher uses to engage students in learning content – in this case, mathematics. It is important to note that mathematics refers to mathematical topics like algebra, geometry, measurement, statistics, probability, and number, and also the mathematical processes students use when engaging with mathematics. These mathematical processes and practices include using representations, making connections, communicating reasoning, creating and critiquing arguments, attending to precision, solving problems, and mathematical modeling (NCTM, 2000, 2014; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010)

Within the didactic triangle many interactions take place. For the purpose of designing our MOOC, we focus on those interactions that are planned and used by the teacher. These include (but are not limited to) pedagogical activities related to 1) the selection and implementation of mathematical tasks, 2) questions teachers pose to push student thinking or probe their understanding, 3) the facilitation of mathematical discussions, and 4) assessment of student learning. We depict these four pedagogical activities at the center of our didactic triangle (See Figure 1).

Figure 1. A sample of activities that take place among students, teachers, and mathematics.

Although not explicitly mentioned, we acknowledge that there are many factors that influence classroom interactions such as classroom culture, norms, attitudes, and beliefs. These all influence the enactment of the pedagogical activities we have placed at the center of the triangle. We depict the addition of technology to the classroom by adding a vertex to expand the didactic triangle and create the didactic tetrahedron.

The Didactic Tetrahedron

To make explicit how one considers the role of technology among interactions with students, a teacher, and mathematics, the didactic triangle was extended by Tall (1986), and more recently by Olive et al. (2010) and Ruthven (2012). We can depict this influence by expanding our didactic triangle to create a didactic tetrahedron with technology as the fourth vertex (See Figure 2). Olive et al. state "the introduction of technology into the didactic situation could have a transforming effect on the didactical situation that is better represented by a didactic tetrahedron, the four vertices indicating interactions among Teacher, Student and Mathematical Knowledge, mediated by Technology" (p. 168).

It is important to define what we mean by technology. For some, technology is any object or tool that allows a user to accomplish a task. Others restrict the use of the term technology to refer to electronic or digital technology. Some researchers make distinctions between artifacts, tools, and

Articles published in the Proceedings are copyrighted by the authors.

instruments. According to Monaghan “an artefact is a material object, usually something that is made by humans for a specific purpose, e.g. a pencil. An artefact becomes a tool when it is used by an agent, usually a person, to do something” (Monaghan, Trouche, & Borwein, 2016, p.6). On the other hand, Trouche discusses the process that is involved in an artefact becoming an instrument. He states, “when an artefact has been appropriated by a user, I will name instrument the mixed entity composed of the artefact and the associated knowledge… a tool is a thing somewhere on the way from artefact to instrument” (ibid, p.8). In this paper technology will be used synonymously with tools as defined by Monaghan.

Figure 2. The didactic tetrahedron which includes technology.

When adding technology to mathematical pedagogical activities it is important for teachers to think about how the use of technology influences representations of mathematics and how the use of technology influences pedagogy.

The Influence of Technology on Representations of Mathematics

Mathematics is abstract and it is only through its representations to which we have access to it. Technology offers new and different representations for students and teachers to interact with and use. For teachers, we emphasize that when evaluating technology there are three important factors to consider (See Figure 3). In particular, it is important to determine if the representations technology offers, determine whether it has mathematical fidelity, and consider if technology will be used with students as an amplifier or as a reorganizer (Pea, 1985, 1987).

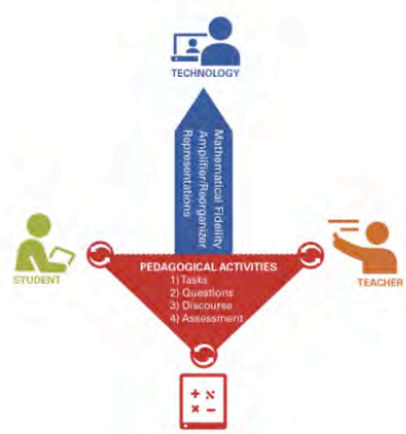

Figure 3. Three factors for teachers to consider when evaluating technology.

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Articles published in the Proceedings are copyrighted by the authors.

Mathematical Fidelity. When choosing technology to use in the classroom, teachers need to make sure that what is represented in the tools are accurate representations of the mathematics. Dick refers to this as, "mathematical fidelity" (Zbiek, Heid, Blume, & Dick, 2007). To illustrate, consider the sketch shown below that was created to show how to compute the slope of a line.

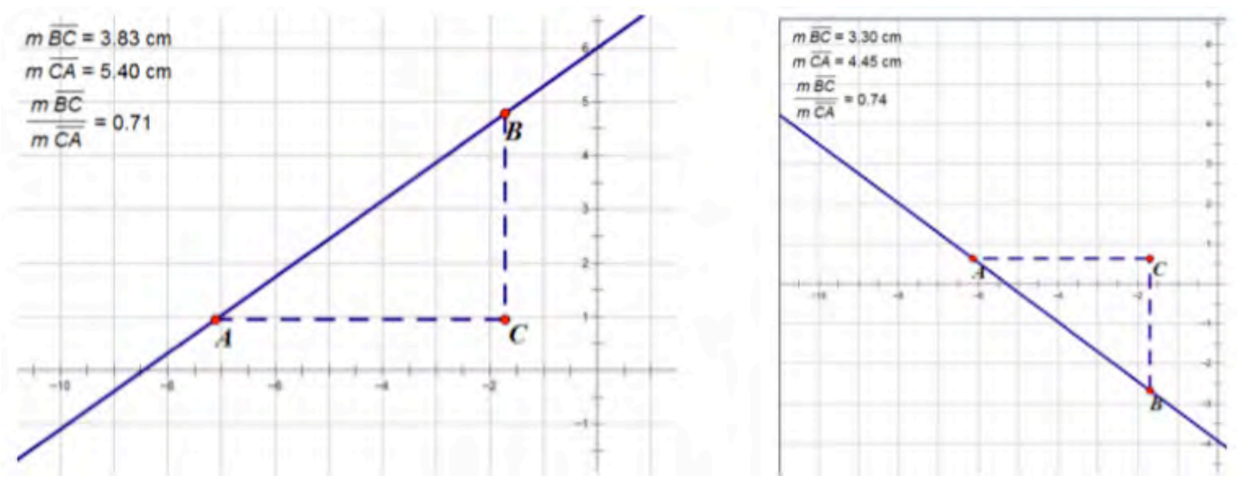

Figure 4. An example of a technology-based activity without fidelity.

Notice that the length of segment *BC* and segment *AC* are used to calculate the ratio of the "rise" to the "run." Segment lengths are always positive and this becomes problematic when the line has negative slope. In this case, the technology calculated the ratio, but cannot account for the negative slope – thus, the calculated ratio is not mathematically correct. The sketch is not faithful to the mathematics and thus lacks mathematical fidelity. It is important for teachers to select tools to use with students that have mathematical fidelity. In addition, teachers should also consider how students will interact with the technology.

Amplifier/Reorganizer. Pea (1985, 1987) used the metaphors of amplifier and reorganizer to describe how technology might be used. As an amplifier, technology performs many of the same actions that could be completed by hand, just more precisely, quickly, and efficiently. The question that is answered using by-hand methods or using technology is relatively unchanged. For example, students might be asked to create a table of values for the function with rule $f(x) = 3x+5$. This could be produced by hand or by using a spreadsheet. The results would be generated more quickly and accurately with the spreadsheet. As a reorganizer, technology changes the way students think about a question or mathematical idea. For example, a student may be provided with the graph of the function with rule $f(x) = 3x+5$ that is linked with a table of values and sliders that dynamically change the values of the slope and y-intercept parameters. By allowing the technology to quickly produce the graph and table, questions can be posed to shift a student's focus from producing the representations to conjecturing and reasoning about how changes in the parameters are related to changes in the graph and table.

When technology is used as a reorganizer, questions can be posed that align with and take advantage of the representations and actions afforded by the tools. Many technology tools that support mathematics learning provide multiple representations of mathematical objects (e.g., numeric, graphic, symbolic, pictorial) and allow the user to interact with the technology to dynamically adjust one representation and see the changes in other representations. This dynamic

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Articles published in the Proceedings are copyrighted by the authors.

linking can influence how students reason with and make connections among different representations of mathematics (Kaput, 1987).

Representations. The use of multiple representations to support students' mathematical thinking has long been recognized as an important pedagogical activity (e.g., Kaput, 1992; NCTM 2000, 2014). Research suggests that the use of multiple representations can assist students in developing deeper understandings of mathematics and become more flexible problem solvers (Ainsworth, 1999). NCTM (2014) claims, "Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving" (p. 24). When one thinks about representations of mathematics, symbols, graphs, and tables often come to mind. Mathematical representations can also include pictures, diagrams, contexts, verbal descriptions, and physical objects. There are many ways to represent mathematical ideas.

Technology tools provide students with easy access to representations and these technology-based representations are often linked. That is, a change in one representation results in a change in other representations. For example, changing the function rule $f(x)=2x+1$ to $f(x)=3x+1$ can result in a corresponding change in its graph. From these interactions, students can better understand slope. However, it is important that the production of multiple representations is not the sole focus of an activity. Rather, multiple representations can be the centerpiece of productive mathematics discussions and making connections within and among different representations an important cognitive activity (NCTM, 2014).

With these important features of technology described, we created questions that teachers might consider when selecting tools to use in the mathematics classroom that are shown in Figure 5.

Technology-Mathematics	*Mathematical Fidelity*. Is the technology tool a faithful and true representation of the mathematics students are to learn? (Dick, 2008) *Amplifier/Reorganizer*. Does the technology allow the teacher and/or student to do the same work more effectively, efficiently, and quickly (amplifier)? Does the technology change the way the student and/or teacher thinks about mathematical ideas (reorganizer)? (Pea, 1985) *Representations.* How does the technology represent the mathematics? Does it provide linked representations for students to use? (Goldin & Kaput, 1996)

Figure 5. Questions teachers may consider when evaluating interactions between mathematics and technology.

The Influence of Technology on Pedagogy

Just as technology can influence the mathematical representations students interact with, it can also impact pedagogy. It is important for the teacher to be aware of the opportunities technology allows and consider how it influences the four pedagogical activities involved in 1) designing tasks, 2) posing questions, 3) facilitating discourse, and 4) assessing student learning.

Researchers have described tasks in terms of their cognitive demand (Henningsen & Stein, 1997) and mathematical richness. Technology can have an influence on both the richness of a task and its cognitive demand. A mathematically-rich task in a paper-and-pencil environment may be a completely different activity when students have access to technology tools. Consider the task of constructing an equilateral triangle. Doing so on paper with compass and straight-edge requires different thinking than doing so with a dynamic geometry program. Similarly solving a question such

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Articles published in the Proceedings are copyrighted by the authors.

as the following requires different thinking if solved using paper and pencil or solved using a dynamic geometry program.

1. In the coordinate plane, line p has slope 8 and y-intercept $(0,5)$. Line r is the result of dilating line p by a factor of 3 with center $(0,3)$. What is the slope and y-intercept of line r?

A. Line r has slope 5 and y-intercept $(0,2)$.

B. Line r has slope 8 and y-intercept $(0,5)$.

C. Line r has slope 8 and y-intercept $(0,9)$.

D. Line r has slope 11 and y-intercept $(0,8)$.

Figure 6. A technology inactive question from https://parcc.pearson.com/resources/Practice-Tests/TBAD/Geo/PC1105806_GeoTB_PT.pdf

If the question is answered using paper and pencil, a student may plot $f(x)=8x+5$. They may then plot the point (0,3). They may determine the distance between (0,5) and (0,3) to be two and then multiply this distance by three to determine that (0,9) is the image of (0,5) under the dilation. This may be repeated for another point to find line r or a student may recall that the slope is invariant under dilation. Using a dynamic geometry program, students need to enact technological procedures, plotting a line, plotting a point, and performing dilation. The thinking needed is different given the different tools students have available for them to use. Thus, when teachers have access to technology, they need to think carefully about the tasks and questions they will pose and the thinking required of students when technology is used. Dick and Hollebrands (2011) stress the importance of questions in the context of technology by stating: "The value of technology to the teacher lies not so much in the answers technology provides but rather in the questions it affords. Indeed, "what questions could I ask that I could not ask before?" is the ruler by which we should judge what technology buys us as teachers of mathematics" (p. xvi).

Technology also allows teachers new tools to use when leading mathematical discussions. Collaborative tools such as Google Docs, Sheets, or Slides allow multiple students to share and discuss their work with the whole class. Mathematics specific technology tools like the TI-Navigator and Desmos allow the teacher to monitor and share student work. Orchestrating discussions using these types of tools requires teachers to focus on their mathematical goals and consider ways that they can reach them by selecting and sequencing students' work to assist them in making connections (Smith & Stein, 2011).

Finally, assessment in mathematics classrooms can look very different when teachers are using technology. Game-like assessment tools like Kahoot! Quizlet and Quizzies can be motivating for students and can provide feedback to teachers about what students know. Diagnostic assessments tied to learning trajectories can provide teachers with information about what students know or do not know and make informed decisions about what to do next to advance students' learning.

We present these as questions a teacher may consider when using technology in the mathematics classroom (See Figure 7).

In addition to considering how technology can influence mathematics and pedagogy there are ways in which technology interacts with the teachers and students that we highlight in our MOOC. These are described in the following sections.

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Articles published in the Proceedings are copyrighted by the authors.

Technology -Teacher Edge. Technology is used by teachers and students in the classroom in a variety of ways. Walk into any classroom and you may find teachers using a document camera, interactive white board, and a laptop equipped with a wide array of software applications. Dick and Hollebrands (2011) use the constructs of conveyance and mathematical action technology to assist teachers in making distinctions among the different technology they have available to use in the classroom.

Type of Technology. Conveyance technologies are used to "transmit and/or receive information" and are not math specific. These include presentation technology (PowerPoint, document cameras, interactive boards, projectors), communication technology (social media), collaboration technology (Google Docs), and assessment technology (clickers, educational games). Even though these technologies are not mathematics specific they can still have a significant impact on a mathematics classroom by providing opportunities for students to consider and critique each other's solutions and justifications.

Pedagogical Activities	**Questions to Consider**
Designing Tasks	What is the cognitive demand of this technology-based task? (Stein & Smith, 1998) How will the student interact with the task and technology? How does technology enhance student learning? What learning goals would be best served by this task? How might I prepare students to engage productively in this task?
Questions	What new questions does this technology allow me to ask? In what ways can I ask questions that will advance student thinking and probe what students are learning? What opportunities does the technology allow for students to pose their own mathematical questions? How might a structure my classes to help students feel comfortable generating and posing their own questions and responding to questions that other students generate?
Discourse	Does the technology allow for different solutions and/or different solution strategies? What would make a discussion of technology-based tasks productive? How can I use technology to facilitate a productive mathematics discussion?
Assessment	What type of feedback does the technology provide to the student? How can I build self-assessment into the tasks? How can I leverage the technology to determine what students are learning? How can I use the technology to assess what the students have learned?

Figure 7. Questions a teacher can consider when examining the effects of technology on pedagogy.

Mathematical action technologies are tools, software, and applets that can "perform mathematical tasks and/or respond to the user's actions in mathematically defined ways" (Dick & Hollebrands, 2011, p. xii). These technologies include: graphing calculators, computer algebra systems, and dynamic mathematical environments (GeoGebra, the Geometer's Sketchpad, Fathom, TinkerPlots). Often these technologies are used to perform computations, graph functions, plot data, and construct geometric figures. However, mathematical action technologies can also be used to allow students access to approaches and tasks that would not be possible without technology. Here technology can

Articles published in the Proceedings are copyrighted by the authors.

be used to develop students' mathematical understanding and support students as they explore patterns. Mathematical action technologies can also offer opportunities for teachers to pose questions and tasks that could not be asked in non-technological environment (Zbiek, Heid, Blume, & Dick, 2007). For example, in a dynamic geometry environment, a teacher can ask students to explore how a particular quadrilateral behaves when one of its vertices is dragged. This question is one that cannot be posed in a paper-pencil environment. Guiding questions a teacher can consider when selecting and evaluating technology tools are included in Figure 8.

Technology Consideration	Issues to Consider and Questions to Pose
Technology-Teacher	Conveyance/Mathematical Action Technology. Will the technology be used for the teacher to convey information to students (e.g., power point, internet)? Will the technology be used to allow students to perform mathematical actions? (Dick & Hollebrands, 2011) Is the technology readily available for the teacher? Is the learning curve minimal for the teacher?

Figure 8. Questions a teacher can consider when selecting technology tools.

Technology-Student Edge. When making a decision about whether to use a particular technology tool, thinking about how students interact with the technology is especially important (See Figure 9).

Figure 9. The edges of the tetrahedron.

Mathematical action technology often include mathematical representations students can directly manipulate. Direct manipulation allows users to use a mouse or their finger to interact directly with the representation of the mathematical object. The way that the object moves is continuous. There is

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Articles published in the Proceedings are copyrighted by the authors.

no lag between the user's own movement and that of the object in the environment. The way that the objects respond is determined by mathematical rules. Thus, through direct interactions the student can observe and infer mathematical properties and theorems. This is one important feature that makes technology tools different from non-technology tools such as base-10 blocks or a ruler. By interacting with technology, students can learn mathematical rules and properties. Conveyance technology tools, on the other hand, typically do not offer mathematics representations and can sometimes be challenging for students to enter mathematics notation. However, these conveyance technologies can be designed by the teacher to provide students with feedback that they can use to gauge their learning and mathematical understanding. Some questions teachers may want to consider when thinking about students' interactions with technology are included in Figure 10.

Technology-Student	Interaction. How does the student interact with the technology? Is the technology available? Is the technology learning curve minimal or steep? Feedback. What types of feedback does the technology provide to students when they are interacting with it?

Figure 10. Questions teachers can consider related to the ways students interact with technology and the feedback it provides.

Conclusion

While technology is rapidly changing and evolving, there are general questions teachers can contemplate when making decisions about whether to select and use a particular technology tool. Teachers should consider whether the technology they are selecting to use is a conveyance or mathematical action technology. They should evaluate the types of representations the technology offers and determine whether those representations are faithful to the mathematics students are learning. They should also assess whether the technology is used to amplify or reorganize students' thinking. How technology effects the design of tasks, questions that can be posed, facilitation of discourse, and assessment of student learning should also be considered. Finally the ways students can interact with the technology and the feedback it provides to support students' mathematics learning should be taken into consideration.

Acknowledgments

The design, implementation, and research of MOOC-Ed courses at North Carolina State University is partially funded by the William and Flora Hewlett Foundation. Any opinions, findings, and recommendations expressed are those of the authors, and do not necessarily reflect the views of the Hewlett Foundation. Thank you to Hollylynne Lee and Gemma Mojica for their contributions in course design and to Tom Dick, Janet Bowers, Pam Buffington, and M. Kathleen Heid for their feedback on the development of a framework for the course.

References

Ainsworth, S. (1999). The functions of multiple representations. *Computers & Education, 33*(2), 131-152.
Brousseau, G. (1997). *Theory of didactical situations in mathematics: Didactiques des mathématiques (1970–1990)*. (trans: Balacheff N., Cooper M., Sutherland R., Warfield V.). Dordrecht: Kluwer.
Dick, T. (2008). Keeping the Faith. *Research on Technology and the Teaching and Learning of Mathematics: Cases and perspectives*. In M. Kathleen Heid & G. Blume (Eds). Volume 2, pp. 333-353.
Dick, T. P., & Hollebrands, K. F. (2011). *Focus in high school mathematics: Technology to support reasoning and sense making*. Reston, VA: National Council of Teachers of Mathematics.
Freudenthal, H. (1991). *Revisiting mathematics education: China lectures*. Dordrecht: Kluwer.

Articles published in the Proceedings are copyrighted by the authors.

Goldin, G. A., & Kaput, J. J. (1996). A joint perspective on the idea of representation in learning and doing mathematics. In L. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), Theories of mathematical learning (pp. 397–430). Mahwah, NJ: Lawrence Erlbaum Associates

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education*, 524-549.

Kaput, J. J. (1987). Representation systems and mathematics. In C. Janvier (Ed.). *Problems of representation in the teaching and learning of mathematics* (pp. 19-26). Hillsdale: Lawrence Erlbaum.

Kaput, J. J. (1992). Technology and mathematics education. In D. A. Grouws (Ed.). *Handbook of research on mathematics teaching and learning* (pp. 515-556) New York: Macmillan.

Monaghan, J., Trouche, L., Borwein, J. (2016). *Tools and mathematics: Instruments for learning.* Switzerland: Springer International Publishing,

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: NCTM.

National Governors Association Center for Best Practices, Council of Chief State School Officers, (2010). *Common Core State Standards - Mathematics*. Washington, DC: National Governors Association Center for Best Practices, Council of Chief State School Officers.

Olive, J., Makar, K., Hoyos, V., Kor, L., Kosheleva, O., & Sträßer, R. (2010). Mathematical knowledge and practices resulting from access to digital technologies. In C. Hoyles, & J.B. Lagrange (Eds.), *Mathematics education and technology-rethinking the terrain* (pp. 133-177). New York, Dordrecht, Heidelberg, London: Springer.

Pea, R. (1985). Beyond amplification: Using the computer to reorganize mental functioning. *Educational Psychologist, 20*(4), 167-182.

Pea, R. D. (1987). Cognitive technologies for mathematics education. In A. Schoenfeld (Ed.), *Cognitive science and mathematics education* (pp. 89–122). Hillsdale, NJ: Erlbaum.

Ruthven (2012) The didactical tetrahedron as a heuristic for analysing the incorporation of digital technologies into classroom practice in support of investigative approaches to teaching mathematics. *ZDM Mathematics Education, 44*, 627-640.

Smith, M., & Stein, M. K. (2011). 5 Practices for orchestrating productive mathematics discussions. Reston, VA: National Council of Teachers of Mathematics.

Speak Up (2014). http://www.tomorrow.org/speakup/

Steinbring, H. (2005). The construction of new mathematical knowledge in classroom interaction: An epistemological perspective. New York: Springer.

Tall, D. (1986). Using the computer as an environment for building and testing mathematical concepts: A Tribute to Richard Skemp. Accessed 5 April 2011 at http://www.warwick.ac.uk/staff/David.Tall/themes/computers.html

Vérillon, P., & Rabardel, P. (1995). Cognition and artefacts: a contribution to the study of thought in relation to instrumented activity. *European Journal of Psychology of Education*, *10*(1), 77–101.

Zbiek, R. M., Heid, K. M., Blume, G. W., & Dick, T. P. (2007). Research on technology in mathematics education: A perspective of constructs. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1169-1207). Charlotte, NC: Information Age Publishing.

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

Articles published in the Proceedings are copyrighted by the authors.