SCHOLARLY PRACTICE AND INQUIRY: DYNAMIC INTERACTIONS IN AN ELEMENTARY MATHEMATICS METHODS COURSE

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This paper represents research that exists at the crossroad of scholarly practice and scholarly inquiry. We share the design, enactment and empirical examination of an elementary methods course activity, Exploring and Supporting Student Thinking (ESST) which engaged 18 prospective teachers in two sessions of one on one problem posing with 3rd grade students. Results mirror outcomes from existing literature on student interviews and letter exchanges.

Keywords: Teacher Education-Preservice, Teacher Knowledge

Research suggests that teachers who understand how students think about particular mathematical ideas will be better positioned to recognize, interpret and support these ideas in their instruction (Brown and Borko, 1992; Fenema and Franke, 1992). Research on Cognitively Guided Instruction (CGI) has demonstrated teacher knowledge of student thinking, reasoning and strategies can lead to gains in student achievement (Carpenter & Fennema, 1992; Carpenter, Fennema, Franke et al., 2000). Ball and her colleagues' work on mathematical knowledge for teaching identified knowledge of content and students as a crucial facet of pedagogical content knowledge necessary for teaching mathematics effectively (e.g. Hill, Ball & Schilling, 2008).

In light of these findings it has become increasingly important for mathematics teacher educators (MTEs) to assist prospective elementary mathematics teachers (PTs) in developing knowledge of children's thinking. Jacobs, Lamb and Phillip's (2010) work on professional noticing of children's mathematics has become a popular framework to explore the ways in which teachers attend to, interpret and respond to students' mathematical thinking. Mathematics methods course activities sometimes provide PTs opportunities to examine and interpret authentic (and/or instructor-generated) samples of student work depicting invented computational strategies or mathematical reasoning as a means to gain experience interpreting and responding to student thinking (e.g. Tyminski, Land, et al., 2014). We term these types of interactions as static, in that there is no student to interact with during the process of interpreting the work, and once PTs have done so, there is no opportunity to respond authentically to students and observe the result. Although we see value in these types of interactions in developing PTs' knowledge of students' mathematical thinking and include examples of them in our methods courses, we sought to design and enact an activity in our early field experience that would foster PTs' understanding of how to elicit and support student's mathematical thinking and which would be dynamic in nature; allowing for a sustained exchange between the PTs and the student.

This paper represents research that exists at the crossroad of scholarly practice and scholarly inquiry. We outline the process in the design, enactment and empirical examination of an elementary methods course activity, Exploring and Supporting Student Thinking (ESST), and answer the question, "What are the experiences of PTs within the ESST activity?"

Literature Review

Scholarly Inquiry and Practice

In methods course activity design and enactment, the authors seek to leverage the interplay between research and practice thorough the processes of scholarly inquiry and scholarly practice (Lee

& Mewborn, 2009). Scholarly inquiry is the exploration of "issues and practices through systematic data collection and analysis that yields theoretically-grounded and empirically-based findings" (p. 3), while scholarly practices are "adapted from empirical studies of the teaching and learning of mathematics and the preparation of mathematics teachers" (Lee & Mewborn, 2009, p. 3). "Scholarly inquiry and practices are interrelated in that MTEs use empirical studies in mathematics education to build practices that are labeled scholarly. In addition, scholarly practices can inform directions for scholarly inquiry regarding PSTs' mathematics teaching and learning" (Kastberg, Tyminski & Sanchez, in press). In order to create the ESST Activity as an example of scholarly practice, we reviewed the literature on existing scholarly inquiry on dynamic interactions in a methods course in order to: 1) synthesize knowledge on the potential impact of these activities on PTs' learning and 2) inform the activity's design by understanding the variation and commonalities of activities described in other researchers' scholarly inquiry.

Dynamic Interactions

Examples of dynamic interaction activities found within the research literature included asynchronous activities such as letter writing exchanges (e.g. Crespo, 2000; 2003; Norton & Kastberg, 2012), as well as face-to-face activities such as interviews with learners (e.g. Ambrose, 2004; Jenkins, 2010), scripted interview protocols (Moyer & Milewicz, 2002), and PTs work with small group of learners (e.g. Nicol, 1998). A brief summary of the activity, context and findings are presented for each example of scholarly inquiry.

In Ambrose (2004), Elementary PSTs worked in pairs to pose open-ended problem solving activities focused on whole number operations and fractions to children. The goal was to impact PSTs' beliefs about teaching, potentially shifting beliefs from teaching as explaining, by leveraging their current beliefs as caregivers. Ambrose concluded PSTs developed new beliefs that were incorporated in existing belief structures.

Jenkins (2010) intervention involved six middle grades PTs working in pairs in alternating roles to pose open-ended tasks focused on patterns and proportions to students. Jenkins searched for evidence of PTs' "interpretive listening skills and awareness of the different ways that middle school students make sense of mathematics" (p. 147). Jenkins reported "the structured interview process fosters an interpretive orientation to listening and initial awareness of the variety of ways that middle school students think about mathematics" (p. 147).

Moyer and Milewicz (2002) engaged 48 PTs in using scripted diagnostic interview protocols focused on rational number tasks to guide their interactions with children. The PTs conducted interviews with children throughout the semester. The final interview was recorded, transcribed, analyzed and reflected upon by the PTs and served as evidence of PTs' experiences and use of questioning. Analysis of the interviews revealed a beginning classification for the types of questioning: 1)"check listing," asking the questions in the protocol with little regard for student responses; 2) "instructing vs. assessing," in which PTs explained mathematics directly to the student with little regard for students' reasoning; and 3) "probing and follow up questions," characterized as PTs genuinely listening to student responses and generating follow-up questions meant to elicit further student thinking.

In Nicol's (1998) activity, 14 PTs were engaged in weekly interactions with small groups of 6th and 7th grade students. The PTs solved problems involving multiplicative reasoning in class and then posed adapted or extended versions of these tasks to students. Nicol examined PTs' abilities to question, listen and respond to students using prospective teachers' journal reflections as sources of evidence for these behaviors. Across the weekly implementations of the activity, PTs began to shift their approaches from those that focused on arriving at a correct answer toward an inquiry-based approach focused on eliciting and understanding student thinking.

Crespo (2000) examined the ways in which elementary PTs listened to the responses of the fourth-grade students in a series of six interactive letter exchanges. PTs' initial interpretations of student work focused on correctness and tended to contain conclusive claims about student understanding based upon small samples of student thinking. Reflective journals were used by PTs to explore their interactions with students. PTs' interpretations began to focus on what the student intended or meant in a solution by the fifth week of the course. Crespo (2003) used the same letter writing activity and data to explore PTs' abilities to pose problems. Initially, PTs attempted to "make their problems less problematic and more attainable to their pupils" (p.251). PTs' questions were worded to avoid student errors or confusion rather than to generate learning opportunities for students or themselves as teachers. Problems included in the last three letters "were puzzle-like and open-ended, encouraged exploration, extended beyond topics of arithmetic, and required more than computational facility" (p. 257). These questions were posed to challenge or extend student thinking and often asked for multiple solutions and explanations.

Methods

Participants and Context

This study examined the experiences of 18 junior-level PTs enrolled in the required elementary mathematics methods course for their university program as they engaged in the ESST Activity. Prior to this course, PTs had completed four mathematics content courses designed for elementary mathematics teachers, including a course on problem solving, and were concurrently enrolled in a fifth content course addressing middle grades mathematics topics. Prior to engaging in this activity, PTs had engaged in activities involving standards documents, CGI problem types and student strategies, responding to students through questioning, number choice and number choice progressions, and opening routines in the grade 2-6 mathematics classroom (Drake, Land, et al., in press).

Exploring and Supporting Student Thinking Activity

The design of the ESST Activity was developed as an example of scholarly practice informed by the literature described above on dynamic interactions. From the literature we identified four contextual factors as potentially supportive in the design and enactment of such an activity: 1) PTs should have opportunities to solve challenging mathematical problems prior to posing them to students; 2) PTs should pose the same problems to students in order to give PTs common experiences to discuss; 3) PTs require opportunities to reflect on their experiences both in a whole group setting as well as through individual, targeted reflection; and 4) MTEs must consistently respond to PTs' reflections.

The ESST activity engaged PTs in solving, planning, and posing a series of 5 tasks. As the instructor of the methods course, the first author provided PTs with 5 tasks designed for use with 3rd grade students. As a class, the PTs and the instructor planned for the enactment of each task using an adaptation of the Thinking Through a Lesson Protocol (Smith, Bill, & Hughes, 2008). During week 5 of our course, PTs visited our partner school where each was paired with a student from a third grade class. During a half-hour session, PTs were asked to pose as many of the five problems as their student could work through, employing extensions and scaffolds as they saw fit. PTs video recorded their session and posted them on the Edthena video tool. PTs were asked to reflect on their own video in terms of their student's solution path and their interactions with the student. They watched and provided feedback for three of their peers, commenting on similar ideas. The instructor also provided feedback on these foci. Following this process, PT synthesized their reflection and feedback into a written plan of ways to improve their facilitation of each task. In week 7 of our course, the PTs returned to our partner school and enacted the same five tasks with a student from a different third grade class. As before, PTs recorded, uploaded, reflected and commented on the videos of this session. The first author provided feedback on these sessions as well. To complete the activity, PTs wrote a reflection paper summarizing their work and what they learned through their enactment and observations. We utilized their reflection papers to make sense of PTs' experience with the activity.

Theoretical Frame

The synthesis of the above examples of scholarly inquiry suggested PTs' dynamic interactions with student thinking can potentially: 1) develop PSTs' knowledge of students' mathematics and strategies (Ambrose, 2004; Crespo, 2000); 2) encourage PSTs to shift their focus in working with students from attaining correct answers to eliciting and understanding student thinking (Crespo, 2000; Jenkins, 2010; Moyer & Milewicz, 2002); and 3) develop PSTs' emerging abilities to use student thinking in crafting responses and posing new problems (Crespo, 2003). We expanded the first code to include not only knowledge of student thinking but any example of what has been defined as knowledge of content and students (Hill, Ball, & Schilling, 2008). We utilized these potential outcomes as our lens as we examined PTs' experiences with the ESST activity, as described in their reflections.

Data Analysis

The authors began the process of data analysis by individually coding the written reflections of the 18 PTs using the three *a priori* codes identified within the theoretical frame. Through several readings of the data and discussion of our existing codes, these three main codes were refined and operationalized using descriptions and sub-codes into our final coding scheme. Excerpts of PTs' written reflection at the conclusion of the activity were taken as the unit of analysis and were coded with both a main code and a sub-code if applicable (Table 1). Inter-rater reliability for the coding was completed demonstrating 74% agreement across 285 units coded. Disagreements were resolved through discussion.

Table 1: Coding Scheme and Number of Coded Units (*N* = 285)

Knowledge of Content and Students (*n* = 48)	Attain Correct Answers or Support Student Thinking (*n* = 120)	Learning to Respond Using Student Thinking (*n* = 117)
Anticipating possible student solution paths 14	Aware – focused on student obtaining correct answer 13	
Conceptions and misconceptions 27	Unaware – focused on student obtaining correct answer 12	
Task difficulty for students 7	Action -- eliciting and understanding student thinking 42	
	Reflection - the goal of the interaction is to focus on student thinking 53	

Beyond the codes developed from our examination of prior scholarly inquiry, we also employed open and emergent coding techniques (Strauss & Corbin 1998) in order to identify other themes within the data. Two main themes emerged from within the PTs' reflection on their experiences: "the importance of unpacking a task for students" and "PTs' tendency to label or evaluate students based on minimal evidence".

Results

In this section we present our findings related to our five main codes and their applicable sub-codes. We include illustrative examples of each in order to demonstrate our analysis process.

Code 1 - Knowledge of Content and Students (KCS)

As anticipated based upon pour literature search, our analysis of the data revealed a number of instances where PTs identified specific mathematics in their student's work and potentially developed knowledge of content and students from those interactions. There were 48 such examples out of the 285 units. To distinguish among the interactions where PTs seemed to sub-codes were developed and utilized to identify three specific categories of KCS (See Table 1).

Anticipating possible student solution paths was utilized when PTs developed new perspectives on how a student might solve a problem. Susan (all PTs and student names are pseudonyms) provided the following evidence of developing such knowledge while interacting with her first student: "Madison struggled with the coin problem trying to use five coins to make 51 cents. I thought it was very interesting while watching the videos that so many students started with putting out five dimes and a penny. My first reaction is always to make 50 cents with two quarters. Perhaps they all started this way because it would get them closer to five coins total." Of the 48 units identified as Knowledge of Content and Students, 14 involved anticipating possible student solution paths.

Instances where preservice teachers described conceptions or misconceptions of students' mathematical thinking were coded as such. There were 27 units within PTs' reflections identifying their attention to student conceptions or misconceptions. In the following quote from Georgia, we see an example of this code.

> I noticed that my student thought in a different way than I was used to and had some misconceptions about regrouping while doing the standard algorithm for subtraction. In the candy problem, she didn't seem to fully understand how to count up by ones. She was counting by 5's and passed the number she was "counting up to." Then, when she started to count by ones she started at 5 then jumped to 10, 11,12,13.

Anna, as another example, discussed the following misconception when working with her student: "I also found it interesting that once he got a new answer from subtracting, he didn't realize or understand what number he needed to change in the addition problem that was suppose to check his answer."

In 7 instances, PTs demonstrated developing KCS through recognizing, most always in retrospect, the potential difficulty of a task for students. Anna's comment was typical of these responses, "This problem was the hardest for me as a teacher because although he comprehended that each person got 9 brownies, he didn't understand that the last brownie got divided into four pieces and became a fraction".

Code 2 –Obtain Correct Answers or Support Student Thinking

Our examination of the existing literature suggested when working with students can support PTs shift from a focus on students attaining correct answers to eliciting and understanding student thinking. There were 120 units coded as examples of these two mindsets. Within these, we categorized PTs' experiences further using sub-codes. In instances where PTs were focused on their student obtaining a correct answer, we differentiated between PTs who were aware of this focus and those who did not seem to recognize it. For units coded as examples of PTs supporting student thinking, we identified two categories: PTs moves we viewed as supporting student thinking and PTs reflections restating the goal of the activity was to elicit, interpret and support student thinking.

The code "obtain correct answers – aware" was utilized when PTs self-identified their tendency to focus on students arriving at a correct answer. Riley's reflection serves as a typical example of this sub-code. "For the second task, the student struggled in understanding how to approach the problem. I made the mistake of telling him that he should possibly add up. Because of the goal of this

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assignment, I should not have suggested a strategy to him. Then, I guided him too much through that strategy. Instead I needed to let him approach the problem on his own." Of the 120 units within Code 2, 13 received this code.

The code "obtain correct answers – unaware" was assigned when PTs seemed unaware of their decision or unconscious tendency to lead the student to the correct answer. Marisa's reflection provides an illustrative example, "She had much more trouble with the Brownie Problem and the Coin Problem. I may have prompted her more through these two problems, but she eventually got to the right answer." There were 12 units identified in which PTs were unaware of their actions leading a student to the correct answer.

Action focused on eliciting and understanding student thinking was assigned when PTs' posed tasks or questions intended to help them to better understand the student's thinking. For instance, Heather discussed the following steps she took when interacting with her student: "For example, I created a road map to follow for each problem depending on the child's responses: whether she chose a successful solution path but could not explain the procedure, struggled and needed scaffolding, or completed the problem successfully and explained her thinking. The questions were not random, but rather flowed with her responses." Kim's reflection also exhibited evidence of her actions intended to focus on student thinking. "When Maggie was answering the problems, I would ask her throughout each one what her thinking was, what ideas she was using and how she was sure that her answers to the problems were correct". There were 42 of 120 instances where PTs focused on eliciting or understanding student thinking.

Reflection stating the goal of the interaction is to focus on student thinking was assigned in instances where PTs reflected on the interaction with their student and reminded themselves to keep their focus on the student's thinking. "This was, and still is, an improvement I need to continue to work on so I am able to better explore and support student thinking, assess what strategies a child knows, and determine a child's overall cognitive ability" (Leah). Heather also commented, "Throughout the coin problem, I did not provide enough wait time and found myself explaining too much instead of letting her explore for a longer period of time". There were 53 of 120 instances where preservice teachers reflected on their interaction with the student and stated that the goal was to focus on student thinking.

Code 3 – Learning to Respond Using Student Thinking

We posited our PTs would have the opportunity to develop their abilities in using student thinking to craft responses and pose new problems based on their interactions with students. There were 117 instances where PTs reflected and offered examples of how they might respond if faced with a similar situation in the future. Georgia discussed her interaction with a student: "I also, could have had him do an extension problem with harder numbers to be able to observe his thinking with more difficult numbers." Georgia's thought to pose an extension problem based on the interaction she had with the student is one example of how PTs considered responses as a result of the activity. Landon shared a similar consideration on based on his interaction with a student: "Instead of saying 'take away the smaller number form the bigger number', which is why he put the smaller number on top of the bigger number in his solution path, I could have said 'you have the bigger number and you want to take away the smaller number from it.' This could have prompted him to complete his solution path without any confusion." Landon reflects on his interaction with the student and how he could have responded differently to help the student better understand the problem. Lawson provided an example of responding using student thinking: "Next I decided that in order to help my next student through the problems that I would need to provide my student opportunities to work with 2 digit numbers if they struggled with 3 digit numbers like my first student did. I also would give my student opportunities to work with 1 digit numbers as well if they needed to." Lawson discusses his responses during the interaction with the student and his goal of helping the student when he or she was stuck or struggling.

Code 4 – The Importance of "Unpacking" a Task for Students

Preservice teachers often discussed the concept of unpacking the problem for their student before allowing the student to explore the problem. The unpacking theme was discovered after many preservice teachers reflected on their interactions with the students. For instance, Haley discussed her goal of unpacking the problem: "Unpacking the problem is one thing that I could have done significantly better - making sure the students understood what the story was about and then the method used to solve it. I worked with two students and found that one was on a much higher math level than the other student causing me to work and speak toward the upper level student in the beginning and the lower student as we began working. When I saw that the higher student understood the problem I was quick to move on without checking the lower students understanding. Although I moved on from unpacking the story quickly I think that both could have benefitted from a more in-depth explanation." Eloise provided an example of her goal of unpacking the problem for her student: "Lastly, I want to work on 'unpacking the problem' more. I think it's beneficial for students because it really sets them up correctly for the problem and gives them the most help to complete the problem." There were 58 instances of PT's commenting on unpacking problems for students.

Code 5 - Evaluating or Labeling Students Based on Minimal Evidence

Preservice teachers also had a tendency to evaluate or label students after a minimal time of working with the student. There were several instances where preservice teachers broadly evaluated a student based on minimum experience. For example, Anna wrote, "I would classify Landon as an above average math student who understands most concepts but gets through problems by going through the motions and performing the standard algorithm." Anna had minimal experience working with Landon, but was quick to classify him as an above average math student based off of a small observation. Margaret evaluated her student after working through a few problems: "I think one of the reasons for this was the fact that my student was very smart and she knew how to do all of the problems, and she for the most part solved them all correctly on the first try." There were 16 instances where preservice teachers worked with their student on a problem and then labeled the student based off their ability of a single interaction.

Discussion and Implications

As we examine PTs' experiences with the ESST activity, we can draw several conclusions about the design of the activity and its potential to foster the types of outcomes suggested by the literature. As presently constituted the ESST activity did not seem to afford PTs the opportunity to develop KCS. Perhaps extending the activity beyond two sessions and utilizing a variety of different problems would be necessary in order to support this development. The activity did seem to provide opportunity for PTs to consider their role of listening to and supporting student thinking as well as to provide opportunity for reflection on the ways in which they did and might respond to students. The additional themes of unpacking and labeling students imply these are areas of our course we need to pay explicit attention to prior to our PTs working with students. The continuing cycle of scholarly inquiry and practice will allow us to further refine and empirically examine this activity.

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