DETECTING MATH ANXIETY WITH A MIXTURE PARTIAL CREDIT MODEL

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The purpose of this study was to investigate a new methodology for detection of differences in middle grades students' math anxiety. A mixture partial credit model analysis revealed two distinct latent classes based on homogeneities in response patterns within each latent class. Students in Class 1 had less anxiety about apprehension of math lessons and use of mathematics in daily life, and more self-efficacy for mathematics than students in Class 2. Moreover, students in Class 1 were found to be more successful in mathematics, mostly like mathematics and mathematics teachers, and have better educated mothers in comparison to students in Class 2. However, gender, attending private or public schools, and education levels of fathers did not appear to differ between the classes. Capturing such fine-grained information extends recent advances in measuring math anxiety.

Keywords: Affect, Emotion, Beliefs, and Attitudes; Middle School Education, Research Methods

Identifying affective characteristics, such as anxiety and depression, that students experience in school settings and dealing with these characteristics are significant challenges for educators. Math anxiety, as one such characteristic, can be defined as "feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations" (Richardson & Suinn, 1972, p. 551).

Math anxiety has been shown to cause low academic performance (Ashcraft, 2002), reduced cognitive information-processing (Young, Wu, & Menon, 2012), and low perceptions of one's own mathematics abilities (Hembree, 1990). Low math abilities and low working memory, as well as non-supportive teachers can also be considered as important risk factors for the existence of math anxiety (Ashcraft, Krause, & Hopko, 2007). As a result, math anxiety can lead to avoidance of selecting career paths involving mathematics (Ashcraft & Moore, 2009). Previous research on mixture item response theory (IRT) models (e.g., Mislevy & Verhelst, 1990; Rost, 1990) has suggested that these models may be useful in detecting latent classes of individuals that differ along one or more cognitive or affective characteristics. Latent classes are statistically determined groupings of individuals who are homogeneous on such characteristics. Latent classes are latent because they are not directly observable as gender or ethnic groups. Previous research has demonstrated that latent classes in a population may differ on multiple kinds of characteristics including problem solving (Bolt, Cohen, & Wollack, 2001), test speededness (Cohen & Bolt, 2005), mathematical knowledge (Izsák, Jacobson, de Araujo, & Orrill, 2012), reading comprehension (Baghaei & Carstensen, 2013) and on personality traits such as depression (Hong & Min, 2007). In view of the negative, long-term impacts of math anxiety, it would be useful to distinguish latent classes of students who differ in their math anxiety. Such an identification of the latent classes would potentially help teachers improve the affective environment in school settings by applying specific interventions based on the needs of students in each latent class.

The purpose of this study was to investigate the utility of a mixture IRT methodology for detection of latent classes of middle grades students' math anxiety. The following research questions were addressed in this study:

1. Are there distinct latent classes of middle grades students that differ in their math anxiety?
2. What does the existence of these latent classes imply about the different response patterns of math anxiety that exist in this population?

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3. What are the effects of manifest variables such as mathematics achievement, gender, liking mathematics, liking mathematics teachers, attending to private or public schools, education levels of mothers and fathers on latent class membership?

The present study makes at least two contributions. First, past research has attempted to identify students' math anxiety levels based on their total scores on a math anxiety scale. Results from the present study suggest that relying on use of total scores may miss important qualitative and quantitative differences in students' math anxiety and for understanding the structure of math anxiety. Second, past research has traditionally focused on explaining math anxiety by measuring its dimensions through exploratory and confirmatory factor analysis (e.g., Baloğlu & Zelhart, 2007; Kazelskis, 1998) and on the structural equation modeling of the relationship between math anxiety and variables such as mathematics achievement (e.g., Harari, Vukovic, & Bailey, 2013; Meece, Wigfield, & Eccles, 1990). However, to our knowledge, there has been no study yet reported in the literature on the detection of different latent classes of the math anxiety population by using relatively new psychometric models, such as mixture IRT models. Therefore, the present study demonstrates that a mixture IRT model can be useful for identifying characteristics of latent classes and for obtaining fine-grained information about particular strengths and weaknesses of middle grades students' math anxiety. Results of this study suggest one potentially useful route for mathematics education research in the future by providing a unique approach on identifying math anxious students in school settings.

Theoretical Framework

The theoretical framework for this study is based on the mixture Rasch model (MRM; Rost, 1990), which is a combination of a latent class model and a Rasch model. Unlike the standard Rasch model, which assumes that the same Rasch model applies to all examinees in the population, the MRM assumes that distinct latent classes exist in the population and that a different Rasch model applies to each. In the MRM, the relative difficulty of ordering the items is determined by a class membership parameter, and the number of items which the examinee is expected to answer or endorse is influenced by a continuous latent ability variable specific to the latent class. For each item, the MRM specifies a separate item difficulty for each latent class and for each examinee, a probability of being a member of a particular latent class.

In contrast to the dichotomous form of the MRM with scoring of an item in two categories such as agree or disagree, the polytomous form of the model can be used when items are scored with more than two categories such as strongly agree, agree, disagree, or strongly disagree, and this form is called a partial credit model (PCM; Masters, 1982). The probability of an answer for the mixture form of this model, the mixture partial credit model (MixPCM), can be written as

$$P(x_{ij}=k|\theta_{jg}) = \frac{\exp[\sum_{r=1}^{k}(\theta_{jg}-\delta_{irg})]}{\sum_{t=0}^{m_i}[\exp\sum_{r=1}^{t}(\theta_{jg}-\delta_{irg})]} \qquad (1)$$

where P is the probability that examinee j gives a response in category k of item i, θ_{jg} is a latent trait of examinee j, and δ_{irg} is a threshold parameter indicating the intersection of adjacent category response curves.

The MixPCM enables one to detect homogeneities in the ways examinees in different latent classes respond to items on a scale. As in equation (1), the relationship between the probability of selecting a response category and the latent trait varies across latent classes. The differences in response patterns to each item of a scale reflect homogeneities in characteristics of members of each latent class. In the MixPCM, the relative difficulty of the ordering of a particular response category among the ordered categories is determined by a class membership parameter, and the number of items answered. In this way, the MixPCM could assign two examinees with similar scores on a scale to different latent classes as a result of the differences in their response patterns.

Methods

Participants

The sample consisted of 244 Turkish 6th and 7th grade students attending public and private schools in Turkey. While the number of male and female students is similar (N=128 and N=116 for males and females, respectively), their range of age was around 13-14 years. A written consent form was obtained from one of the parents of each student before the study.

Instruments

The *Math Anxiety Scale* (MANX; Erol, 1989), is a four-point Likert type scale written in Turkish with options for each item ranging from “never” to “always.” There were 45 items yielding minimum and maximum scores of 45 and 180, respectively. Higher scores demonstrate a higher math anxiety level. An internal consistency reliability estimate of .90 was obtained in this study. This was consistent with previous results of .92 on a sample of 754 middle school and high school students (Erktin, Dönmez, & Özel, 2006). Erktin et al. detected four factors, which were *test and evaluation anxiety*, *apprehension of math lessons*, *use of mathematics in daily life*, and *self-efficacy for mathematics*.

Demographic information was also obtained regarding students’ mathematics grade at the end of the previous semester, their gender, whether or not they liked mathematics and mathematics teacher, the type of school they attended, and parents’ education levels.

Data Analysis

The data were analyzed using the MixPCM as implemented in the computer program WINMIRA (von Davier, 2001). First, different numbers of latent classes were estimated in separate models to determine the relative fit of each model. That is, the MixPCM was estimated with one class, two classes, three classes, and four classes. Second, three indices for each model were compared to select the best fitting model: the Akaike’s information criterion (AIC; Akaike, 1974), the Bayesian information criterion (BIC; Schwarz, 1978), and the consistent AIC (CAIC; Bozdogan, 1987). These indices are defined as AIC = $-2 \log L + 2p$; BIC = $-2 \log L + p (\log N)$, and CAIC = $-2 \log L + p (\log N + 1)$ where L is the maximum likelihood value, p is the number of estimated parameters, and N is the sample size. AIC, BIC, and CAIC all include penalty functions to modify the -2 log likelihood for either the number of parameters or the sample size or both. BIC has been found to more accurately select the best fitting model for dichotomous mixture IRT models (Li, Cohen, Kim, & Cho, 2009). In this study, the model with the smallest BIC value was selected as the best fitting model. Next, we analyzed the characteristics of each latent class by focusing on places where item locations differed significantly by latent class and places where members of one latent class considered items to be easier or harder to endorse than other latent classes. In addition, independent sample t-tests and chi-square tests were conducted to examine the relationships between manifest variables and latent class membership.

Results

Unidimensionality for the Scale

An exploratory factor analysis using maximum likelihood estimation as implemented in the SPSS 16.0 software (SPSS Inc., 2007) indicated eigenvalues of the first three factors as 14.1, 2.6, and 2.5.

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The total variance explained by the first factor was 31.4%. Reckase (1979) reports that if the amount of variance explained by the first factor is 20% or more, then the scale can be considered as essentially unidimensional. Based on these results, the MANX was considered to be essentially unidimensional.

Number of Latent Classes

The information indices for model selection are given in Table 1. Minimum values for AIC, BIC, and CAIC of 12883.82, 13705.72, and 13978.72, respectively, all suggested a two-class solution in the data. Class 1 had 126 students (51.5%) and Class 2 had 118 students (48.5%).

Table 1: Model Fit Indices of the Mixture Rasch Model

Model	AIC	BIC	CAIC
One class	13757.02	14166.47	14302.47
Two classes	**12883.82**	**13705.72**	**13978.72**
Three classes	13091.45	14325.81	14735.81
Four classes	13335.07	14981.88	15528.88

Note. AIC = Akaike information criterion; BIC = Bayesian information criterion; CAIC = Consistent Akaike information criterion; the smallest information criterion index is bold.

Item thresholds indicate the point on the trait scale between each adjacent score category and indicate the relative ease of endorsing each item in each latent class. Item thresholds for each class are plotted in Figure 1 and Figure 2. Thresholds lower on the scale (e.g., -3, -2) indicate that examinees had a greater propensity to endorse that response category. Similarly, thresholds higher on the scale (e.g., 2, 3) indicate that examinees had a greater propensity to endorse a higher response category. Thresholds may differ by latent class, meaning that relative propensity for endorsing a category of an item is specific to each latent class. Because the MANX has four response categories ranging from “never” to “always,” there are three possible thresholds that can be used to interpret the math anxiety level for each item as follows:

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Categories: “never”       “sometimes”       “usually”        “always”
Scores:          (1)               (2)               (3)               (4)
Thresholds:      |--------T1--------|--------T2--------|--------T3--------|
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For example, if an examinee’s trait level is smaller than T1 (i.e., the first threshold), then the response is expected to be “never.” If an examinee’s trait level is smaller than T2 (i.e., the second threshold) but larger than T1, then the response is expected to be “sometimes.”

Figure 1 and Figure 2 present plots of the item thresholds for Class 1 and Class 2. It is clear that students in Class 1 were more variable in endorsing or agreeing than students in Class 2, with thresholds ranging from -7.41 to 9.00. Students in Class 1 also had lower tendency to endorse items above threshold 1, and greater tendency to endorse items above threshold 3 than students in Class 2. On the other hand, students in Class 2 were more constrained in endorsing items with the range of thresholds from -2.186 to 2.249.

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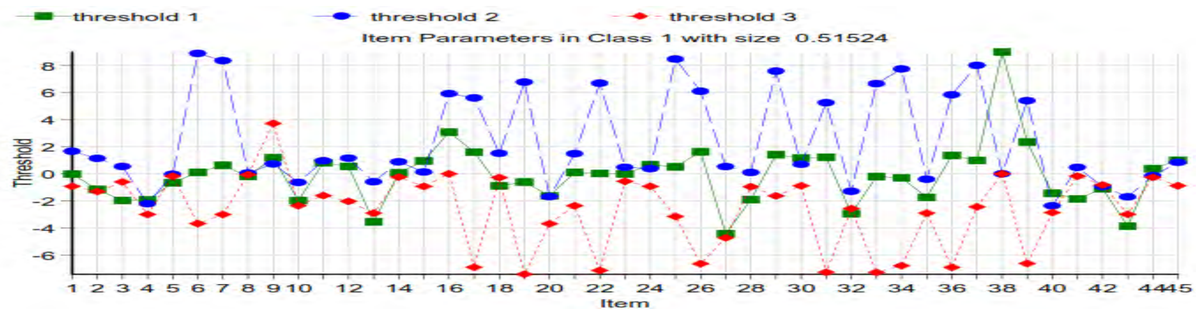

Figure 1. Item thresholds for class 1.

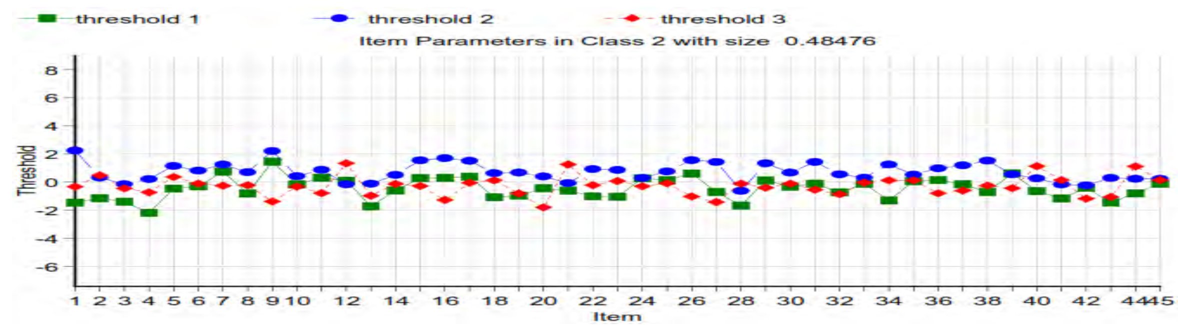

Figure 2. Item thresholds for class 2.

Analyses of Item Locations and Item Response Distributions

The item location is the mean of all item thresholds for an item. Higher mean thresholds indicate lower propensities of endorsement (Masters, 1982). Thus, item locations suggest which items cause differences in response patterns between latent classes.

In addition to the analysis of item locations, item response distributions between the two latent classes were compared to examine similarities and differences in item responses for each latent class. Analyses of item locations and item response distributions led to three main results: (1) Students in Class 1 were less anxious than students in Class 2 in terms of having anxiety about *apprehension of math lessons* and *use of mathematics in daily life*, (2) students in Class 1 had more *self-efficacy for mathematics*, and (3) students in both latent classes had similar levels of *test and evaluation anxiety*.

Figure 3 presents item locations for each latent class. Based on the figure, it can clearly be said that Class 1 had different and more variable item locations than Class 2.

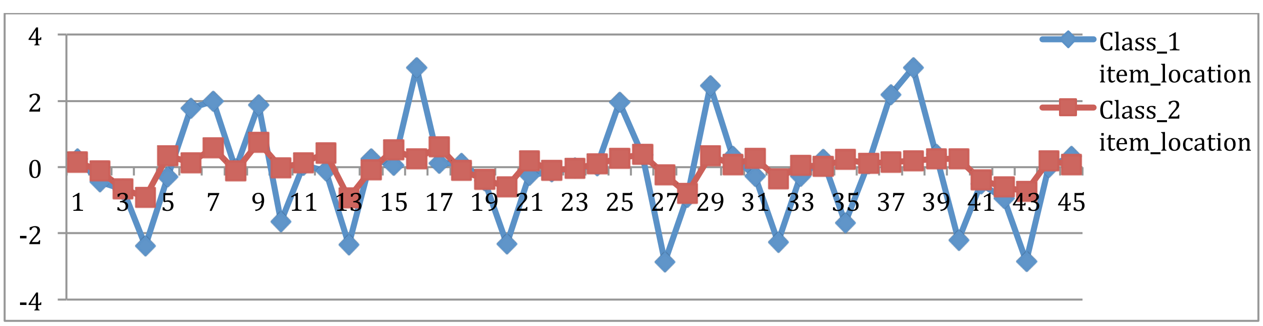

Figure 3. Item locations for class 1 and class 2.

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Items with a difference on the scale of 1 logit or more were considered as indicating differences between the two latent classes. Based on the item locations for the two latent classes (see Figure 3), items 4, 6, 7, 9, 10, 13, 16, 20, 25, 27, 29, 32, 35, 37, 38, 40, and 43 appeared to have different response propensities for Class 1 and Class 2.

Items reflecting anxiety about *apprehension of math lessons* (i.e., Items 6, 7, 16, and 37) were more difficult to endorse for Class 1 than Class 2 (see Figure 3). For example, for Item 16, “Math book bothers me,” the item location for Class 1 was 3 but for Class 2, it was .25. The proportions selecting the options of “never” and “always” in Class 1 were 98.7% and 0% respectively, in contrast to 53.3% and 15.1% in Class 2, respectively. On items such as Item 6 and Item 7, which asked students to identify how much they panic when a lot of mathematics problems are given as homework and how uncomfortable they feel when studying a hard mathematics topic (For Item 6 and Item 7, Class 1 item locations were 1.78 and 2; Class 2 item locations were .14 and .58), students in Class 1 mostly agreed with the option “never” (80.2% for Item 6 and 87.1% for Item 7) than students in Class 2 (38.6% for Item 6 and 64.2% for Item 7).

On the other hand, students in Class 1 endorsed more easily items with positive statements such as enjoying numbers (i.e., Items 4, 10, 13, 20, 32, 35, and 40). For example, for Item 40, “Opening any book on math and looking at one of its pages full of mathematics problems makes me happy,” (Class 1 and Class 2 item locations were -2.21 and .25, respectively), the proportion selecting “always” was 70.3% in Class 1 as opposed to 7.8% in Class 2.

Items focusing on anxiety about *use of mathematics in daily life* (i.e., Items 9, 29, and 38) were harder for students in Class 1 to endorse than for students in Class 2 (see Figure 3). For Item 29, “When I am asked to help a primary school student with his/her homework, I may refuse to help because I feel afraid that there may be some problems that I could not solve” (Class 1 item location was 2.46; Class 2 item location was .35), almost all students in Class 1 (93.5%) and half of the students in Class 2 (50.5%) selected the option “never.” On items that asked students to rate their ideas about *test and evaluation anxiety* (i.e., Items 2, 3, 8, 11, 14, 18, 19, 21, 22, 24, 25, 28, 30, 33, 41, 42, and 44), item locations as well as the distributions of responses were similar across choices for both classes.

Finally, on items involving *self-efficacy for mathematics* (i.e., Items 27, and 43), item locations and the distribution of responses were different for the two classes. For Item 43, “When I think I succeeded at a math exam, I feel relaxed and peaceful while waiting for the announcement of the results” (Class 1 item location was -2.84; Class 2 item location was -.73), the proportion selecting “always” in Class 1 was 72.8% as opposed to 39.5% in Class 2.

The Relationships Between Manifest Variables and Latent Class Membership

To obtain detailed information about the characteristics of each latent class, we examined the relationships between manifest variables and latent class membership using independent sample t-tests and chi-square tests. Regarding mathematics achievement, students in Class 1 were significantly more successful than students in Class 2 (t (df = 111) = 3.71, $p < .01$). In terms of gender, there was no significant association between the two classes ($\chi^2(1) = .98$, $p = .32$). The associations between students’ liking mathematics and liking their mathematics teachers, and latent class membership were significant ($\chi^2(1) = 11.83$, $p < .01$ and $\chi^2(1) = 6.30$, $p < .01$, respectively). However, there was no significant association between the type of school attended and latent class membership ($\chi^2(1) = .57$, $p = .45$). Finally, education level of mothers was higher for students in Class 1 than Class 2 (t (df = 136) = 2.36, $p < .02$), but there was no significant difference in terms of education levels of fathers (t (df= 136) = 1.07, $p = .29$).

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Discussion

In this study, we examined the utility of a mixture IRT methodology, named MixPCM, for detecting latent classes of middle grades students' math anxiety. With respect to the first research question, two latent classes were detected with distinct patterns of math anxiety. With respect to the second research question, Class 1 consisted of students who were reported being less anxious about *apprehension of math lessons* and *use of mathematics in daily life*, and as having more *self-efficacy for mathematics* than students in Class 2. However, there did not exist any difference between Class 1 and Class 2 in terms of *test and evaluation anxiety*. With respect to the third research question, students in Class 1 were found to be more successful in mathematics, mostly like mathematics and mathematics teachers, and have better educated mothers in comparison to students in Class 2. Moreover, there was no significant association between the two classes in terms of gender, attending private or public schools, or education levels of fathers.

The results reported here on the relationships between math anxiety and the manifest variables were consistent with the findings in the literature. Similar to the previous findings indicating that math anxiety was negatively related to mathematics achievement (e.g., Hembree, 1990), students in Class 1, in the present study, were reported being less anxious and more successful in mathematics. Previous research on the effects of positive attitudes and education levels of mothers on math anxiety has led to a consensus that positive attitudes towards mathematics and education levels of mothers were negatively associated with math anxiety (e.g., Engelhard, 1990; Meece, Wigfield, & Eccles, 1990). In this study, students in Class 1 were found to be less anxious but be more likely to have positive attitudes such as enjoying mathematics and liking their mathematics teachers, and to have mothers with higher education levels than students in Class 2. Including the analysis of manifest variables along with results from the MixPCM and obtaining consistent results with previous research strengthen the validity of the interpretations about the characteristics of each latent class reported in this study.

The results of this study have important implications for teachers and researchers. First, it may be misleading to compare all students based on their total scores on a scale of math anxiety. Rather, within a population of students, there exist latent classes that differ in their math anxiety. Relying on only single total score, therefore, might hinder gaining insight about particular characteristics of students. In this regard, the MixPCM was found to be a useful tool for identifying those students with different patterns of math anxiety in classroom settings. This, in turn, could help teachers make interventions specific to the needs of each student. For example, they can focus on reducing some particular students' anxiety levels towards mathematics lessons by not calling on these students to solve a problem at the board; engage some students with more mathematics related activities in daily life by presenting simulated real-life situations and asking word problems in a real-life context; and help some students build self-confidence for mathematics through asking mathematical problems from simple to more complex.

In conclusion, the present study was the first study that examined the utility of the MixPCM for detection of distinct latent classes based on different patterns of math anxiety. The results reported here provide initial evidence that the MixPCM, when applied to a scale like the MANX, can provide fine-grained information about latent classes of middle grades students population and their characteristics of math anxiety. Future studies should continue on conducting similar studies with other popular math anxiety scales in different populations.

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