TIME AS A MEASURE: ELEMENTARY STUDENTS POSITIONING THE HANDS OF AN ANALOG CLOCK

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Time is an area of measurement that is difficult for children. This interview study addresses the question: What are children's solution approaches to position the hands of an analog clock? To explore this, we investigated problem solving when using a clock manipulative with mechanically linked hands. We compare overall success rates among students in Grades 2 (n=24) and 4 (n=24) in positioning hour versus minute hands. We then present a qualitative analysis of solution approach for both hour and minute hands. Results indicate successful students may use the linked hands without overt consideration of the measurement structure of the clock.

Objective

Children have difficulty with the topic of time (Earnest, 2017; Kamii & Russell, 2012; Williams, 2012). Despite the fact that this topic is a staple of early grades mathematics instruction (NGA Center & CCSSO, 2010), little empirical research exists that documents problem solving in the context of common tools for telling time. Because time measure underlies mathematics of change in later grades as well as serving as an independent variable for STEM-related investigations, the present study seeks to reveal how younger students make meaning of units and unit relations on an analog clock, a prevalent cultural tool for time. In the present analysis, we investigate children's strategies for positioning hands on an analog clock to indicate particular times. The objective of our investigation is to reveal patterns across children's strategies and, because time is an area of measure, to consider how different strategies reflect measurement ideas related to unit and scale (Lehrer, Jaslow, & Curtis, 2003).

Theoretical Framework

We frame our study with two lenses. First, we consider mathematical aspects of time units as related to children's developing *theory of measure* (Lehrer et al., 2003). Second, we consider the mediating role of the clock manipulative itself. First, children develop a theory of measure through everyday examinations of the attributes of objects or events (Lehrer et al., 2003). In investigating the world in such a way, children gradually attend to such attributes as length, area, weight, and duration, each of which may be measured formally or informally. As children compare and contrast and, eventually, quantify such attributes, they grapple with mathematical ideas particular to measure (Lehrer et al., 2003). These ideas include conceptions of unit (such as the need for identical or equal units) as well as conceptions of scale (such as any point serving as an origin or zero-point).

As a measurement tool, the analog clock features 12 equal hour intervals, with the numeral 12 marking both a zero-point and ending point depending on that to which a user attends. As with any standard tool for measure, the analog clock provides equal intervals that are arranged end-to-end without gaps or overlaps. Of course, the clock does not represent only hours; rather, the same 12 intervals reflect both minutes and seconds. As in other areas of measure, individuals may draw upon tools for measure in procedural ways unrelated to their mathematical properties (Moore, 2013; Stephans & Clement, 2003). One concern in the present study was not only to identify students' approaches as they position the hands, but to understand them in relation to ideas related to unit and scale.

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Second, the present study positions thinking and learning as inextricably linked to cultural practices (Cole, 1996; Earnest, 2015, 2017; Sfard, 2008), with conventional tools (i.e., a digital or analog clock) serving a mediating role in problem solving. Analog and digital clocks represent time and its properties in different ways, with the analog clock's intervals of time translating duration into spatial distance (Lakoff & Nuñez, 2000; Williams, 2012). Digital time provides a precise time to the minute without reflecting part-whole relations of minutes and hours. The digital time 2:50, for example, provides a quick and precise numeric representation of time. In contrast, for the analog clock's hour hand to show 2:50, one may interpret its position as not just showing the "2" as with digital notation, but its displacement from 2:00 to 2:50 as well as the length remaining length for the ten minutes from 2:50 to 3:00. Our study involved a particular clock manipulative featuring mechanically linked hands; based on our perspective of thinking and learning, this material property is consequential to children's solution approaches.

Related Research

Classrooms in the United States typically feature classroom clock manipulatives for teaching time, though the functionality of clock features—and how unit relations are thereby supported—vary. Of two common clock manipulatives, one features mechanically linked hands such that movement in one hand provokes the proportional shift in the hour hand; this specific clock is the focus of our study below. For example, on a clock displaying 7:00, if one were to move the minute hand clockwise to show 7:30, the hour hand would proportionally move as well (see Figure 1). With this tool, a user may note the proportional shift in hand movement; alternatively, one may not attend to this particular feature at all, as such proportional movement is not dependent on the user's intentions in hand positioning. A different clock manipulative features independent hands, such that a user must deliberately position each of the two hands to indicate a particular time. On this clock, given 7:00, if one were to move the minute hand clockwise to reflect 7:30, the hour hand would remain at the 7 and thereby reflect a time that does not exist in our system (Figure 1b). In particular, we seek to understand if one manipulative is more helpful for children in terms of connections to measurement (not necessarily just in terms of accuracy).

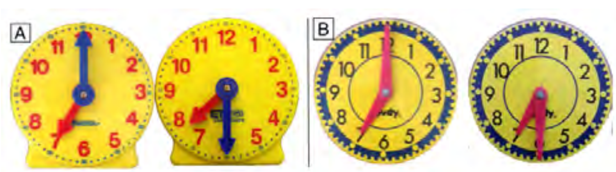

Figure 1. Manipulatives with (a) mechanically linked hands or (b) independent hands.

Given our concerns related to a theory of measure together with how tools mediate thinking and communication, our work has investigated problem solving related to time in the context of the two clocks in Figure 1. A recent analysis revealed that elementary students performed differently as a result of the clock available to them (Earnest, 2017), and in particular students were more successful when the hands were mechanically linked. To understand why students may have performed more poorly when the clock featured independent hands (Figure 1b), we further analyzed students' performances with this specific tool. We found that students' incorrect approaches often did not overtly reflect concerns for unit and/or a continuous scale (Earnest, Gonzales, & Plant, 2017). One such approach included treating intervals as containers; for example, treating the 2-3 interval as representing a container for the 2 o'clock hour (see also, Williams, 2012). In another approach, students matched a number from the digital time (i.e., the 2 of 2:50) to the numeral on the clock (i.e., positioning the hour hand on the 2).

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Although less common, approaches reflecting concern for unit and scale often led to success (Earnest et al., 2017). One approach involved treating the two hands—functionally independent on the clock manipulative (Figure 1b)—as coordinated; for example, to show 2:50, one student explained how the hour hand would be in the 2-3 interval but close to the 3 because the minute hand was only ten minutes from the top of the hour. Other students identified a zero-point on the clock. For example, to position the minute hand for 2:50, students began with the 12 as a zero-point and counted by 5s to reach the accurate position; such a concern for zero-point has been identified as a key idea related to scale (Lehrer et al., 2003). Overall, students were statistically more successful with the minute hand as compared to the hour hand, which, because hands were not linked on the manipulative, each student had to deliberately position.

Research Questions

Our prior analysis focused on a manipulative with independent hands (Figure 1b), yet we found students were more successful overall when using the clock with mechanically linked hands (see Figure 1a) (Earnest, 2017; Earnest et al., 2017). We wondered if the success among students using linked hands reflected different solution pathways for students as compared to those using independent hands. If so, results could illuminate how the linked hands support disciplinary ideas related to time. Alternatively, the linked property of the hands may support pathways towards accurate hand placement through mechanically accomplishing this mathematical work on behalf of the user.

The present study investigates the questions: Are children more successful positioning one hand over the other? And, what are the solution approaches children apply to position each hand? In particular, we were concerned with how such strategies reflected treatments of unit and interval consistent with geometric measure.

Methods

Participants included students in Grades 2 (n = 24) and 4 (n = 24) from six elementary schools in diverse areas (urban and rural) of western Massachusetts. All schools were identified as having a high percentage of children from low-income families. Interviews were conducted in 2015. Grade 2 students were selected because standards indicate children in this grade have already mastered time to the hour and half hour and are currently working on time at the 5 minutes (NGA Center & CCSSO, 2010). Grade 4 students were selected because, according to standards, time concepts including elapsed time have been mastered in prior grades, and their performances therefore illuminate any persisting differences in performance on problems involving time.

Based on an assessment administered to a larger group of students in the six focal schools, we identified a range of students with permission in each classroom and assigned them to one of three clock conditions; our focus in this paper is on the condition featuring an analog clock with mechanically linked hands (Figure 1a; see Earnest et al., 2017, for a similar analysis involving an analog clock with independent hands).

Interviews lasted approximately 30 minutes. Our analysis here focuses on seven particular tasks specific to positioning hands on the clock, as these tasks reveal students' treatments of unit and interval (see Earnest, 2017, for all interview tasks). Hand Positioning tasks were designed based on prior literature (Kamii & Russell, 2012; Williams, 2012) and ongoing piloting of tasks. The seven times included in tasks were: time to the hour (7:00), time to the half hour (4:30), time on the first half of the clock (10:10), time on the second half of the clock (2:50), and time with a minute value less than 10 (9:03), along with two relative time tasks using hour units only (half past 11, quarter past 8). To present tasks, the interviewer provided the clock positioned to an unrelated time and asked the student to show the target time (e.g., "Show me what 2:50 looks like on this clock."). The interviewer

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also turned over a card on which the same question was printed (Figure 2). Once the student positioned the hands, the interviewer asked that student to explain her/his thinking.

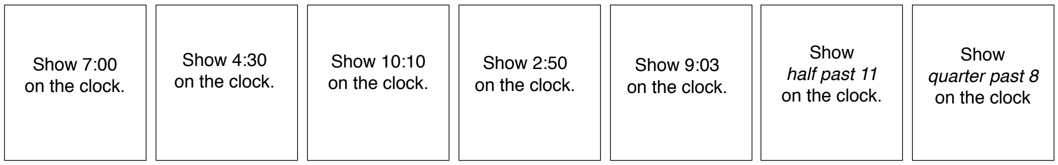

Figure 2. Playing cards for administering the Hand Positioning tasks.

We conducted both a quantitative and qualitative analysis. First, using video and transcript, we coded each hand position separately as correct (1) or incorrect (0) for placement, enabling a comparison in performance between the two hands. To do so, we identified an interval for the two hands for each problem (i.e., 4.3-4.7 for the hour hand for 4:30), outside of which a response was considered incorrect. All responses were double coded, and any discrepancies in hand positioning accuracy were resolved in team meetings. Second, we open coded video and transcripts using the constant comparison method (Corbin & Strauss, 2008). Based on rounds of coding to identify particular solution strategies in data, we generated a codebook. Three coders then double-coded all data, with any discrepancies discussed until reaching consensus on a final code.

Analysis and Results

The analysis is presented in two parts. We first present quantitative results speaking to whether students had similar success at positioning hour and minute hands. Following this, we present a qualitative analysis to identify strategies students applied**.**

Performances on Hand Positioning Tasks

We first compared performances for hour and minute hands: Were students more successful at one hand over the other? Means and standard deviations are provided in Table 1, and in general mean performances out of 7 problems show that students were quite successful. A Two (Hand) ✕ Two (Grade) repeated measures analysis of variance (ANOVA) revealed a main effect for Hand, $F(1, 46) = 11.560$, $p = .001$, with better performance with the minute hand as compared to the hour hand. A main effect also emerged for grade, $F(1, 46) = 7.676$, $p = .008$, with Grade 4 students outperforming Grade 2 students. There was no significant Hand ✕ Grade interaction, suggesting that the discrepancy in performance was roughly equivalent across grades ($p = .168$). A post hoc Tukey's HSD showed that the difference for Grade 4 students in hand positioning was not significant ($p = .135$) and with a low effect size ($d = .194$) (Cohen, 1988). For Grade 2 students, the difference was found to be significant ($t(23) = 3.14$, $p = .005$), yet with a low to medium ($d = .338$) effect size, suggesting only a low or low to moderate practical significance of the result.

Table 1: Means and Standard Deviations for Each Grade for Correct Performance on the Seven Hand Positioning Tasks

	Grade 2	Grade 4
	Mean (SD)	Mean (SD)
Hour Hand	5.04 (1.681)	6.17 (1.129)
Minute Hand	5.54 (1.250)	6.37 (0.924)

This finding is different than the comparable analysis for the analog clock with independent hands, for which both grades were more successful with the minute hand and both with a large effect size (Earnest et al., 2017). In the present analysis, when using the clock with linked hands, there was little detectable difference in students' positioning of hour versus minute hands. Given our interest in children's approaches to showing time on the clock, we question how the linked hands may differently support such accuracy and whether such success owes to conceptual understandings students applied or, alternatively, the mathematical achievements underlying the manipulative's functionality. To address this, we turn to our second question; given the high level of success with this clock, what are students' strategies when positioning the hands, and how are such strategies related to key ideas within measurement?

Children's Strategies to Position Hour and Minute Hands

In this section, we provide an overview of solution codes that emerged in our analysis for the seven Hand Positioning tasks (Figure 2). The role of qualitative data analysis involving children's strategies is to further contextualize performance results above that suggests there was little difference in the challenge of placing the two hands and high overall accuracy in hand positioning. We first present the six codes that emerged from analysis of video and transcript. After this overview, we present our analysis of strategies for each hand across all Hand Positioning tasks in both grades among the 48 students using the clock with mechanically linked hands.

Our analysis of the 48 students' solutions resulted in six strategy codes (Table 1): Container, Number Matching, Hand as Lever, Number as Floor, Origin, and Coordination, with idiosyncratic or unclear strategies coded Other. Table 1 features code names with examples as well as the frequencies across the 672 possible instances (7 problems with 2 hands per problem for 48 students) and, given all instances for just that code, the percent correct for each of the two hands. We first consider four strategies that (in our determination) did not overtly relate to unit or scale followed by two additional strategies that reflected some aspect of these measurement ideas.

First, 50 responses were coded as Container. Consistent with Williams (2012), children treated a particular interval as a container, with any point in that interval the same as any other point (see examples in Table 2). Second, Number Matching refers to a strategy to match the number from the time in the prompt with a number on the clock (e.g., "It's 2:50, so the hour hand goes on the 2.". At times, this included the application of a fact that remained unexplained in the interview (e.g., "I know 6 is 30"). Third, Hand as Lever involved children treating the focal hand as a mechanism to move the other hand; only nine students applied such a strategy, and in all cases this involved treating the minute hand as a lever to move the hour hand. Fourth, Number as Floor involved students finding the position of the hour hand for the top of the hour (e.g., when solving for 2:50, first finding 2:00 so the hour hand points to 2) and then applying continued movement to the minute hand clockwise resulting in further movement of the hour hand. In these cases, after finding time to the hour, students focused exclusively on positioning the minute hand without any overt further consideration of the hour hand. Unlike Number Matching, for which students' goals involved positioning the hour hand as close as possible to the target number (e.g., indicating 1:50 on the clock when matching the hour hand as close to 2 as possible), students found the top of the hour and then relied on the functionality of the linked hands. This occurred 65 times in interviews.

We identified two strategies related to key ideas of measurement (Lehrer et al., 2003): Origin and Coordination. Origin involved strategies in which a student identified a zero-point on the clock—typically the 12—with the target position as a path from that starting point. Unlike Number as Floor, which involved finding the top of the hour to locate the hour hand and then (based on available data) turning their attention to the minute hand, Origin involved a starting point with an explicit follow-up strategy (see example in Table 2). This code was applied 76 times, yet we note that it was applied only for the minute hand and not once to the hour hand. In almost all applications (96.1%) of this strategy, students were accurate. We also coded strategies as Coordination (n = 105), which involved treating the position of the focal hand as dependent or related to the position of the other hand. In

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doing so, such cases involved the proportional relationship of hours and minutes. These two measurement strategies were more successful than the prior four strategies, yet arose less frequently in our data.

Table 2: Strategy Codes for Positioning Hour and Minute Hands (N = 672)

Strategy (frequency)	Example
Container (*n* = 50)	*To position the hour hand for 4:30.* "It's 4, so the hour hand is in the 4-space.
Number Matching (*n* = 263)	*To position the minute hand for 4:30.* "I know 6 is 30."
Hand as Lever (*n* = 9)	*To position the minute hand for half past 11.* "I placed this here because I wanted this hour hand to be between 11 and 12."
Number as Floor (*n* = 65)	*To position the hour hand for 2:50.* "I first found 2:00, and then I knew that the hour hand would be after the 2."
Origin (*n* = 76)	*To position the minute hand for 4:30.* "I started here [at 12] and counted by 5s until I got to 30.
Coordination (*n* = 105)	*To position the hour hand for 2:50*. "I know this goes there because it's close to the 3, and the minute hand is only 10 minutes away from 3:00."
Other (*n* = 104)	Idiosyncratic or did not respond.

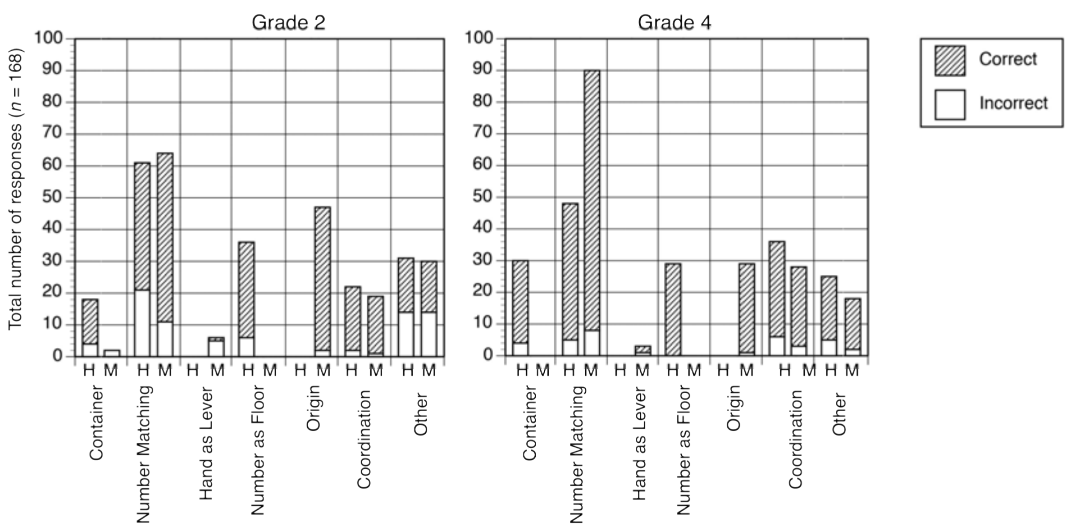

Figure 3. Strategy use for hand positioning when using clock with linked hands.

Figure 3 displays bar graphs for each grade to indicate frequencies of particular strategies for each hand along with whether such uses were correct or incorrect. Despite quantitative results above indicating little difference in success between the two hands, the strategies behind their positioning were often different. In particular, students' application of Container and Number as Floor were almost always for the hour hand. Although we would expect in general that strategies reflecting unit and scale would lead to greater accuracy (see Earnest et al., 2017), the material properties of the tool might have accomplished important mathematical work on behalf of the users applying Container, Number Matching, Hand as Lever, or Number as Floor strategies. We further note that the Origin strategy was applied exclusively to the minute hand. Such a result leads us to question whether, for

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children, the hour hand has an obvious zero-point like the minute hand does. We also note here that Hand as Lever was employed only in the context of relative times (i.e., half past 11) and, although we did not consider this to reflect unit or scale, any problem in which a student's minute hand position was coded as Hand as Lever received a Coordination code for the hour hand.

Concluding Remarks

With limited existing research focusing on children's understanding of time, we contend that the results of the present analysis are an indication that children may be developing an understanding of the inner workings of the clock—specifically how it reflects units and unit relations—in ways that are unrelated to mathematical properties of unit and scale. Further data is required to examine the extensiveness of these implications. Based on available data, if we were to look at overall performances among students using the clock with linked hands (Table 1), we may have concluded that this clock is a useful and productive manipulative for children to learn about time; however, our present analysis suggests that the material properties of the tool may be doing some important mathematical work on behalf of students.

Broadly speaking, what are our instructional goals related to time, particularly given that digital clocks are pervasive? We contend that instruction ought to move beyond procedures of clock-reading. Considering the role of time as a parameter in later mathematics, children ought to engage more deeply with the underlying meaning of clock features related to unit and scale. This research identifies a potential change in route with respect to how the field has framed learning and instruction related to time. We contend that more research is necessary in the area of how children come to understand time and common representations and tools for time. Although clocks are certainly pervasive in culture, results of this study underscore that children's ideas of time may be unrelated to the mathematical ways in which we measure it.

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