TEACHERS' QUANTITATIVE UNDERSTANDING OF ALGEBRAIC SYMBOLS: ASSOCIATED CONCEPTUAL CHALLENGES AND POSSIBLE RESOLUTIONS

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While the mathematics education community encourages teachers to support students in developing a more meaningful contextual understanding of algebraic symbols, very little is known about teachers' quantitative understandings of algebraic symbols themselves. The goal of this study was to fill this gap and examine secondary teachers' ability to contextualize algebraic symbols, in particular notation that results from algebraic generalization. The results led to the identification of various conceptual hurdles that teachers encountered as they endeavored to articulate the underlying quantities as well as various conceptualizations they invoked, both productive and unproductive, in their attempt to overcome these challenges.

Keywords: Mathematical Knowledge for Teaching, Algebra and Algebraic Thinking

Introduction

Traditionally, algebra instruction in the United States has focused on symbol manipulation. Teachers tend to emphasize formal methods, involving abstract mathematical symbols, over other approaches that involve representations that are more closely grounded in context such as diagrams, tables, and graphs (Kieran, 2007; Smith & Thompson, 2007; Yerushalmy & Chazan, 2002). Unfortunately, such an approach has failed to meet the needs of many, if not most, students. Struggling to cope with abstract notation, abruptly introduced and presented as detached from a coherent system of referents, students often fail to develop meaningful interpretations of algebraic symbols and the associated operations (Kieran, 2007; Harel (2007); Knuth, Alibali, McNeil, Weinberg, & Stephens, 2005; Sfard & Linchevski, 1994).

The ability for students to not only manipulate symbols, but interpret the contextual quantities that expressions represent has been emphasized as a core component to algebraic thinking. The Common Core State Standards (2010) underscores this understanding, including it as one of the eight practice standards (SMP 2: Reason abstractly and quantitatively) as well as a high school algebra content standard (HSA.SSE.A.1). Likewise, many scholars have articulated the significance of this understanding. Kaput and colleagues (2008) noted that without such an understanding, students' actions are guided strictly by the rules of the notational system without support from the previously learned structure of the reference field. As such, knowledge is more fragile with students tending to overgeneralize symbolic rules such as $(a + b)^2 = a^2 + b^2$.

To support students in developing a contextual understanding of symbols, researchers have advocated for the introduction of algebra through inquiry-based activities grounded in more concrete representations such as tables, situations, and words (Koedinger & Nathan, 2004; Nathan, 2012). One example of such an approach is through figural pattern generalization. These are tasks in which students are provided drawings of sequential stages and asked to find subsequent stages and eventually write an expression to model their understanding of a general stage. Exploring these patterns affords students the opportunity to convey their generalizations through a variety of increasingly abstract representations, leading to a more meaningful interpretation of the eventual symbolic forms.

In order to support students in developing a quantitative understanding of the notation, teachers must possess specialized content knowledge that goes beyond simply the ability to write expressions (Ball, Thames, & Phelps, 2008). They must understand how to relate, with precision, the various

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mathematical representations to the contextual quantities they represent. Although several researchers have investigated students' understanding of representations in algebra (e.g., Knuth, 2000; Nathan & Kim, 2007), less attention has been given to examining teachers' understandings of algebraic notation and their ability to draw connections to the context. Stylianou (2010) studied middle school teachers' beliefs about the instructional use of multiple representations, but not their knowledge. Harel, Fuller, and Rabin (2008) documented ways in which teachers failed to support students to develop meaningful interpretations of symbols, but without exploring teachers' symbolic reasoning or other potential causes for the failure.

While the field has emphasized the need for students to develop a contextual understanding of symbolic representations, we know very little about teachers' understanding in this area. Having a better image of the specific challenges teachers face and how to overcome these challenges will inform teacher educators how to better support teachers in working with their students to develop this ability. Therefore, the goal of this study was to examine secondary teachers' understandings of the quantitative meanings of algebraic symbols, in particular notation that results from algebraic generalization. The results led to the identification of various conceptual hurdles that teachers encountered as they attempted to make sense of and connect the underlying quantities and quantitative relationships as well as various conceptualizations they invoke, both productive and unproductive, in their attempt to overcome these challenges.

Theoretical Perspective

Although there is a lack of empirical studies addressing teachers' understandings of mathematical representations, considerable thought has been devoted to establishing the importance and role of representations theoretically. Multiple scholars have developed theoretical rationales to explain why the ability of expressing the meaning of numeric and algebraic figures is foundational to the understanding of mathematical notation.

Quantitative Reasoning

In order for students to be able to contextualize algebraic notation, they must possess a strong understand of the quantities the symbols represent. Therefore, a key component to possessing a deep understanding of algebraic symbols is quantitative reasoning. According to Thompson (1994), "quantitative reasoning is not reasoning about numbers, but reasoning about objects and their measurements (i.e., quantities) and relationships among quantities" (p. 8). As such, problem solving is not about determining the sequence of operations that will result in the correct answer, but about developing a conceptual understanding of how the quantities in a given problem are interrelated and how they combine to create new quantities. By focusing on the relationship between quantities, students develop a deeper understanding of the problem situation. Smith and Thompson (2007) argue that students must possess a sophisticated enough understanding of the structure of the problem to warrant the use of algebraic tools. Without a grasp of the quantities that shape the problem situation, students are unable to see algebraic notation as a representation that communicates quantitative relationships and consequently are left interpreting symbolic expressions as simply a tool that serves to calculate numerical values.

Symbolization

While understanding the contextual situation is foundational for developing meaning of algebraic expressions, for such an understanding to become embedded in abstract symbolic forms and for students to see notation as communicating the quantitative structure, various cognitive developments must take place. Kaput, Blanton, and Moreno-Armella (2008) described a process they refer to as *symbolization,* in which through one's experience in working with mathematical ideas, their related understandings become infused in the mathematical objects used to represent the phenomenon. They

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noted that over time and with multiple iterations of reflection, students' understanding of the context becomes instilled in more and more densely compressed forms of symbolization. Initially, students use more contextually connected representations such as oral, written, and drawn descriptions to express their experiences. They then use these representations to reflect on this same experience. This process leads to a newly mediated conceptualization of the mathematical phenomenon and possibly to new representations. Each interaction with the mathematical phenomenon, whether individual and or socially mediated, results in a new conceptualization. Eventually these conceptualizations converge into a conventional and compact symbolic form, establishing a rich, densely packed interpretation of the mathematical phenomenon. Kaput and colleagues noted that in the end, instead of the symbols' representing the referent as a separate entity, the two become interpreted as one. Actions applied to the symbols are construed as actions on the referent itself. At this stage, a student does not look *at* symbols, but *through* them, seeing the mathematical phenomenon and the notation as one.

Connections Between Representations

The role of multiple representations in mathematics and the importance of teachers to engage students in making connections among mathematical representations has been recognized by many scholars (National Council of Teachers of Mathematics [NCTM], 2014). Several studies have demonstrated the ability to translate between representations as a characteristic of more robust and flexible knowledge (e.g. Pape & Tchoshanov, 2001; Stylianou & Silver, 2004). In particular, Lesh, Landau, and Hamilton (as cited by Lesh, Post, & Behr, 1987) observed that students working through mathematics problems seldom came to the solutions successfully using a single representational mode. Explaining this phenomenon, Tripathi (2008) noted that using these "different representations is like examining the concept through a variety of lenses, with each lens providing a different perspective makes the picture richer and deeper" (p. 439). Extending this idea, Dreyfus and Eisenberg (1996) argued that representations differ not only in the way information is expressed, but also in terms of the information itself. They maintained that "any representation will express some, but not all of the information, stress some aspects and hide others" (pp. 267). Subsequently, mathematical ideas are not embodied by a single representation but rather lie, at the intersection of these representations. Finally, Lesh et al (1987) asserted that establishing a relationship from one representational system to another supports students in developing a stronger understanding of the various properties within the situation as they are encouraged to focus on what structural characteristics are preserved in the mapping.

Methods

To investigate the various conceptual hurdles associated with teachers' quantitative understanding of the algebraic notation used to describe figural generalizations, I engaged four 8th grade teachers each in a 1.5-hour individual semi-structured clinical interview (Ginsburg, 1997). Wanting to identify particular challenges associated with connecting algebraic notation to quantities as well as productive conceptualizations teachers formulate to overcome these difficulties, I chose teachers with significant experience with algebraic generalization. The teachers selected all had previously participated in multiple days of professional development focused on algebraic generalization as well as significant experience teaching and interviewing students in this area. Although a study of a more representative group of teachers might provide more generalizable information, choosing more knowledgeable participants allowed me to investigate in detail the subtleties of contextualizing algebraic notation.

During the interview, the teachers were presented two different figural generalizing tasks (see Figure 1). These afforded many different decompositions including interpretations of groups of

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varying sizes and overlapping groups. Also, the two patterns picked differed in that one was more conducive to being construed as consisting of a constant number of groups of increasing size, while the other could be more readily understood as comprising of an increasing number of groups of constant size. During the interview the participants were asked to provide numerical expressions for specific stages and a general algebraic expression for the n^{th} stage. After each expression they formulated, I asked them to explain what each symbol represented. In addition, I asked the participants to analyze the quantitative meanings of students' work to examine their understanding of decompositions that might differ from their own. Throughout the interview, questions focused on the teachers' understandings of individual symbols and collections of symbols. In addition, I asked participants to comment on their interpretations of various initial symbolic rules as well as on intermediate expressions that arise through syntactical manipulation.

Each interview was videotaped and transcribed. Teachers' responses were reviewed using a grounded theory approach (Strauss & Corbin, 1994) in which I used open coding and the constant comparison method to analyze their responses. I began by identifying particular areas of difficulty across the four teachers. I then compared the actions and comments in these areas among the participants as well as among similar items on different problems.

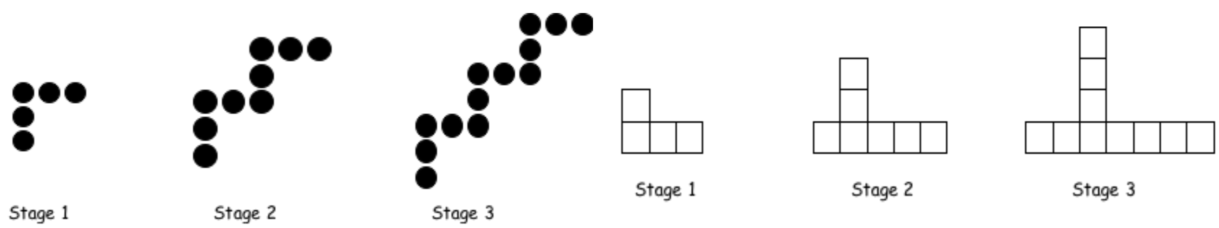

Figure 1. Figural Generalizing Tasks

Results

All four teachers approached the generalizing tasks quantitatively. That is, rather than using a procedure based on numerical values to arrive at a correct linear expression, they began by decomposing the figures into various quantities and then formulating expressions to express their understanding of the quantities they saw. In addition, all of the participants were successful in writing different expressions that corresponded to distinct decompositions of the pattern when asked to analyze the pattern differently and were able to explain possible interpretations of the pattern when exposed to students' expressions that differed from their own. That being said, while the teachers were able to connect the expressions to the quantities in the pattern in general, they struggled articulating the precise contextual quantities that symbols represented. In the end two different challenges emerged along with 3 different conceptualizations to overcome each challenge, one unproductive and two productive.

Challenge #1. Interpreting the Coefficient of *x*

The first conceptual difficulty centered on the participants' understandings of the coefficient of *x* and its relationship to the variable. Initially, all four participants described the coefficient and the variable together as representing groups of a particular size (i.e. $5x$ represents the number of groups of 5), but struggled to disentangle the two and articulate the specific meanings of the symbols independently.

Unproductive conceptualization: Detaching meaning of the symbols from details of context. To overcome this challenge one participant, Denise, reconceptualized the coefficient as the constant difference between stages even when such a construal was inappropriate for the context. To illustrate this type of thinking, I describe Denise's explanation of the expression $3x+1$ for the second task.

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Initially, she stated that the *3* and the *x* both represented number of groups of 3. When asked to clarify, she began vacillating between various interpretations (the number of groups of three, three tiles, the number 3, the three added on) before ultimately concluding that it represented the three tiles that were being added at each stage. When asked to highlight "the three added on" in the figure, she seemed to imposed her notion of adding three on the diagram, selecting tiles that did not correspond to how the pattern changes between stages. Initially she said it did not matter which ones, circling what seemed like arbitrary groups of 3 dots (see left image of Figure 2), before eventually deciding the three additional tiles each stage were the one far left tile and the two far right tiles (see right image of Figure 2).

Denise's vague and even problematic explanations of the various symbols' referents are evidence that she was not using symbols to communicate her interpretation of the figure. Instead, she seemed to reinterpret the coefficient as representing a decontextualized growth factor and attempted to improvise a quantitative interpretation on the figure.

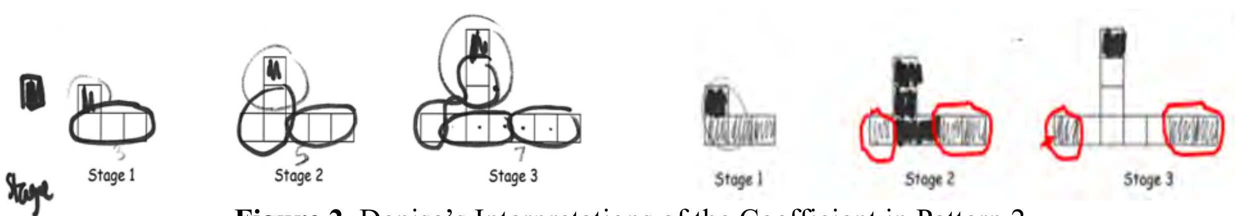

Figure 2. Denise's Interpretations of the Coefficient in Pattern 2.

Productive conceptualizations: Interpreting the variable as the number of groups and coefficient as a ratio of tiles per group or vice versa. While the other three participants also struggled identifying the meaning of the coefficient and variable separately, they eventually disentangled their meanings, describing the variable as the number of groups and coefficient as a ratio of dots or tiles per group for the first task. Notable was their explanation of the symbols in the expression $3(x - 1) + 4$ for the second task. While two of the participants switched their interpretation of the symbols relative to the first task, with the coefficient now representing the 3 constant groups and the variable corresponding to their varying size, the third teacher did not. To make sense of the 3, he imagined orbits of 3 tiles being added to each stage. Such a conceptualization matched his previous interpretation of the variable as the number of groups with the coefficient representing its size. While all three participants had quantitative interpretations of the symbols, the first two flexibly adapted their interpretations of the symbols to accommodate their quantitative understandings of the figure, while the third had a more fixed view of the symbols, reconceptualizing the quantities in the figure to match his previously formulated understanding of the symbols.

Challenge #2. Interpreting Expressions Where the Variable Appears More Than Once

The second conceptual challenge that emerged for the teachers was negotiating the meanings of variables that appeared more than once in a single expression or between expressions after algebraic manipulation. Such a situation exists in the first pattern when decomposing the figure into overlapping groups of 5, first with the expression $5x - (x - 1)$ and then in the subsequent simplified expression $4x + 1$. Initially all four participants interpreted the various *x*s in these expressions as representing different quantities in the figure. They understood the *x* in $5x$ as the number of groups of 5, the *x* in $x - 1$ as the number of overlapping dots, and the final *x* in 4x as the number of groups of 4. Expecting a single variable to a have a consistent meaning, they struggled to explain this apparent conflict.

Unproductive conceptualization: Imposing the interpretation of one variable onto another.

Three of the participants, in an effort to coordinate the symbols' referents, initially imposed an interpretation of the *number of groups of five* on the x in the expression $x – 1$. In doing so they then incorrectly reinterpreted the minus *1* as accounting for the difference in sizes of the groups of five and the groups of four (i.e., the difference in the number of dots) rather than the difference between the total number of groups of five and the number of overlapping dots. While all three teachers devoted at least 5 minutes to this incorrect construal, eventually they all noticed their inappropriate interpretation. Of these three teachers, two then formulated productive conceptualizations of the variables to overcome this problem, while the third participant did not. Instead, in an attempt to resolve this inconsistency, she oversimplified the symbols' referents, arriving at a final interpretation of all the *xs* as simply a dot. Accordingly, she construed $5x$ to mean 5 dots and $4x$ to mean 4 dots, but was unable to indicate which exact dots in the figure. Such a conceptualization of x as a dot essentially treats the variable as a label and the coefficients as decontextualized numbers, removing any quantitative meaning of the symbols and failing to explain any quantitative relationships between the symbols.

Productive conceptualizations: Coordinating numerical values and reinterpreting symbols to align quantitative meanings. Two of the participants were able to articulate viable, yet different solutions to reconcile the diverging meanings of the symbols. Although both teachers initially tried to make sense of the *x*s by using a literal translation of the words like the participant described previously, they eventually formulated productive conceptualizations.

The first participant did so by a**ssociating the quantities numerically.** By evaluating the expressions multiple times and stating the quantities and their numerical relationships, she was able to see that the number groups, initially of 5 dots, is always equal to the number of overlapping dots to be removed, which is equal to the number of groups of 4 dots. In the end, although she continued to interpret the same variable as different referents, she realized that the numerical value of each of these quantities is always equal.

The second productive conceptualization resulted through a reinterpretation of the quantitative meanings of the variables. Similar to the previous participant, the third teacher verbalized his understanding of the various variables in the expression $5x–(x+1)$ and $5x–x+1$, while carefully examining their values. This process helped him to not only coordinate the values of the two quantities (groups of 5 and overlapping dots), but also to connect them physically, noticing that the overlapping dots were members of the same groups of 5. In addition, he had added, apparently somewhat serendipitously, a coefficient of *1* in front of the second expression (resulting in $5x–1x–1)$. Together, these various semiotic acts supported him in reconceptualizing the overlapping dots as group of size 1. This conceptualization allowed him to reinterpret x as the purely the number of groups, without attaching a size, and the varying coefficients as the size. In the end, he was able to formulate an understanding of the variable so that it retained consistent quantitative meaning throughout as well as explained the varying coefficients.

Discussion

As this analysis reveals, contextualizing algebraic notation is challenging, even for experienced teachers. There are many nuances that experts overlook when they use algebraic expressions to solve problems and communicate their generalizations. In this final section I will revisit some of these challenges and discuss implications that I see stemming from these results.

Conceptual Complexities of Interpreting Algebraic Expressions

To highlight the complexity of contextualizing algebraic notation, I want to revisit a particular conceptualization that emerged. This example serves to not only illustrate the sophisticated understanding necessary, but also emphasizes that the quantities the teachers came to see in the

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notation were not intrinsic properties in the figure but rather, mental constructs that they themselves created.

One challenge identified in this study was articulating, with precision, separate meanings for the variable and coefficient that explained the relationship between the two. To overcome this difficulty, participants not only interpreted x as the number of groups and the coefficient as the size of each group, but also reversed this mapping and conceptualized the coefficient as the number of groups and the variable as the group size. To see both ways requires an abstract and flexible interpretation of the symbols. To illustrate the abstraction of perceiving both ways, I will use an alternative figure (see Figure 3) in which the transition between these two views requires only a subtle, cognitive shift in defining the group. As I point out, while this pattern can be modeled by the expression of $3n$, depending on your perspective, the 3 and then n can take on different meanings. In interpretation 1, the n indicates the number of groups of size 3 and in interpretation 2, the n represents the number of dots in the constant 3 groups.

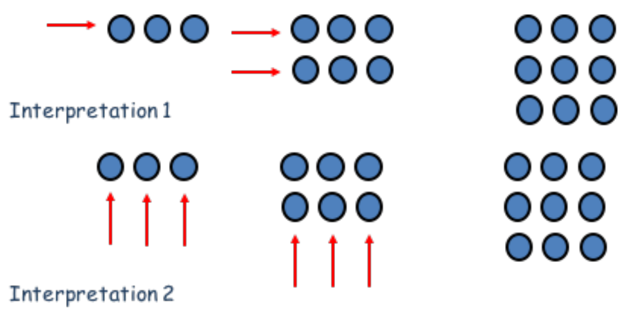

Figure 3. Flexible Conceptualization of Variable and Coefficient.

As this example illustrates the quantitative structure of the pattern is not an inherent characteristic of the figure or of the corresponding notation used to communicate it. The capacity to interpret symbols in multiple ways is an understanding that must be explicitly developed.

Implications

While I see several implications that stem from this study, I will highlight two which are interrelated. As noted in the introduction, algebra classrooms are dominated by a symbolic focus without attention to meaning. While only a few studies have specifically tackled this issue from the teacher's perspective, the consensus seems to be that the primary cause is teachers' orientations. The results of this study indicate that the challenge to transform the current symbolic focus in algebra classrooms is not simply an issue of beliefs. By detailing teachers' struggles with the complexity of this topic, this study demonstrates that, at least in part, the difficulties teachers experience in shifting their instruction is connected to their knowledge bases. Consequently, a second, related implication is the need for teacher preparation programs to explicitly develop this understanding. While definitely a daunting task, the results of this study contribute to this endeavor by identifying both conceptual hurdles and conceptual resources on which to focus instructional attention to support teachers in developing this knowledge and ultimately helping their students foster a deeper, quantitative understanding of the notation.

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