MATHEMATICAL MODELING: CHALLENGING THE FIGURED WORLDS OF ELEMENTARY MATHEMATICS

Megan H. Wickstrom
Montana State University
megan.wickstrom@montana.edu

This article is a report on a teacher study group that focused on three elementary teachers' perceptions of mathematical modeling in contrast to typical mathematics instruction. Through the theoretical lens of figured worlds, I discuss how mathematics instruction was conceptualized across the classrooms in terms of artifacts, discourse, and identity. I then highlight, through four themes, how mathematical modeling challenged the ways in which both the teachers and students understood what it means to know and do mathematics. Findings suggest that the practice of mathematical modeling allowed for access, empowerment, and real world connections that were typically not present in classroom instruction. In addition, it challenged student positioning in the classroom in terms of who was framed as capable of doing mathematics.

Keywords: Elementary School Education, Equity and Diversity, Modeling

Introduction

Mathematical Modeling, a standard of mathematical practice in the Common Core State Standards, is a process in which students use mathematical tools to reason about, represent, and make decisions surrounding a real world scenario (Lesh & Doerr, 2003). The process of modeling is cyclic and it begins when the modeler translates the scenario into the mathematical world by posing a question. Using knowledge and mathematical tools, the modeler proposes solutions and translates them back to the real world to determine if they are appropriate or if modifications need to be made. In this paper, mathematical modeling refers to the entire process rather than the end product.

Although mathematical modeling has traditionally taken place in secondary and college classrooms, researchers (Carlson, Wickstrom, Burroughs, & Fulton, 2016) have argued that it is equally as important for elementary students to engage in the process. Modeling supports mathematical literacy (Steen, Turner, & Burkhardt, 2007) and allows students to draw on their own backgrounds and experiences in framing the mathematical problem (English & Watters, 2005). Modeling also promotes productive attitudes toward mathematics (Lesh & Yoon, 2007), and fosters integration across mathematical content and practices (Lehrer & Schauble, 2007).

The study of mathematical modeling in the elementary classroom is a relatively new field of study. The purpose of this paper is to add to existing literature by describing an elementary modeling task and the ways in which it challenged teachers' and students' perceptions of what mathematics is and what it means to do mathematics as well as the students' and teacher's roles within the classroom. Through the theoretical lens of figured worlds, in this paper I address the following research questions:

1. How does mathematical modeling press on or extend the boundaries of what it means to know and do mathematics in the elementary classroom?
2. In what ways, if any, does mathematical modeling challenge positionality and roles in the elementary classroom?

Theoretical Framework: The Mathematics Classroom as a Figured World

This work is framed through the theoretical lens of Holland, Skinner, Lachicotte, and Cain's (1998) concept of figured worlds. They define a figured world as, "a socially and culturally

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

constructed realm of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others." (p.52) Holland et al.'s work addresses the idea that each individual's thoughts, behaviors, and ways of interpreting the world are often influenced by culture, power, and status. In addition everyday activities act as figured worlds that build, inform, and continually define individual's identities. In this paper, I argue that the mathematics classroom functions as a figured world. In the elementary classroom, there are routines that define what it means to know and do mathematics. In addition, both the teachers and students take on different roles and identities that are continually formed across the school year.

Figured worlds consist of three key elements: artifacts, discourse, and identity. I begin the paper by discussing a typical day in the teachers' classrooms in response to artifacts that contributed to the figured world of mathematics instruction, discourse surrounding how the three teacher's interpreted doing mathematics, and the identities and roles the teachers perceived in the classroom. Next, I identify and describe four themes that arose while mathematical modeling that challenged the established norms or figured worlds.

Methods

Participants

Three teachers participated into this study, Ms. A, Mr. B, and Ms. C. Ms. A was a fifth-grade teacher. Mr. B was a fourth-grade teacher and Ms. C was a third-grade teacher. The teachers were participants in a NSF-funded professional development on integrating mathematical modeling in the elementary classroom that took place in a school district in the Rocky Mountain West. As part of the professional development, the teachers attended a weeklong professional development on mathematical modeling and pedagogical practices in the summer. During the summer, they designed a modeling task to implement in their respective classrooms. Following summer professional development, the teachers participated in a teacher study group in which they met seven times across the fall semester to debrief and discuss the modeling task with a university faculty member. The three teachers were chosen for this study because they were grouped together in the same study group and enacted the same modeling task. I, the researcher, took on the role of their study group facilitator.

Modeling Task

This section is meant to give a brief overview of the task. Specific examples of students engaging in the task will be given in the results section. In designing the modeling task, each of the teachers discussed that they led some type of community-building lunch at the beginning of the school year for students to get to know one another. Instead of designing the activity themselves, they decided they would use this as a real-world scenario to engage students in mathematical modeling. Teachers presented the following scenario to students, "Building community in our classrooms is very important. The university has given us money to support a community building lunch." After presenting the scenario, they asked students to consider 1) What do we need to know? and 2) What tools could we use to help us? Students decided that they needed to address broad questions like "What should we have for lunch?", "What activities should be included to build community?", "Will what we want fit into our budget?". Students also discussed that there are other factors to consider like allergies and personal preferences in food selection.

The teachers worked with students on this task across 3-4 weeks, visiting the task a few times a week. Initially, the students worked on determining what should be served at lunch and quantity. Students primarily used surveys, multiplication, counting, and measurements as mathematical tools to aid them in making decisions. Once students had determined what should be served and how much, they needed to determine where the food would come from and if the meal was in budget. The

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

teachers helped by providing grocery store and restaurant ads. Again, the students primarily used multiplication and repeated addition in determining the total cost.

Data Collection

I, the researcher, observed all three teachers across implementation of their modeling tasks visiting each teacher for 3-4 lessons. During observations, I took qualitative notes of what occurred in the classroom including what the teachers said or did, students' progress in the task, and students' remaining questions or concerns. In addition, I facilitated and video-recorded seven meetings in which the three teachers debriefed about the modeling task and their work as teachers. Following the fall teacher study groups, the teachers individually participated in a one-on-one interview that lasted for about 45 minutes. The purpose was for teachers to first describe the structure of a typical mathematics lessons including routines, student activities, and teacher activities. In the second half of the interview, the teachers were asked to describe their experiences enacting mathematical modeling. This included describing key features of and comparing mathematical modeling to a typical lesson. In addition, they were asked to describe what the process was like for their students and for them, as teachers. The primary data source for this article are the one-on-one interviews with observational notes used as triangulation.

Data Analysis

Interviews with teachers were audio recorded. Each of the interviews was transcribed verbatim resulting in about 10 pages of typed transcript per teacher. Classification and coding took place first related to the research questions and theoretical framework (Miles, Huberman, & Saldana 2014). I analyzed the transcripts first looking for statements that helped to contextualize and describe components of a typical mathematics lesson. Then, I analyzed the data looking for statements related to the three main components of figured worlds: artifacts, discourse, and identities. Finally, I looked for statements, in describing the modeling task, that were in contrast to the figured world of the mathematics classroom. I analyzed and grouped the statements to generate themes regarding the figured world of the mathematics classroom and the ways in which mathematical modeling challenged or reinforced this world.

Results

The Figured World of the Mathematics Classroom

Artifacts. Artifacts are objects that act as the "means by which figured worlds are evoked, collectively developed, individually learned, and personally powerful." (Holland et. al, 1998, p.61) Across the three classrooms, there were three artifacts that supported how mathematics instruction was conceptualized: group/carpet area, worksheets, and journals. Each of the teachers began the lesson by bringing the class together in a communal area or a carpet at the front of the classroom. The purpose was for the students to be able to express themselves in a communal environment. The teachers described that they first presented an idea or a problem to students in pairs or groups. Students were asked to compare and discuss their solutions with their partner and then solutions were shared allowed. In describing this part of the lesson, Mr. B stated:

> So they'll (the students) work independently, compare their answers with their learning partner and then I'll call, usually using the equity sticks, I will call several people up…often I just look and see a variety of different problem solving methods and always emphasize that the students learn more from each other than me.

Each of the teachers discussed this as time for students to voice multiple solution strategies, build community, and learn from one another.

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

The second artifact that shaped mathematics instruction was the worksheet. After students discussed and shared solution strategies, they moved back to their desks and were given different types of practice problems. All of the teachers used the worksheet as a way to individually assess if students understood the material as well as time to talk individually with students. Ms. C described, "I give them a few problems and I just want to see, I rotate from group to group to see if they are getting it." The emphasis of this part of the lesson was for students to practice the mathematics and try out different strategies independently.

The journal was the final artifact that shaped mathematics instruction. All of the teachers either had students write in a journal or respond to a reflective prompt describing their learning and successes or challenges they faced. Ms. A described,

> After we have been doing any sort of activity…they open their journals and they reflect for a few minutes and I have them identify a success or a challenge that they had and we talk about it. Or sometimes I ask them to give advice to the next class on what they learned.

In examining the artifacts, students participated in mathematics through daily routines. Through the carpet space, mathematics was communal and open to discussion. Through the worksheets, mathematics became an individual endeavor in which teachers could examine students' thinking and skills. Finally, mathematics became a reflective process through the mathematics journal.

Discourse. Discourse accounts for the ways in which people interact with one another and discuss a particular topic in their setting. Through classroom discourse, teachers and students are able to shape and define what it means to do mathematics. There were four themes that arose surround discourse from the teachers' perspectives: student voice, multiple strategies, problem solving, and mistakes.

All of the teachers discussed student voice in defining what it means to do mathematics in their classrooms. The students were expected to share ideas with one another and this fostered the second theme of multiple strategies. When multiple students are able to share out, the teachers stressed that there are multiple ways to do a mathematics problem. For example, Ms. C described different student hand signals she employs so students can respectfully disagree or add to another students' thought. She stated,

> As we talk we have different signals, like (one for) something to add when someone is explaining something or if they don't think the answer is right. So there are different hand signals so that we establish a community that's equitable and everyone's voice is heard and we share different ways. So kids are prompted to think about if someone did in a different way and they can learn from each other.

For each of the teachers, mathematics was more than knowing facts or solving problems correctly, it was perseverance in problem solving. They each discussed that they wanted students to leave their classrooms with the confidence to attempt problems and apply what they knew. In addition, related to this theme, they also discussed making mistakes as part of problem solving. Mr. B stated,

> I want students to be sort of fearless. They don't worry about failing. They just jump right in and, you know, my biggest thing is taking what you know, how can I approach a problem with what I know. So I want my students to be truly confident and believe that they can do this or don't mind failing.

Across the four themes, mathematics was framed as an activity in which multiple voices should be heard and where multiple strategies could lead to a valid solution. In addition, students were

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

encouraged to view mathematics as a problem solving activity in which it was normal to make mistakes.

Identity. Identity is the roles that teachers and students take on during mathematics instruction. The teachers each discussed that they expected students to take on different roles. They wanted students to learn from each other and be comfortable presenting mathematical strategies. Students took on roles of learner, presenter, and teacher with varying levels of engagement in each of these roles.

The teachers primarily envisioned themselves as facilitators rather than instructors. They discussed that they observed, listened to students, and had students explain their thinking rather than instructing students on how to solve the problem or having them complete several practice worksheets. Their role was to observe student reasoning over time and help students make progress both collectively and individually.

Pressing on and Extending the Boundaries of Mathematics through Modeling

As the three teachers engaged their students in the process of mathematical modeling, they described that the process was in contrast to typical mathematical instruction. Four themes emerged in relation to pressing on or extending boundaries: access, empowerment, real world connection, and positioning.

Access. Each of the three teachers discussed that the modeling task provided access and differentiation across the class that was not typically present during mathematics instruction. At the beginning of the modeling unit, when discussing the theme of a community luncheon, all students had questions and ideas that were important to them and they wanted to investigate. Based on past experiences, all students were able to contribute and posed broad ideas like we need to consider cost, likeability, and number of people but also more personal factors like food, allergies, and best places to shop. They each had experiences that they could draw from to start the conversation across multiple perspectives. In describing this, Mr. B stated,

> I think the thing that is most incredible about modeling is watching students use what they know to solve problems, you know? Watching each individual or each group come up with a completely unique way to solve a problem and bring their individual strengths to be part of the solution. That has been really powerful.

The teachers commented that the students, by grade level, determined the mathematics they would use and how far to pursue the task. For example, when planning the luncheon, the third-grade students decided to investigate the cost of a main and side dish while the fifth-grade students planned drinks, a main dish, a dessert, a game following lunch to help build community, and how they would allocate their time across each. Individually within grade levels, students could also access the task and apply mathematical concepts that were appropriate to their understanding. For example, in fourth-grade, students decided they wanted pizza for lunch. When determining the number of pizzas needed, some students used repeated addition while others used multiplication. In describing access, Ms. C stated, "It differentiates itself just by design and kids that are at different levels can be successful at it."

Ownership and Empowerment. The second theme that emerged was the idea of ownership or empowerment. The teachers identified that the students were able to make choices in the process of modeling. For example, when determining what beverage(s) to serve at the lunch, students surveyed one another using Google documents and found that students wanted the following: 28% root beer, 25% orange soda, 17% lemonade, 10% Cool Aid, 10% apple juice, and 8% milk, and 2% Caprisun. At first, some students proposed that they should just serve root beer because it had the highest percentage. Other students disagreed and stated that less than half of the class wanted root beer, so

Galindo, E., & Newton, J., (Eds.). (2017). *Proceedings of the 39th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Indianapolis, IN: Hoosier Association of Mathematics Teacher Educators.

they should have multiple choices. In the end, the class decided they would offer the top three choices so that more people would be happy in their beverage choice. In describing the process, Ms. A stated that she tries to give students choice, but the process of modeling provides greater opportunity for student choice and ownership. She stated,

> Well (modeling) it's all about choice. I mean they choose what path they want to do or take and how they go about solving it. I try to have a lot of choice in here (my classroom) but I can only have so much, right? And modeling is different because it (the choices made) are mostly theirs and when it wasn't theirs, they didn't know that. They had this empowerment that it was them controlling where they were going.

In addition to having choices, students understood that the choices they made mattered. Whatever they decided upon actually happened. For example, in the fifth-grade class, students decided on pizza for lunch, but ran out of Hawaiian pizza before everyone who ordered it was served. In describing the situation, Ms. A., stated,

> Like the little girl who didn't get her pizza. She just assumed her math was right and when I began to think about it, I just assumed that the kids had been sneaky, but maybe their math wasn't right and we didn't order enough of that kind of pizza?... It is pretty powerful and something that we could have a future discussion about.

Although this example highlights a negative outcome, through taking on the responsibility of planning, teachers commented that students felt confidence in making decisions and then seeing their decisions become reality.

Real World Connection. A third theme that emerged was the concept of math as reality. In describing the launch of the task, Ms. C stated, "I think a lot of them didn't realize they were doing math." Because the problem was situated in reality, all three teachers discussed that students engaged and related to the mathematics with more excitement and perseverance. Mr. B discussed that students were self-motivated when investigating the lunch problem. He stated, "for fourth-grade…our lunch was really successful, you know? It's been really motivating. There is no work I have to do, you know, no encouragement I mean. We just start the process and they are excited and want to attack the problem." It is interesting to note that the lunch modeling task took about a month to complete with students working on the task a few times a week. At no point did they loose motivation or interest to finish the project. Ms. C discussed that during a typical mathematics lesson there is limited connection to the real world. She felt that the process of modeling added an additional layer of meaning to the problem. She stated,

> The real worldliness of what we were doing was key. Because a lot of math that I teach on a daily basis I feel like has no connection to the real world. I mean, maybe you can stretch it to where we are talking about candy or in a story problem dividing it up, but it kind of loses something because it's not connected to a real-world thing that means something to the kids.

The process of modeling highlighted that mathematics is more than different strategies to a particular problem. It can be used to reason about issues students face.

Positioning. The three themes above, access, ownership, and real world connection, all involve how the students perceived and engaged with mathematical content. Mathematical modeling also served as a tool to question positioning in the classroom. All teachers described that it helped to challenge students who they labeled as gifted. The modeling process often takes time and there is no one right answer which was frustrating for some students. Ms. C. stated,

> I also like that it is challenging for the kids who are considered gifted. I have a couple (gifted) kids in my class that when they wrote their reflection, they were like, "I don't like this" because

> they are so used to, even though they are only third-graders, they are so used to being right and getting the right answer…and this was out of their comfort zone. I think that was a good thing.

The process helped to challenge the idea that mathematical thinking is not always about solving problems quickly and correctly. Mathematics can be interpretive. Some students commented that they usually did not like mathematics, but they enjoyed this process.

In addition, teachers commented that modeling fostered mathematics as a community activity. Everyone could feel included and that their ideas mattered. Students could bring knowledge and experiences from outside of the classroom in to help them make decisions about the task. In describing positioning in the classroom, Ms. A stated,

> The most amazing thing to me is that everybody is able, no matter who you are, can enter the (modeling) process where you need to enter it. I just, my entire life, as a person, I have always had a hard time not including everyone and not having everyone feel like they are valued and important. And, I've, when I decided to become a teacher, as much as we like to think public education is inclusive, it's not. We have groups, pullouts and things because we need to service everybody. I totally understand, but it has always made me a little uncomfortable because I see the dynamics because of that. Roles are created. Status is created within the classroom. It's just reality and so this was the first time that I had that that "aha" moment in the class this summer when we were reading those articles. If this is how math could be in my classroom where everyone was doing mathematics and didn't have a role in this or as the really smart kid or the not so smart kid. We would all just have a part in it.

All three teachers described that modeling allowed for all students to feel that they could actively contribute to solving the problem.

Discussion and Concluding Remarks

The teacher interviews and study group notes suggest that mathematical modeling can act as means to extend and redefine students' notions of what it means to know and do mathematics. In classrooms that already valued multiple solution strategies and community-based discussion, modeling acted as a means for all students to feel that they had something to contribute. This is similar to statements made by English and Watters (2015) that students draw on their own experiences to frame the problem. Students were also empowered to view mathematics as a tool rather than seeing mathematics as the practice of skills. The mathematical choices students made mattered. In addition, this study highlights that modeling pressed on the idea that mathematics is bound to classroom instruction. Students were able to see that mathematics could help them make choices about real world decisions. Lastly, modeling challenged perceptions regarding who was capable of doing mathematics and what it meant to be successful in solving a mathematical task. In closing, if mathematical modeling adds to students' understandings of what it means to know and do mathematics, it is important to investigate ways to provide opportunities for modeling across the k-12 system as well as investigate the impact from the student's perspective.

Acknowledgements

This material is based upon work supported by the National Science Foundation under Grant No. 1441024.

References

Carlson, M. A., Wickstrom, M. H., Burroughs, E., Fulton, E. W. (2016). A case for mathematical modeling in the elementary classroom. In C. Hirsch (Ed.), *Annual perspectives in mathematics education: Mathematical modeling and modeling mathematics.* (pp. 121– 130). Reston, VA: National Council of Teachers of Mathematics.

English, L. D., & Watters, J. J. (2005). Mathematical modeling in the early school years. *Mathematics Education Research Journal, 3*, 5879

Holland, D., Lachicotte, W., Skinner, D., & Cain, C. (Eds.) (1998). *Identity and agency in cultural worlds.* Cambridge, MA: Harvard University Press

Lehrer, R. & Schauble, L. (2007). A developmental approach for supporting the epistemology of modeling. In W. Blume, P. L. Gabraith, H. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI study.* (pp. 153-160). New York, NY: Springer.

Lesh, R. & Doeer, H., M. (2003). Foundations of models and modeling perspective on mathematics, teaching, learning, and problem solving. In R. Lesh (Ed.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching.* (pp. 3-34). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R. & Yoon, C. (2007). What is distinctive in (our views about) models and modeling perspectives on mathematics problem solving, learning, and teaching? In W. Blume, P. L. Gabraith, H. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI study.* (pp. 161-170). New York, NY: Springer.

Miles, M. B., Huberman, A. M., & Saldaña, J. (2014). *Qualitative data analysis : a methods sourcebook.* Thousand Oaks, Californinia : SAGE Publications, Inc.

Steen, L. A., Turner, R., & Burkhardt, H. (2007). Developing mathematical literacy. In W. Blume, P. L. Gabraith, H. Henn, & M. Niss (Eds.), *Modeling and applications in mathematics education: The 14th ICMI study.* (pp. 285-294). New York, NY: Springer.