CONNECTION BETWEEN TOPIC-SPECIFIC TEACHER KNOWLEDGE AND STUDENT PERFORMANCE IN LOWER SECONDARY SCHOOL MATHEMATICS

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The interpretive cross-case study focused on the examination of connections between teacher and student topic-specific knowledge of lower secondary mathematics. Two teachers were selected for the study using non-probability purposive sampling technique. Teachers completed the Teacher Content Knowledge Survey before teaching a topic on division of fractions. The survey consisted of multiple-choice items measuring teachers' knowledge of facts and procedures, knowledge of concepts and connections, and knowledge of models and generalizations. Teachers were also interviewed on the topic of fraction division using questions addressing their content and pedagogical content knowledge. After teaching the topic on division of fractions, two groups of 6th grade students of the participating teachers were tested using similar items measuring students' topic-specific knowledge at the level of procedures, concepts, and generalizations. The cross-case examination using meaning coding and linguistic analysis revealed topic-specific connections between teacher and student knowledge of fraction division. Results of the study suggest that student knowledge could be reflective of teacher knowledge in the context of topic-specific teaching and learning of mathematics at the lower secondary school.

Keywords: Teacher Knowledge, Rational Numbers, Mathematical Knowledge for Teaching

Purpose of the Study

In the last several decades, research on teacher knowledge initiated by work of Shulman (1986) has focused on teacher knowledge as a major predictor of student learning and achievement. Since then the field benefited from numerous studies that substantially advanced the conceptualization of teacher knowledge. Scholars (e.g., Chapman, 2013; Izsak, Jacobson, & de Araujo, 2012) examined different facets of teacher knowledge without explicitly emphasizing its connection to student learning. Studies also stressed the importance of the kind of knowledge a teacher possesses because it impacts his/her teaching (Steinberg, Haymore, and Marks, 1985). Another line of research (e.g., Hill, Rowan, & Ball, 2005; Baumert et al, 2010; Author, 2011) specifically targeted the effects of different types of teachers' knowledge on student achievement.

Recently, scholars have advanced the field by examining teacher knowledge in variety of domains including number sense (Ball, 1990), algebra (McCrory et al., 2012); geometry and measurement (Nason, Chalmers, & Yeh, 2012), and statistics (Groth & Bergner, 2006). However, the field lacks research that provides an in-depth analysis of the various facets of teacher knowledge and its connection to student knowledge at *a topic-specific level*. To know what kind of teacher knowledge impacts student learning in the topic-focused context is an important issue worth of studying. Considering the importance of topic-specific knowledge, this study was guided by the following *research questions*: (1) Does what a teacher knows matter in regard to her students' topic-specific knowledge and performance? (2) What is the nature of topic-specific connections between teacher and student knowledge?

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Conceptual Frame: Topic-Specific Content Knowledge

Division of fractions is one of the topics in lower secondary school mathematics curriculum for grade 6th in Russia (Ministry of Education and Science of Russian Federation, 2004) where the study was conducted. Scholars (Ball, 1990; Ma, 1999) found that the teachers have limited topic-specific content knowledge and they lack conceptual understanding of the topic. One of the main reasons is that the topic of division of fractions is traditionally taught by using the “flip and multiply” or “cross-multiply” procedure (e.g., invert-and-multiply algorithm) without helping learners to understand why it works (Siebert, 2002).

Although some teachers consider the traditional algorithm as one of the efficient procedures to divide fractions, they lack understanding of its connection to the inverse nature of division and multiplication (Flores, 2002; Lamon, 1999). Moreover, profound understanding of division of fractions requires connections to other topics such as measurement and sharing/ partitioning interpretations of division (Ball, 1990; Flores, 2002). Other important meanings of fraction division are “finding a whole given a part”, “missing factor problem” interpretation of fraction division (Flores, 2002), and “the common-denominator algorithm” (Sharp and Adams, 2002).

In order to develop students’ knowledge and comprehension of fraction division teachers themselves need to understand underlying meanings of the algorithms and procedures (Ball, 1990) to make their mathematical knowledge connected and conceptual (Ma, 1999). To be connected topic-specific teacher knowledge should address different *cognitive types*: knowledge of facts and procedures, knowledge of concepts and connections, and knowledge of models and generalizations (Author, 2011).

Analyzing cognitive types of teacher knowledge and its connection to student knowledge within a topic-specific context will contribute to the field of mathematics education and provide tools to enhance teacher education and professional development in order to improve student learning.

Methodology

The interpretive cross-case study (Merriam, 1998) focused on the topic-specific connections between teacher and student knowledge of lower secondary mathematics. Two teachers were selected for the study. Teachers completed the Teacher Content Knowledge Survey (TCKS) before teaching a topic on division of fractions. The TCKS consisted of 33 items measuring teachers’ knowledge of facts and procedures, knowledge of concepts and connections, and knowledge of models and generalizations. Teachers were also interviewed on the topic of fraction division using questions addressing their content and pedagogical content knowledge. After teaching the topic on fraction division, students of the participating teachers were tested using similar items measuring students’ knowledge of procedures, concepts, and generalizations. The cross-case examination was performed using meaning coding and linguistic analysis techniques (Kvale & Brinkmann, 2009) to report connections between teacher and student knowledge of fraction division.

Participants

The study participants were selected using non-probability purposive sampling technique based on the following set of criteria: 1) selected teachers should represent upper and lower quartiles of the total scores on the TCKS; 2) selected teachers should have similar teaching experience; 3) selected teachers should have similar teaching assignments; 4) selected teachers should teach at similar school settings.

The TCKS was administered to the initial sample of lower secondary (grades 5-9) mathematics teachers (N=90) in Russia (Author, 2015) and then the sample was subdivided by quartiles using teachers’ overall TCKS scores. The maximum teacher score on the TCKS was 27 (out of 33) and the minimum score was 13. With regard to the first criteria, the overall sample of teachers was reduced

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to 43 teachers: 17 of which represented the upper quartile with the range of total TCKS scores 24-27 and 26 teachers represented the lower quartile with the range of total TCKS scores 13-18. After applying the remaining set of criteria 2)-4) we identified two teachers who met the requirements of the purposive sampling (names of the teachers are changed to keep the data anonymous) - Irina (with a total score of 25 on the TCKS) and Marina (with a total score of 17 on the TCKS). Both selected subjects are experienced lower secondary mathematics teachers and both of them are females of the same ethnic origin. Irina has 33 years of teaching experience and Marina - 21 years of teaching experience. Participants have similar teaching assignments - 5-8 grade mathematics with content addressing the following main objectives: Arithmetic, Algebra and Functions, Probability and Statistics, and Geometry. They both teach at urban public schools with similar student population concerning its' ethnic distribution and SES level.

We purposefully selected two contrasting cases with regard to teachers' mean scores on different cognitive types of content knowledge to closely examine the impact of teacher topic-specific knowledge on student performance while solving a set of problems related to division of fractions. The cluster sample of N=55 6th grade students of participating teachers (29 students in Irina's group and 26 students – in Marina's group) was used for collecting student level data after they studied a topic on division of fractions. The topic was a part of the chapter on operations with rational numbers placed in the 6th grade mathematics curriculum at the beginning of the fall quarter (Ministry of Education and Science of Russian Federation, 2004). Additionally, both Irina and Marina were teaching mathematics to the participating cohorts of students for the second consecutive year starting at 5th grade. Therefore, one may say that they established a certain teaching and learning "history" with these students.

Data Sources

The study used the following data sources: 1) TCKS to collect data on cognitive types of teacher knowledge; 2) structured teacher interviews on the topic of division of fractions; and 3) student data on solving three tasks related to the topic of division of fractions.

TCKS is the instrument that was designed to assess teacher content knowledge based on the cognitive types identified above. The survey consisted of multiple choice-items addressing main topics of lower secondary mathematics: Arithmetic, Algebra and Functions, Probability and Statistics, Geometry and Measurement. Specification table along with item analysis was performed to ensure content and construct validity of the TCKS along with its' reliability measured by the Cronbach alpha coefficient at *.839* (Author, 2011).

Teachers were interviewed using two sets of questions related to the topic of fraction division. First set of questions was aimed at tapping into teachers' pedagogical content knowledge whereas the second set was focused on different cognitive types of teacher content knowledge.

Students' written work on solving three tasks related to similar questions on division of fractions was collected and analyzed to examine connections to teacher knowledge. We purposefully used similar questions for teachers and students in order to trace linguistic, procedural, and conceptual traits in their reasoning as well as to analyze non-parametric quantitative trends in student topic-specific knowledge.

Data Analysis

For the qualitative phase of analysis, the teacher interviews were audio recorded and transcribed. Student written work was collected after the completion of the unit on division of fractions. In order to respond to the research questions we conducted meaning coding and linguistic analysis (Kvale & Brinkmann, 2009) of teacher narratives as a primary method of analysis. The data-driven meaning coding technique was used for the purpose of "breaking down, examining, comparing,

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conceptualizing and categorizing data" (Strauss & Corbin, 1990, p. 61). The linguistic analysis technique addressed "the characteristic uses of language, … the use of grammar and linguistic forms" (Kvale & Brinkmann, 2009, p. 219) by participating teachers and students within the topic-specific domain of lower secondary mathematics. The meaning coding and linguistic analysis was performed and cross-checked by two of the co-authors of this paper.

Considering ranked nature of the quantitative data collected in the study, we employed non-parametric technique (Chi-square test of goodness of fit) to detect group differences using frequency data in student responses.

Results

In this section, we will present major findings of the study starting with teacher responses on pedagogical content knowledge questions. Then we will report data on questions and tasks focused on teacher content knowledge and student knowledge of fraction division. Finally, we will present results of the quantitative analysis of students' performance on selected tasks.

In the qualitative phase of the study, we conducted structured interviews with two of the study participants – Irina and Marina. Irina's mean scores on the TCKS items are as following: score on items measuring knowledge of facts and procedures – 80%, items measuring knowledge of concepts and connections – 46%, and items measuring knowledge of models and generalizations - 30%. Irina's total TCKS score is 51%. Marina's mean scores on the TCKS are as following: knowledge of facts and procedures – 90%, knowledge of concepts and connections – 69%, knowledge of models and generalizations – 70%, and total score – 75%.

The qualitative phase of the study included two stages: (1) teacher interview and (2) student problem solving. The teacher interview consisted of the following two sets of questions: a) the subset of questions 1)-2) tapping into teachers' pedagogical content knowledge and aiming at teachers' understanding of learning objectives for the topic of fraction division; and b) the subset of questions 3)-6) focusing on teachers' possession of cognitive types of content knowledge. The first subset included the following questions: 1) When you teach fraction division, what are important procedures and concepts your students should learn? 2) What is the meaning(s) of division of fractions?

The second subset consisted of the following questions: 3) What is the fraction division rule? 4) Divide two given fractions 1 3/4 and 1/2. 5) Construct a word problem for the fraction division from the previous question. 6) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ *(a, b, c,* and *d* are positive integers) ever true?

Responses were audio recorded and teachers were provided with a scratch paper. We used open coding followed by axial coding technique (Strauss & Corbin, 1998) applied to the transcribed narratives to analyze meaning expressed and language used in teachers' responses. Below we present teachers' narratives to the first two pedagogical content knowledge questions.

Teachers' Responses to Pedagogical Content Knowledge Questions

Participants' responses to the question 1 is transcribed below. Based on Irina's response to the first question, it is evident that she capitalizes on her procedural knowledge with little or no attention to concept development. There is a slight indication of applying the rule in "standard situations" (lines 8-9 of Irina's interview excerpt for the question 1) with no further clarification on the nature of this application. Whereas Marina extends the application of the fraction division rule to the "non-routine problem solving situations" (lines 6-7 of Marina's interview excerpt for the question 1). Also, we thought that Irina's reference to "factorization of polynomials" in teaching fraction division was not further elaborated by her and, therefore, was confusing.

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IRINA

Before introducing the fraction division, I would like my students to recall the topic on factoring a polynomial, recall the rule of fraction multiplication, and recall reciprocals. After the lesson on fraction division I expect my students to know fraction multiplication and division rules, acquire skills to use these rules *in standard situations* [emphasis added], as well as apply factorization of polynomials.

MARINA

When I teach fraction division, first of all, I expect students to learn fraction division rule as it applies to the case of common fractions. Then, I expect them to know how to apply the rule to mixed fractions. Further, students need to understand how to use the fraction division *in routine and non-routine problem solving situations* [emphasis added]. Pedagogy wise, I always support students' motivation through engaging students in small group work and classroom discussion.

Responding to the question 2, Irina used the part-to-whole interpretation of fraction division whereas Marina offered two different but somehow related interpretations of the meaning for division of fractions.

IRINA

Well… there are two main problems in school arithmetic: finding a part of a whole and finding a whole knowing its' part. Said that, the meaning of *fraction division is finding a whole knowing its' part* [emphasis added].

MARINA

From my perspective… There are at least two meanings for division of fractions. First meaning is based on the interpretation of *division as operation opposite to multiplication* [emphasis added]. In other words… to divide a fraction A by a fraction B means to find a fraction C such as A=B x C. For example, 5/6 divided by 1/6 means that there is a fraction C such as 5/6 = 1/6 x C. Or 5/6 ÷ 1/6 =5. On the other hand, *division is kind of "sorting"* [emphasis added]. For instance,1/2 = 1/4 + 1/4 = 1/4 x 2 meaning that 1/4 goes 2 times into 1/2 whereas 1/2 goes 1/2 times into 1/4. Hope it makes sense… [smiles]

Teachers' Responses to Content Knowledge Questions

Irina's response to the question 3 further confirmed that she has well-established procedural knowledge of the fraction division rule.

IRINA: The rule of fraction division is reduced to the rule of fraction multiplication. Therefore, you need to multiply the *first fraction* [emphasis added] by the reciprocal of *the second one* [emphasis added].
INT: What do you mean by reduced to fraction multiplication?
IRINA: As students say, *cross multiply* [emphasis added] fractions.

Marina's response to the question 3 is depicted below. Surprisingly, Marina used a similar conclusion connecting fraction division to multiplication as Irina did in her response to the same question.

MARINA: In order to divide fractions, you need to multiply *dividend* [emphasis added] by the reciprocal of the *divider* [emphasis added]. For example, $\frac{15}{4} \div \frac{3}{10} = \frac{15}{4} \times \frac{10}{3} = \frac{25}{2}$ (writes on a scratch paper). Generally speaking, fraction division "boils down" to multiplication.

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Irina's response to the question 4 consisted of the solution only (she wrote it on a scratch paper) without any commentary: $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2} = 3.5$.

Unlike Irina, Marina supported her response to the question 4 with step-by-step comments.

MARINA: First, we convert given mixed fraction 1 3/4 to common one 7/4. Notice, here the numerator is larger than denominator. Then, we replace division by multiplication *reversing the divider* [emphasis added]. Hence, $1\frac{3}{4} \div \frac{1}{2} = \frac{7}{4} \times \frac{2}{1} = \frac{7}{2}$ (writes on a scratch paper).

With regard to the question 5, it took a while for Irina to think about the problem. Then Irina clarified whether she can write down the word problem she came up with on a scratch paper.

IRINA: May I write down the problem on the paper?
INT: Yes, of course.
IRINA (writes on a scratch paper): Area of a rectangle is equal to 1 3/4 cm^2, its length is equal to 1/2 cm. Find width of the rectangle.

Irina was consistent in applying the part-to-whole interpretation in her response, more specifically – using "the missing factor problem" as a meaning for division of fractions (Flores, 2002).

After some thinking, Marina offered the following word problem in her response to the question 5.

MARINA: Here is my word problem: an automated machine packs butter in 1/2 kg bricks. How many bricks one can make out of 1 3/4 kg of butter? May I draw a picture!?
INT: Sure.
MARINA: (draws a picture on a scratch paper)

We noticed that Marina herself offered drawing a picture to illustrate her word problem.

In Irina's answer to the question 6, she basically repeated her response to the question 2.

IRINA: The given statement is not correct. In order to divide fractions you need to multiply the *first one* [emphasis added] by a reciprocal of the *second one* [emphasis added].

Question 6 was the most challenging to Marina. Nonetheless, she confessed that she liked it.

MARINA: I like this question. It makes me think.
INT: Good.
MARINA: Alright, notice that in order to solve this problem *ac/bd* should be equal to *ad/bc*. Right?
INT: So…
MARINA: Therefore, $c/d = d/c$. This is possible only if $c = d$.

Students' Responses to Fraction Division Questions

At the stage of student problem solving, we asked groups of 6th grade students of participating teachers (Irina's group had n=29 students and Marina's group n=26) to solve a subset of questions corresponding to different cognitive types of content knowledge similar to those presented to teachers: 1) Divide two given fractions. 2) Construct a word problem for fraction division from the previous question. 3) Is the following statement $\frac{a}{b} \div \frac{c}{d} = \frac{ac}{bd}$ *(a, b, c,* and *d* are positive integers) ever true?

Questions were presented to students after they completed a chapter on the basic operations with fractions. Student wrote down their responses on a paper and student work was collected for further analysis. The number of correct students' responses on questions 1-3 along with the chi-square statistic comparing student performance between groups on each question is presented in the table 1 below.

Table 1. Chi-Square Analysis' Results of Student Responses on the Fraction Division Questions

	Question 1	Question 2	Question 3
Irina's Group (n=29)	28	12	1
Marina's Group (n=26)	25	15	10
Chi-square and *p*-value (df=1)	χ^2=.41 *p=.522*	χ^2=.88 *p=.348*	χ^2=8.43 *p=.0037*

As we mentioned before, questions selected for the student problem solving session reflected different cognitive types of content knowledge on the topic of division of fractions.

Discussion: Does What a Teacher Knows Matter?

The most important finding of the study was the evidence collected and analyzed in support of the first research question: what a teacher knows matters in regard to his/her students' topic-specific knowledge. As we expected based on similar teachers' scores on cognitive type 1 items (measuring knowledge of facts and procedures), there was no difference observed between student performances in two groups on the procedural question 1. There was some difference, not significant though, observed on the question 2 measuring knowledge of concepts and connections (in favor of students in Marina's group). The most evident difference between student performance in two groups was observed on the question 3 (measuring knowledge of models and generalizations) was statistically significant (χ^2*=8.43, p=.0037).* We were surprised by the partially-correct student's response from Irina's group considering the fact that Irina herself was not able to correctly solve the question. Overall students' responses were reflective of their teachers' knowledge: student performance in Marina's group was stronger than in Irina's group, particularly in solving questions 2 and 3 with difference in responses to question 3 being statistically significant.

The data collected and analyzed to respond to the second research question - What is the nature of topic-specific connections between teacher and student knowledge? – revealed that teacher's mastery of cognitive types of content knowledge is associated with the students' topic-specific knowledge. Thus, findings of this study contribute to the body of research claiming that teacher content knowledge is critical for student learning (Hill, Rowan, & Ball, 2005; Baumert et al., 2010). Teacher interviews and students' problem solving helped us to closely look at the nature of the relationship between teacher knowledge and student performance.

We are cognizant that the study had its limitations such as teacher sample size, multiple-choice format of the teacher content knowledge survey, to name a few. Following on the discussion about complexities of assessing teacher knowledge (Schoenfeld, 2007), we are aware of the limitations of the multiple-choice format in test construction and assessment of teacher knowledge (p. 201). Therefore, we included the qualitative phase of the study to zoom further into teacher knowledge and understanding. We are also cognizant that classroom observations could be another source of data in this study. However, to explicitly address the research questions we purposefully focused the study

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on the link "teacher knowledge-student performance" in the topic-specific context. Considering these limitations, we are sensitive enough to not overgeneralize the results obtained in the study. The major findings of this study open an opportunity to discuss the importance of different cognitive types of the topic-specific teacher knowledge and its potential impact on student knowledge and learning.

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