

INTERPRETING AND REPRESENTING STUDENTS' THINKING IN THE MOMENT: PRESERVICE TEACHERS' INITIAL NUMBER STRING LESSONS

Lizhen Chen
Purdue University
chen1905@purdue.edu

Laura Bofferding
Purdue University
lbofferd@purdue.edu

Being able to effectively interpret students' thinking and respond effectively in the moment are important skills that preservice teachers (PSTs) need to learn. This study zooms in on 14 PSTs' planning, teaching, and reflections involved in number string lessons, and investigates to what extent PSTs anticipate their students' strategies and incorrect answers. Further, through a lens focused on PSTs' talk moves, we explore how PSTs supported students' understanding of the strategies and concepts, and handled incorrect answers and unanticipated responses (strategies). Results show that PSTs were capable of anticipating many answers and strategies but need more improvement in making full use of various talk moves in their questioning. Implications are provided for mathematics methods instructors to plan activities to better support PSTs.

Keywords: Teacher Education - Preservice, Classroom Discourse, Instructional Activities and Practices

The ability to know students' thinking is a key factor in mathematics teachers' ability to conduct group discussions and improve their classroom teaching (Stein, Engle, Smith, & Hughes, 2008). Especially when teaching practices emphasize students' freedom of exploration and use of mathematical processes, the importance of facilitating discussion around students' solutions to problems are paramount (NCTM, 2006). Teachers' roles in facilitating discussion rely on being able to anticipate what students will say and have strategies for dealing with unanticipated responses. Our study helps characterize this process with preservice elementary teachers during number string lessons, which involve presenting a series of related arithmetic problems that highlight an overarching strategy or pattern (Stein et al., 2008).

Theoretical Framework

Responding to Students' Thinking in the Moment

Even though much literature focusing on PSTs offers teacher educators resources to educate PSTs before they actually teach, responding to students' spontaneous thinking during number string lessons is a challenge for PSTs and a demanding task for teacher educators (Bofferding & Kemmerle, 2015). Responding to students' spontaneous thinking is complex, involving PSTs' mathematics knowledge, students' mathematics knowledge, the classroom learning culture, teacher-student interactions, and instructional representations (Ghousseini, 2015). Further, PSTs must make quick actions to respond to students' ideas, especially when unanticipated (Stein et al., 2008), under the condition where they lack a rich knowledge-and-experience base to handle everything in the classroom (Jacobs, Lamb, & Phillip, 2010). Because of this complexity, there are no textbooks to tell PSTs how to anticipate or determine the best way to respond to the variety of situations they might encounter (Zeichner, 2012).

Before responding to students' thinking in the moment, PSTs must unpack and interpret students' reasoning, a process that is aided by the use of restatement and other talk moves to ensure mutual understanding among the class (Chapin, O'Connor, & Anderson, 2009). They make use of symbols, pictures, gestures and mathematical discourses to represent chosen students' thinking and assess students' strategies (Leinhardt & Steele, 2005). During this process, PSTs must judge the relative

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importance of different ideas. The judgment is particularly pertinent to what Ghouseini (2015) defined as “mathematical sensibilities” (p. 343), i.e., PSTs’ mathematical knowledge and abilities to attend to what proper interpretations to make and how representations are selected and used.

Anticipating Students’ Problem-solving Strategies

There are a few steps PSTs should take before engaging students in solving a mathematics problem. They should know the correct answer (or answers) and should at least solve the problem themselves. However, knowing only one solution to a problem is insufficient to teach students with a wide array of answers and strategies for the same problems (Stein et al., 2008). Full preparation for possible emergent student thinking is beneficial (Jacobs, Lamb, & Philipp, 2010); without such preparation, PSTs might be inclined to feel unprepared and have limited ideas about ways how to handle unanticipated answers (Smith, 1996). Therefore, it is important for PSTs to anticipate multiple student strategies (both correct and incorrect), different reasoning supporting those strategies, and representations utilized by students at different levels of mathematical understanding (Stein et al., 2008).

Talk Moves

Aside from anticipating possible student strategies and reasoning, PSTs also need to utilize this knowledge together with appropriate questioning prompts in order to structure the sharing of students’ mathematical thinking to the class (Stein et al., 2008). To solicit students’ thinking in the moment, PSTs have access to some discursive tools that are helpful in encouraging students to say more about their ideas, support positive teacher-student interactions, and align with instructional goals. Much literature has documented such discursive tools. Chapin, O’Connor, and Anderson (2009) presented five talk moves that support students’ thinking and help maintain a collective learning environment, i.e. teacher revoicing, student restating, applying one’s own reasoning to someone else’s, further prompting opportunities and using of wait time. In a similar vein, Ghouseini (2015) elaborated the discursive tools from a perspective of discourse routines and divided them into five categories, which serve as our lens for analysis. These categories are revoicing (the teacher repeats, rephrases and translates students’ saying), orienting (the teacher puts someone’s idea on the spot and asks other students to comment on and contribute to that idea), pressing (the teacher pushes students to talk more about their reasoning), negotiating (the teacher connects different students’ strategies and tries to involve students in the discussion about the similarities and differences), and making certain aspects of the discourse explicit.

In the face of errors or mistakes, many teachers cannot help correcting students (Stein et al., 2008) and tend to use show-and-tell discourse (Ball, 2001). Especially when these errors are unanticipated, PSTs might feel out of control and at a loss for how to appropriately respond to students’ ideas (Son, 2016). This study is aimed to contribute to the literature by looking at the intersection of PSTs anticipating and use of talk moves in number string lessons. Investigating PSTs’ actual and spontaneous responses to students’ strategies and identifying what discursive tools PSTs usually fall back on when encountering unanticipated student errors will advance our understanding of this intersection. The following research questions guide us in this exploration.

1. To what extent do PSTs anticipate their students’ mathematical strategies and incorrect answers during number string lessons?
2. What talk moves do PSTs use, and what potential do they have for supporting students’ understanding of the mathematical strategies and concepts?
3. How do PSTs handle incorrect answers and unanticipated responses (strategies), if they occur?

Methods

Participants and Setting

Participants in the study included 17 female PSTs taking an elementary mathematics methods course at a Midwestern university. Of these, we selected 14 for further analysis by excluding PSTs who worked with only one student, a life skills class, or did a topic besides addition, subtraction, or multiplication. Between two sections of the methods course, twelve PSTs came from one section and two came from the other. As part of their course, the PSTs each spent nine afternoons teaching, doing interviews with students, helping out, and teaching three mathematics lessons and two to three science lessons at an elementary school. Our focus—a number string lesson—was one of the three mathematics lessons. All but two of these PSTs shared a classroom with a second PST, and their placements varied from Kindergarten to Grade 5.

Design and Materials

The methods course aimed to raise PSTs' awareness of students' strategies and attune them to ways of building on students' mathematical strategies through talk moves and the use of representations. In the class, PSTs were asked to read chapters from two books to help them think about students' arithmetic strategies: Fosnot and Dolk's (2001) *Young Mathematicians at Work: Constructing Multiplication and Division* and Wright, Stanger, Stafford and Martland's (2006) *Teaching Numbers in the Classroom with 4-8 Year Olds*. With the purpose of interpreting and representing students' thinking in number strings, PSTs read about students' strategies for solving addition, subtraction, and multiplication problems. Drawing on these reading materials, PSTs designed their number string teaching plans (as negotiated with their placement teachers in the elementary schools).

Their plans, which were co-written if they shared a classroom with another PST, required them to identify the targeted strategies they hoped students would use in the lesson or pattern they hoped students would notice, list the problems they would pose, anticipate strategies and answers (both correct and incorrect) for each problem, draw possible representations PSTs would use to illustrate students' thinking, and list connections they anticipated making among students' strategies. Given PSTs' instructor's written feedback to the teaching plans, some PSTs also revised their plans. PSTs' teaching plans and revised versions serve as one data source.

Then PSTs implemented their teaching plans in elementary schools, and lessons lasted around 15-20 minutes. They audio-recorded their lessons and took pictures of any representations they made on the board during their lessons. The course instructor (the second author) was on site with PSTs and noted down their representations and students' uses of strategies. Together, the transcripts of PSTs' teaching and the course instructor's notes served as a second data source. Finally, PSTs each wrote a teaching reflection focused on the actual strategies students used, how the PSTs handled errors, how they used representations, and any changes they would make to the lesson. Therefore, the data includes 8 lesson plans, some with revisions, audio and pictures of the representations used from the lessons, and 14 lesson reflections.

Expected Take-Away from Course Readings

Fosnot and Dolk (2001) and Wright et al. (2006) discussed in depth strategies of solving addition, subtraction and multiplication used by K-5 students (see Table 1 for addition examples). We expected PSTs to think about these strategies when they anticipated what their students would do as part of their teaching plans and draw on them during their number string lessons as well as their reflections. Aside from the strategies discussed in the readings, PSTs also had their own strategies, such as the use of standard algorithms, to use.

Table 1: Strategies of Solving Addition Mentioned in Course Readings

Strategies	Explanations
Counting by ones	14 + 5: Count all or count on starting at either number.
Doubling	7 + 7: Double same numbers
Jump Strategies	14 + 7: Start at one number and make strategic jumps to add on, e.g., 14 + 6 = 20, 20 + 1 = 21.
Split Strategies	24 + 13: Split apart the numbers into tens and ones and add similar place values, then add the results together, e.g., 20+10=30, 4+3=7, 30+7=37.
Split-jump	24 + 13: Split apart the numbers into tens and ones, add the tens, and then make jumps to add on the ones, i.e. 20+10 = 30, 30+4=34, 34+3=37.
Compensation	26 + 17: Change one number to make the addition easier, and then adjust for it later, e.g., change 17 to 20, then do 26 + 20 = 46, 46 – 3 = 43.
Manipulatives/tools	Use ten frame, double ten frame, fingers, cubes, pictures, number line, etc. to help counting

Analysis

With PSTs' teaching plans and actual teaching transcripts available, we began our analyses by identifying the anticipated vs. unanticipated strategies and answers. We looked at PSTs' planning documents for answers and strategies they anticipated students would have for each problem, and compared these to the answers and strategies students used in the lessons. We organized these by anticipated answers (anticipated strategies or unanticipated strategies) and unanticipated answers (anticipated strategies or unanticipated strategies) with the purpose of revealing the consistency between PSTs' anticipation and actual student action. We looked at totals for each category as well as how many of the anticipated and unanticipated answers generated were incorrect. Correct answers were assumed as anticipated if not explicitly discussed in PSTs' lesson plans.

To better understand PSTs' talk moves around anticipated and unanticipated answers and strategies, we focused on two addition number strings and coded each talk turn within each lesson episode using Ghouseini's (2015) framework. An episode is defined as a round of teacher-student exchanges (talk turns) in which a student's reasoning for his/her answer is sought out in detail. Episodes occurred after initial answers were elicited, and certain problems had more episodes if there was more variety in how students solved the problems (i.e., the teacher asked several students how they solved the problem). If a PST asked a student a question, the student responded, and the PST followed up before moving onto a different strategy, that was three talk turns and one episode. Some talk turns had no codes if no talk moves were used, and some had multiple codes if more than one talk was used. One researcher completed the coding, and a second researcher checked a portion of the codes; disagreements were discussed until there was agreement. Finally, we compared and contrasted the frequency of different talk moves in order to get a general profile of PSTs' discourse when they responded to students' ideas.

Results

Discrepancies Between PSTs’ Anticipated and Unanticipated Answers and Strategies

First of all, we focus on the results of PSTs’ anticipation of students’ answers and strategies across the addition, subtraction and multiplication lessons. As Table 2 shows, 53.5% of students’ strategies (54 out of 101) were anticipated by PSTs while 33.7% (34 out of 101) were not anticipated by the PSTs’ regardless of their successful anticipation of students’ answers. PSTs were able to anticipate answers most of the time but missed out on the variety of strategies students used to get the same answer. In other words, the PSTs focused mainly on one way to get an answer, even though thinking about multiple ways was modeled heavily in their methods course. For instance, for $8+8+4+4$, PSTs in the 3rd grade addition class succeeded in anticipating that students might use doubles to get 16 and 8 and then add 16 to 8 to get the final answer; however, the PSTs did not anticipate a popular strategy, 8×3 , arising from the third graders’ recent focus on multiplication. Even though PSTs could anticipate a general strategy, they did not typically anticipate all variants of the strategy. In the case of $19+21$, PSTs expected students to use a compensation strategy, i.e. $20+21-1=41$; $41-1=40$. One student, nevertheless, knew the answer of $19+22$ and thus compensated differently.

Table 2: Anticipated Answers versus Unanticipated Answers

Grade Levels \ Strategies	Anticipated Answers		Unanticipated Answers	
	Anticipated	Unanticipated	Anticipated	Unanticipated
2nd Addition	8 (1) *	2	1 (1)	3 (3)
3rd Addition	6	9	3 (3)	0
Kinder Subtraction 1	3	3 (3)	0	1 (1)
Kinder Subtraction 2	6	7	0	1 (1)
Kinder Subtraction 3	5	2	0	2 (2)
3rd Subtraction	0	7	0	0
3rd Multiplication	16 (1)	1	1 (1)	1 (1)
5th Multiplication	10 (1)	3	0	0
Overall	54	34	5	8

Note:

*: “8” means the number of student strategies PSTs anticipated; the number “1” in the bracket refers to the number of those that were incorrect answers.

In consideration of PSTs’ different responses to correct and incorrect answers, we found that they mostly anticipated correct answers. A plausible explanation might be that PSTs applied the strategies they had learned in the methods class to their teaching plans. When predicting the split-jump strategy, PSTs in the second grade addition class anticipated two different usages of this strategy and were able to smoothly handle students’ reasoning. By contrast, PSTs without a perception of students’ strategies in advance had a more difficult time making connections when questioning students. For example, in the third grade addition class, two co-teaching PSTs did not expect students to use multiplication (or arrays) in addition problems. When four students claimed that they used arrays in three addition problems, PSTs took a lot more talk turns to figure out what students’ arrays looked like. For example, given $16+16+4+4$, a student claimed to use an array of 8 down, 5 across, i.e. 5×8 . PSTs repeatedly asked questions about this to ensure they understood what the array looked like. Once getting the idea of 8 down, 5 across, PSTs concluded the conversation and moved to the next one without making a reasonable connection between the addition and the multiplication. The

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PSTs later reflected, “Arrays are a drawing strategy taught by teachers to encourage multiplication.” Overall, PSTs only anticipated 6 out of 19 incorrect answers. Another 5 incorrect answers resulted from students’ improper uses of PSTs’ anticipated strategies and the remaining 8 were unanticipated answers preceded by unanticipated strategies.

PSTs’ Talk Moves in Addition Problems

PSTs had to utilize different talk moves for anticipated versus unanticipated answers and strategies. The two groups that did addition number strings demonstrated an overwhelming use of revoicing and pressing talk moves in both classes, i.e. 29 in the 2nd grade class and 62 in the 3rd grade class (see Table 3). Also PSTs used more talk moves all together in situations of anticipated answers than those in situations of unanticipated answers (2nd: $19 > 16$, 3rd: $45 > 22$). Since the number string lessons were meant for PSTs to elicit and represent students’ mathematical thinking, PSTs were advised not to tell students how to solve the problems. Consequently, they often repeated students’ answers or strategies and pressed for more information to get clarity from students. This is consistent with Baxter and Williams’ (2010) results that PSTs deprived of their most familiar show-and-tell teaching mode likely lean toward silence and avoid telling students anything, expecting students can make use of their questioning to discover the right way to correctness.

Table 3: PSTs’ Talk Moves in Two Addition Classes

Strategies	Anticipated Answers		Unanticipated Answers		Total	
	Anticipated	Unanticipated	Anticipated	Unanticipated		
2nd Addition	revoicing	10	1	3	5	19
	pressing	4	2	3	1	10
	orienting	0	0	0	0	0
	negotiating	2	0	2	2	6
	Total		19		16	35
3rd Addition	revoicing	11	16	7	0	34
	pressing	5	12	11	0	28
	orienting	0	1	2	0	3
	negotiating	0	0	2	0	2
	Total		45		22	67

The aforementioned situation was applicable when students themselves could clearly articulate their reasoning. Then PSTs could pretend not to know the strategy and press students to articulate their reasoning. In the 3rd grade addition class, for example, a student proposed multiplication (8×3) to solve $8+8+4+4$ because of three “8’s.” A PST argued that she did not see three 8’s, pressing the student into saying more about the strategy. The student then made it clear that two 4s were put together first in his strategy. This is a successful episode where the PST used pressing to make the student’s reasoning clearly accessible to the whole class. On the other hand, revoicing and pressing did not move students’ reasoning forward when their reasoning was ambiguous. Students with difficulty presenting their strategies logically and meaningfully often revealed more helpful information when PSTs oriented the class to these students’ ideas or negotiated with them around alternative strategies. Solving $4+4+4+4+2+2$, one student relied on “8 times 3” to get his 24 but could not offer where three 8’s came from. Then the PST negotiated by asking, “So you thought it was like the last problem ($8+8+8$)?” A “yes” answer was given. Under such a circumstance, the PST was able to look far beyond students’ reasoning and associate possible mathematical evidence with students’ reasoning so that more information was elicited from the student through the PST’s appropriate questioning.

Similar to the function of negotiation mentioned above, PSTs were inclined to turn to orienting and negotiating talk moves when they addressed students' mistakes. In the 3rd grade addition class, there were three episodes dealing with wrong answers and PSTs either invited the involvement of the class to discuss the problems in question, negotiated an alternative with students, or negotiated by soliciting other students' explanations.

Conclusions & Implications

Overall, most PSTs did anticipate a majority of students' answers and strategies after the instructional reading in their methods course. But there were still many alternative strategies to the same problem that PSTs failed to anticipate. Therefore, we as teacher educators need to guide students to try out various solutions to a problem, in particular multiple ways to get the same answer, especially considering students' knowledge at different grade levels (i.e., such as the third graders knowing multiplication). The results from this study also indicate that the revoicing and pressing talk moves prevailed in PSTs' classroom discourse. PSTs need to analyze the benefits and use cases of the talk moves in more depth; students' problem-solving procedures were well explored by means of revoicing and pressing but student reasoning was ignored to some extent. Admittedly, revoicing and pressing helped PSTs continue on in the exploration of student reasoning when they encountered something unexpected; but these strategies are far from being sufficient to elicit students' thinking and create learning opportunities that students can take up later. In this regard, the negotiating and orienting talk moves provide students with opportunities to reconsider and compare their strategies with others.

In the case of errors, the fact that most incorrect answers were not anticipated by the PSTs during their planning called our attention to the need for PSTs to think more about common incorrect answers, especially how students might make mistakes using strategies they anticipated. When students presented incorrect strategies, they could not easily jump out of their reasoning and discover the expected path to correct strategies when the only talk moves PSTs used were revoicing and pressing. One possible way to better support students is for PSTs to use negotiating and orienting question prompts to target students' reasoning, and bring students face to face with their emerging strategies. These discursive interactions with students allow PSTs to help students notice unproductive steps in their strategies. Further, an important next step is to support PSTs in helping students move beyond addressing their mistakes to building on their mistakes (e.g, when a student did $19+19$ by adding only 9 to 9, PSTs should move beyond eliciting what the student was thinking when making the mistake, have students build on what they did successfully ($9+9$), push students to fix their mistakes (help them figure out how to make use of the tens).

When we delve further into PSTs' interpretations and representations of students' thinking, it is evident that there is room for teacher educators to orient the methods course with how to build on student mistakes and encourage PSTs to try the orienting and negotiating talk moves. Therefore, teacher educators can encourage PSTs to embrace the uncertainty of what students will say and see if the class can collectively make sense of and build on places where students' strategies are breaking down. This could be done through such pedagogical approaches as rehearsals (Grossman, Hammerness, & McDonald, 2009), creating opportunities of modeling real classroom teaching to help PSTs practice dealing with some specific mathematical topics like unanticipated strategies and student mistakes. By means of these rehearsals, PSTs could possibly enhance their teaching skills and mathematics expertise when teacher educators focus PSTs' attention on those specifics of teaching as well as those "variations of the practice as it relates to particular students and mathematical goals" (Lampert, et al., 2013, p. 238).

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