TOWARD MULTIMODAL POETIC ANALYSIS: A CASE OF PROPERTY NOTICING

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This paper uses multimodal discourse analysis to show that discursive form, in addition to words, gestures and sound dimensions of speech, is an important linguistic resource for expressing mathematical meaning. During a collaborative task, a student spoke an insight about an algebraic property five times over a few minutes, in slightly different ways. He consistently used repetition of grammar and words, a speech form known as poetic structure. These poetic structures were marked elaborately through discursive modes such as pause, intonation and gestures, suggesting that they form a meaning-making mode that is real to the speaker.

Keywords: Classroom Discourse, Problem-Solving, Algebra and Algebraic Thinking

Introduction

This paper contributes a methodological example to scholarship on language as a resource for learning mathematics. Use of home languages and the interplay of academic and informal discourses are known to be central resources (Barwell, 2015; Planas & Setati-Phakeng, 2014). This paper suggests that the discursive form of students' mathematical statements is also an important resource for learning. Grammatical repetitions within students' sentences are just as important as the words themselves in focusing attention on key issues within a task.

This study reports on a task in which two students, Sheila and Joseph, were asked to find a formula for the perimeter of a string of *n* hexagons. The key issue is that the interior sides of a hexagon string do not contribute to the perimeter. One of the students, Joseph, noticed this property (Pirie & Kieren, 1994), and over the course of about four minutes, stated the property five times before his partner agreed to it. Over these moments, we can consider which discursive strategies Joseph conserved and which he changed as he tried to make his point. Poetic structures, elaborately marked with pauses, changes in intonation and with gestures, were a discursive strategy that Joseph used in each property noticing attempt. This paper proposes that poetic structures can be considered as a discursive mode in multimodal discourse analysis.

What Is a Poetic Structure?

A poetic structure occurs when a speaker repeats a phrase or sentence that was spoken previously, while retaining some of the syntax of the prior statement, and at least one word (Staats, 2016a, 2016b). Repeating grammar helps students talk about math because the syntax establishes relationships among small mathematical ideas or images. Repeating the phrase lets students conserve or modify these relationships. Looking for repetitions that retain at least one word from the previous statement helps the researcher focus on continuity of topic.

Sometimes a poetic structure occurs within one student's turn at talk, as an "internal" poetic structure (Staats, 2016a). Here, Sheila is describing each diagram in Figure 1, from the n = 1 case to the n = 4 case, in which the number of interior sides are 0, 2, 4, and 6, respectively. In the layout of the transcript, I have used indentation to call attention to the grammatical repetitions so that they are arranged roughly in columns, one column for the phrase *would be*, one for *the total number of sides* and one for the number of sides that she sees:

So that would be	like a formula, right?
So this would be	6L,

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and then this one would be, uh,
the total number of sides minus 2 .
And then this one would be
the total number of sides minus 4 .
This would be
the total number of sides minus 2, 4, 6

Poetic structures can also occur between students, or "across" students, when one student repeats and perhaps modifies a phrase that another student said in the past (Staats, 2016a; 2016b). For example, a short time after Sheila's statement above, over several of her turns at talk, she spoke fragments of a formula while trying to figure out how to subtract interior sides. At turn 76, she says, *…So, like in statistics the number of cases would be n and that would be your number minus 2…* and at turn 77, she says, *…Number of hexagons would be 1, 2, 3, 4. 4, uh, times 2….* At turn 78, Joseph uses an "across" poetic structure to recombine two of Sheila's phrases in order to request clarification, *Times 2 or minus 2?*

In this way, poetic structure analysis can focus on the form of a single student's mathematical commentary, or it can focus on the relationships of commentary that students share, sometimes over long stretches of a conversation. This study is concerned with the way Joseph used discursive form to explain a property of the hexagon diagrams, and so here we focus on "internal" poetic structures. As we see in the examples above, poetic structures can be shown on the page through indentation and through underlining. In order to conserve space in this research report, in the following examples, I use several forms of underlining instead of indenting to highlight repetitions within Joseph's property-noticing statements.

Theoretical Foundation

In recent research reports, I have outlined a methodological foundation for identifying and interpreting poetic structures in collaborative mathematical conversations. I've shown that poetic structures can facilitate activities such as organizing data, generalization, and shifting from a spoken mathematical formula to a written one (Staats, 2016a, 2016b). In this research report, I discuss how poetic structures contribute to a student's property noticing statements.

In Pirie and Kieren's theory of the dynamic growth of mathematical understanding, three of the early stages are image making, image having and property noticing (Pirie & Kieren, 1994). A student in the property noticing stage can coordinate multiple abstract images or ideas in order to identify a new property of a mathematical object or activity. Martin and Towers have extended this theory to describe collective property noticing, in which the insights are distributed across different students, with no single student expressing the full idea (Martin & Towers, 2015). Throughout the hexagon task, Sheila and Joseph had collaborated closely, and Sheila had voiced many key insights. But in the selections below, Joseph was in an individual stage of property noticing. Joseph seemed to be making a bid to have his insight validated by Sheila and brought into their collective work. The theoretical foundations advanced by Pirie and Kieren on one hand and Martin and Towers on the other allow us to locate Joseph's commentary just at the boundary of individual and collective property noticing. Joseph used multimodal discourse in varying ways over five moments of property noticing to gain collective engagement, with only some success.

Participants and Task

Sheila and Joseph were two undergraduate students who had recently completed a university class in precalculus. They participated in a paid, one hour video and audio recorded problem-solving session. The hexagon task was based on Wilmot et al (2011, p. 287). The task includes diagrams for

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strings of 1 to 4 hexagons, shown in Figure 1. It asks the students to fill a table of values for n = 1 to n = 5 hexagons with the corresponding perimeters. A correct answer is $p = 4n + 2$. Sheila and Joseph developed an answer of the form $\#H(6) - 2(N - 1) =$ in which H and N both stand for the number of hexagons (Staats, 2016a; 2016b).

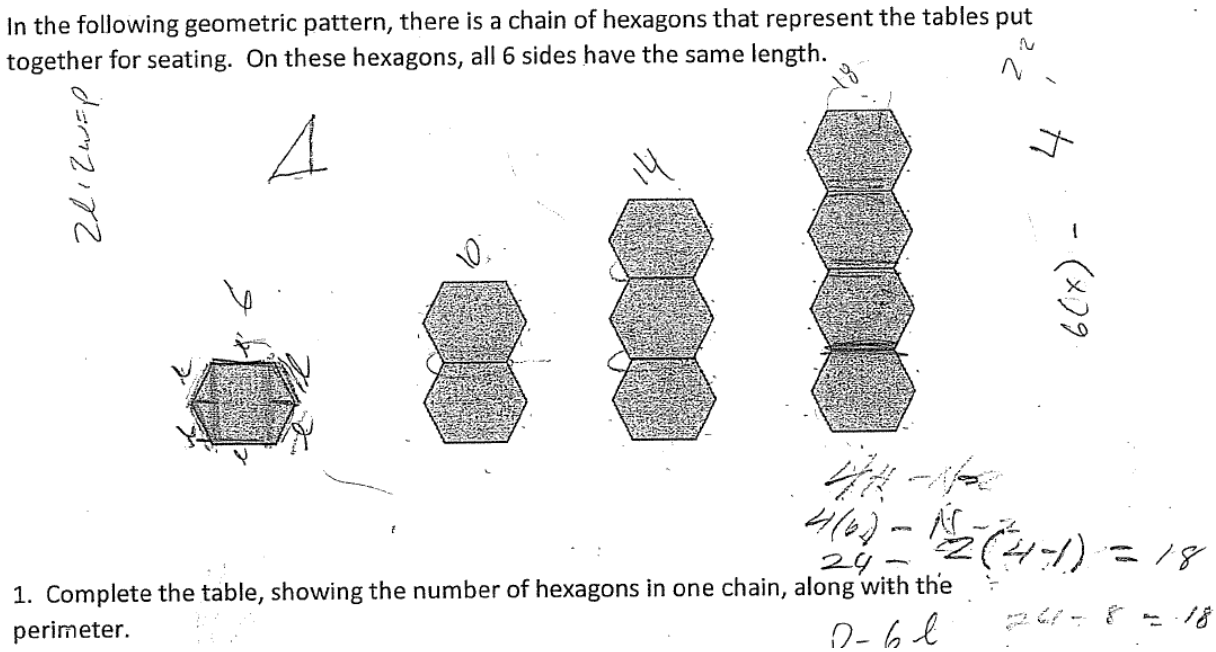

Figure 1. Task diagrams for n = 1 to n = 4 hexagons.

Methods

This study uses multimodal discourse analysis to understand a student's bid to propose a property as a collaborative activity. Because this paper is concerned with patterns in syntax and sound, as well as gesture, I use an approach to discursive multimodality that is drawn from conversation analysis (Bergmann, Brenning, Pfeffer, & Reber, 2012). This approach takes prosody—patterns of sound in language such as pitch, intonation, pause or loudness—combined with gesture and syntactic structures, as interactional modes that allow people to create meaning. Syntactic units are sometimes marked multimodally through prosody and gesture (Szczepek Reed, 2012). Multimodal analysis in mathematics education has shown that prosody, gesture, sounds created through gesture, and rhythm can be coordinated to express mathematical understanding (Bautista & Roth, 2012; Radford, Bardini, & Sabena, 2007). Alibali, Nathan, et al (2014) show that the coordination of speech and gesture is particularly important in linking mathematical ideas, particularly for new material. In this paper, I introduce poetic structure as an additional discursive modality for creating meaning.

When Joseph explains the property and Sheila fails to agree with it, he must explain it again but change something, in other words, he must deploy a new combination of discursive resources. Speakers could shift among many different discursive strategies to try to highlight important information for their idea. I consider four elements of speech that a speaker could combine in various ways to create a new strategy for explanation: introducing academic vocabulary; elements of prosody; gesture; and poetic structures. I represent these elements according to the transcription key in Table 1. This mode of transcription doesn't use punctuation such as commas, periods or question marks unless they are defined in the key as features of the sound. In the selections below, I transcribe Joseph's property noticing statements closely, and I present other statements in ordinary prose with ordinary punctuation. To identify poetic structures within one turn at talk, I looked for phrases that

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were repeated and that share some grammar and at least one word. If I could identify alternative ways of parsing the statement into poetic structures, I chose the approach that accounted for the longest stretch of words.

Table 1: Transcription Symbols

Symbol	Definition
↑ or ↓	High pitch or low pitch, respectively, compared to nearby words. Could occur at beginning, middle or end of word.
Bold text	Emphasis on a word through loudness, vowel lengthening, or stress
(x.y)	Pause, estimated, in seconds.
Underlining styles	Single, double or bold underlining represent the different poetic structures in one turn at talk
[*italics*]	Descriptions of gestures

Five Attempts to Explain a Property

In this part of the conversation, the students had already developed a numerical method for solving the hexagon problem that corresponds to the formula that they had written, *#H(6) – 2(N – 1) = .* The task now asks them to explain their method in terms of the diagram. After a brief discussion of whether this refers to the table of values or the images of hexagons on the handout, Joseph states a property of the hexagon diagram for the first time in the conversation.

First Attempt, Turn 171

171 J: Then this must be the diagram (1.0) so ↑how can we use ↓this (0.2) to prove our point (1.4) [*Pencil taps vertically on the diagram at each "this."*]

so the **hex**agon has ↑six sides (1.4) but <u>when you put a hexagon in a **chain**</u> they share **two** sides (0.6) so <u>you're taking away **two** sides</u> (0.8) from the chain↓ (1.6) so <u>each time you **add** another hexagon in a chain</u>↑ [S: But it's not asking, this one] <u>you're losing two sides</u>↑ [*During part of this segment, Joseph's hand obscures the paper, so we don't know precisely whether he used small gestures.*]

172 S: This one is not asking about the diagram. It says, how do we know this is true? Why does it work? Because the formula matches the diagram and table.

There are two repeated phrases in Joseph's first statement of the property, which can be summarized as: *you/put, add/hexagon in a chain* and *you're/taking away, losing/two sides*. The beginning and ending of each of these four underlined phrases is fairly strongly marked through prosody, either through pause, intonation or emphasis. For example, *so you're taking away **two** sides* is bounded on either side by a distinct pause. The poetic structure *when you put a hexagon in a chain* is marked as a bounded unit as well, but perhaps less distinctly, with an opening pause and a small level of sound emphasis on the word *chain*. It is important to notice that the beginnings and endings of many of the following examples of poetic structures are marked as a distinct unit, primarily through intonation or pause, and sometimes through less prominent forms of emphasis that are noted with bold text.

The poetic structure in Joseph's first explanation introduces a strategy that he uses in some of the following explanations: a cause/effect or if/then form of argumentation. Fairly often, the first

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repeated phrase is a cause or action in some sense, and the second one is an effect. This argumentative strategy is conveyed primarily through Joseph's poetic structures.

Though gestures are obscured, it appears from Joseph's hand position that there were probably some small gestures. Still, they did not result in Sheila's acknowledgement of his statement. In the next few turns, the students discuss what it means to explain a method in terms of a diagram. Sheila wonders if they need to draw a new diagram, which leads to Joseph's second statement of the property.

Second and Third Attempts, Turns 177 and 181

177.1 J: ↑I don't think so I ↑think you just have to explain so (1.8) a ↑hexagon has six sides↓ (0.4) [*just before "a hexagon", a heavy tap on the base of the n = 1 diagram*]

177.2 and as you **add** (1.2) an additional hexagon (1.2) [*at "add," light touch on the base; at "additional," light touch on top*]

177.3 you add six↑ (0.8) *[touch base before "six" and touch top after]*
177.4 because they ↑share two sides↑ (0.6) [*circling the top of the n = 1 diagram*]

177.5 you subtract↓ two sides↓ [*pencil is above the page. At "subtract," there's a beat in the air near the top of the hexagon and at "sides," there's a beat near the base*]

177.6 from their total↓ (0.8) number of sides↓

178 S: Um. I think this is, I think this is –
179 J: Which is essentially this.
180 S: The, the number of tables has six sides. And with that multiplied you minus, as they join together they lose one side. So for every-

181 J: One side for (0.2) **one** (0.2) hexagon. [*Overlapping with Sheila's 180 to 181. No gestures; Joseph's hand is away from the paper*].

182 S: -For every, for every two tables, one side is lost. So for the, for the, well, uh, the formula just states it, right? How do you state the formula?

At turn 177, Joseph realized that his property noticing was not taken up by Sheila, and so he made several prominent discursive changes. He used two mathematical poetic structures, each one advancing an "action/result" kind of argument: 177.2-177.3, and 177.4-177.5. Each of these four lines was strongly bounded by prosody, particularly pause, but also with some intonation and emphasis, except the end of 177.5 which was only bounded by intonation and gesture. In 177.2, 177.4 and 177.5, each was accompanied by a two-fold pointing gesture that emphasized the words *add*, *six*, and *subtract…sides*, respectively. Line 177.3 was marked by a different gesture, circling near one side of a hexagon. Generally, the poetic structures in attempt 2 included elaborate coordination with gesture and prosody. Joseph also shifted to the more precise mathematical terminology of *add* and *subtract* with this statement. These layers of multimodal cues suggest that these poetic structures were real to the student, rather than a researcher's analytical imposition.

Once a speaker establishes a unit of speech through a poetic structure, it opens up additional expressive possibilities. If we think of poetic structure repetitions as a kind of rhythm in speech, then

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prosody and gesture sometimes form a sub-rhythm that highlights words within the poetic structure in ways that enhance the mathematical idea. This is well known for gesture, but multimodal poetic structure analysis shows more clearly that gestures and prosody can express comparisons and contrasts in mathematical entities that are defined through the poetic structure.

For example, a subtle sub-rhythm emerges in 177.2, with a slight emphasis on the word *add*. With this word, Joseph introduces a new strategy in attempt 2, to use more precise computational terminology. The computational word *add* is a new idea, and this is enhanced with prosody (emphasis and pause) and the first part of a two part pointing gesture. In the repetition in 177.3, however, Joseph shifts the emphasis from *add* to *six*, with intonation, pause and a two part pointing gesture. This sub-rhythm controls and shifts focus within the poetic structure words. It shifts the hypothetical action on the diagram of *add/additional hexagon* and towards the computational technique of *add/six*. The poetic structure of 177.4 and 177.5 also contains a sub-rhythm that is coordinated with the repeated words of the poetic structures. Rising intonation on *share/sides* is contrasted with falling intonation on *subtract/sides*, which, like 177.2 and 177.3, emphasizes a shift from action to computation but also realizes that cause/effect argumentation structure. The sense of cause/effect was heightened as the circling gesture at 177.4 shifted to a two part beat gesture on *subtract/sides*. Within elaborately marked and bounded poetic structures, sound and gestures help Joseph to highlight mathematical actions and relationships.

Sheila's response to turn 177 was ambiguous: *as they join together they lose one side*, and so Joseph clarified again at turn 181. With each attempt to explain the property, Joseph uses a different combination of discursive tools. Here, his statement, *One side for one hexagon,* relied on a prominent poetic structure with no gestures. His strategy in attempt 3 was minimalism, to explain using as few words as possible. A poetic structure allowed him to do this. A subtle sub-rhythm used pauses and a small bit of emphasis to highlight the clarification of ***one*** *hexagon*.

Fourth Attempt, Turn 185

183 J: Well they want you to use the diagram. So using this we have to explain how that formula works. So there are six sides, which is, which corresponds to –

184 S: Let's see. Let's do 4. For every, for all these tables, four tables, there's only three intersections [*Joseph takes a piece of paper and writes "4."*]

185.1 J: But they **all** lose two sides at an intersection (1.6) so (0.4) a **hex**agon has six sides, right↑ (0.6) so 4 times 6 is 24↑ [*Joseph has now written 4·6 = 24*] (1.2)

185.2 um but each (1.0) table when each table touches the other one (1.2) [*Joseph has taken a second piece of paper, and at "when," he holds his hand up with palm down*]

185.3 they lose two sides↓ [*At "lose," he closes his palm into a fist*]

186 S: Oh, that doesn't work out mathematically.

Joseph's fourth attempt to assert property noticing used poetic structures less prominently, possibly because he was in the middle of introducing the new modality of writing mathematical formulas. The small level of repetition that exists still expresses the cause/effect argumentation structure, and this is supported through gesture. The open-handed gesture at 185.2 seems to be a gesture of asserting, and the closed fist at 185.3 seems to grasp a conclusion.

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Fifth Attempt, Turn 187

187.1 J: Because ↑here's one and ↑here's one (0.8) **here's** one **and here's** one (0.4) **here's** one **here's** one (0.4) so you lose **six** sides (0.8) [*Joseph's hand has obscured the view again, but we can hear him scratching over the interior pairs of sides, and we can see him moving from the bottom pair to the middle pair to the top pair, making the marks that are visible on the interior sides of the n = 4 case in Figure 1. His hand comes away after the last "here's one."*]

187.2 so **24** (0.8) [*He writes a negative sign in 4·6 = 24 -]*

187.3 would be the number of ↑sides if they weren't touching (0.4) but because they're ↑touching (0.4) you lose six↓ [*He writes 6 after the negative sign*]

187.4 and that comes down to 18. [*Joseph has now written 4·6 = 24 – 6 = 18.*]

188S: Yeah. Okay, go ahead and write it down. You're better at that than I am.
189J: I'm just more visual.

The first part of turn 187 has a strong poetic structure based on *here's one and here's one*, in which the beginning and ending is marked and bounded off with prosody and a combination of gesture and drawing. Alibabi, Nathan, et al (2014) consider this action as a "writing gesture," though this is distinguished from writing linguistic or mathematical text. The various gestures that Joseph made near the base and top of a hexagon in several previous statement were here made lasting and tangible through a writing gesture.

At 187.2, when Joseph writes the minus sign, *4·6 = 24 -*, he creates a sense of incompletion. We could consider this as a sub-rhythm that is not completed until he writes the *6* as he says a part of the poetic structure at 187.3, *you lose six* and then completes the calculation of *18*. There is a slight dramatic feel to this moment, because he performs the subtraction while he completes the poetic structure. Another sub-rhythm is associated with the poetic structure at this moment, the rising and falling intonations on *touching* and *lose six*. The poetic structure, writing mathematical text, pause and intonation are all coordinated to reproduce the cause/effect argument that Joseph used throughout his property noticing statements. At 188, Sheila responded with a long, thoughtful sounding *Yeah*. It sounded as if she decided that she agreed with Joseph.

Conclusion

Across his five property-noticing statements, Joseph combined expressive modes into distinct discursive strategies that finally achieved acceptance if not extended collective engagement. Poetic structures were present in all attempts, prominently so in four of them. The poetic structures were very commonly marked near the beginning and end through prosody, especially pause and changes in intonation. This analysis shows that as syntactic units are repeated, they may continue to be marked multimodally, amplifying their expressive quality.

Poetic structures put units in parallel position into relationships of similarity— *as you add an additional hexagon/ you add six*; and of difference— *because they ↑share two sides↑ you subtract↓ two sides↓*. Bautista and Roth have similarly observed that sounds produced through gesture mark important mathematical similarities or differences (2012). Joseph capitalized on poetic structures to add layers of sub-rhythms through gesture, pause and intonation that did quite a lot of the mathematical work in Joseph's five property noticing statements. The co-occurrence of poetic

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structure and several modalities is strong evidence that poetic structures are not simply abstract grammatical mappings applied by a researcher, but rather, highly expressive tools of meaning-making. Poetic structure analysis is a powerful method to trace the moment-by-moment construction and expression of precise mathematical ideas through informal language.

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