LEARNING INTEGER ADDITION: IS LATER BETTER?

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We investigate thirty-three second and fifth-grade students' solution strategies on integer addition problems before and after analyzing contrasting cases with integer addition and participating in a lesson on integers. The students took a pretest, participated in two small group sessions and a short lesson, and took a posttest. Even though the results reveal significant gains for both grades from pretest to posttest, second graders gained significantly higher than fifth graders. In this paper, we explore students' treatment of the negative sign and describe this gain difference.

Keywords: Number Concepts and Operations; Elementary School Education; Cognition

When students encounter new concepts, they try to apply their prior knowledge in an effort to make sense of the new information. Consequently, children's negative integer knowledge builds upon their whole number understanding (Bofferding, 2014). The transition from working with whole number concepts to interpreting new number classes appropriately requires substantial time and considerable conceptual change (Vosniadou, Vamvakoussi, & Skopeliti, 2008). However, there is a noteworthy time gap between when children learn about whole number concepts and the introduction of negative numbers. While whole numbers are introduced at an early age, negative numbers and operations are currently not taught until sixth and seventh grade (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). Introducing negative numbers in upper elementary and middle school grades could serve as a barrier to student learning. As a result of this large gap, students may interpret negative numbers in various ways given their whole number knowledge, which may influence their solution strategies in problems involving negative numbers. For instance, students may treat the negative sign as a subtraction sign (e.g., solving -4 + 7 as 7 – 4) (Bofferding, 2010). In fact, Murray (1985) found that students continued to explain that you could not take larger numbers away from smaller ones (e.g., 5 – 8) even though they could solve other negative integer problems.

Even first graders can move toward having more formal mental models of integers, integer order, and integer values by engaging in instructional activities that help them focus on the meaning of the negative sign and evaluate the numbers in relation to each other (Bofferding, 2014). Similarly, instructional activities can help students make specific connections between the results of operating with negative and positive numbers. One way to encourage these connections is via the analysis of two carefully chosen, contrasting integer problems shown as worked examples. Studies have identified contrasting cases as a powerful instructional tool that has promising results in students' developmental knowledge in the mathematics classroom (e.g., Rittle-Johnson & Star, 2007; 2009; 2011). As one of the promising ways to help students conceptualize negative number operations, this method can help students leverage their prior knowledge of whole number addition to making sense of negative number addition problem types.

However, it is not clear to what extent analyzing contrasting cases of integer addition problems could benefit younger students compared to older students. Although younger elementary students can reason productively about integers, they may need more intense experiences in order to facilitate their conceptual change. On the other hand, older students may be less willing to modify their conceptions. In this paper, we analyze the performance of younger and older elementary students before and after they compare sets of integer addition problems. Further, we explore how their interpretations of negative number addition change.

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Specifically, we ask the following research questions:

1. How do second and fifth-grade students perform in solving integer addition problems before and after analyzing contrasting cases involving negative numbers?
2. How do students' interpretations of the negative sign and addition operations change before and after analyzing the contrasts?

Theoretical Framework

Interpretation of the Minus Sign

Students engaged in working with algebraic equations experience difficulty when manipulating negative numbers due to the different perceived meanings of minus signs (Vlassis, 2004). The three primary meanings of the minus sign are unary, binary, and symmetric and also play a role in students' interpretations of integer addition and subtraction problems. The binary meaning of the minus sign corresponds to the subtraction operation (Vlassis, 2004), which students with a whole number mental model will interpret to mean getting less or smaller (Vosniadou, Vamvakoussi, & Skopeliti, 2008). Students with only a binary understanding of the minus sign might ignore negatives or treat them as a subtraction sign (Bofferding, 2010). For example, -8 + 6 can be turned into a subtraction problem as 6 – 8 or 8 – 6 and might be answered incorrectly or correctly. The symmetric, or opposite, meaning of the minus sign indicates an operation of multiplying by -1. In other words, the symmetric meaning illuminates the opposite positions of negative and positive numbers (Vlassis, 2004). Students rely on the symmetric meaning when they add integers as if they were positive and make the answer negative (Bofferding, 2010). For instance, -2 + 5 might be solved as 2 + 5, with students adding the negative sign at the end, and answering -7. Finally, the unary meaning of the minus sign involves seeing the negative sign as attached to a numeral, a negative number (Vlassis, 2004). Students with a strong unary understanding will often start at a negative number and count towards or away from the negative direction (Bofferding, 2010).

Conceptual Change

In prior research, students' difficulties in interpreting minus signs were primarily reflected in situations involving successive signs in arithmetic operations. From a conceptual change lens, this is unsurprising. Until students encounter negative numbers, they would not see two successive minus signs. Their unawareness of the multiple roles of the minus sign resulted in the mistreatment of the signs and signaled partial conceptions (Vlassis, 2008). In terms of conceptual change, students were trying to make sense of new information in light of their prior knowledge. During this process, they likely formed synthetic mental models (Vosniadou & Brewer, 1992), conceptions that blend their prior understanding with new hypotheses about the meaning of the new signs. Different studies have explored students' various ways of reasoning in their arithmetic solution strategies. They framed how students' understanding of the multiple meanings of the minus sign correspond to their integer arithmetic solutions (e.g., Bofferding, 2010, Lamb et al., 2012; Murray, 1985, Vlassis, 2008). Lamb, Bishop, Philipp, Whitacre, and Schappelle (2016) claimed that no single best model exists that could be applied in students' ways of reasoning. We build on this literature by using a conceptual change lens to investigate how students' conceptions of the minus sign – and consequently their strategies – change as they explore contrasting integer addition problems.

Methods

Participants and Settings

Both second and fifth-grade students engaged in this study. We chose to work with second graders because they represent students who typically have not had formal instruction in negative numbers but have whole number understanding. We chose fifth graders because they often have heard about negative numbers and would learn about them formally in the following school year. We recruited fifth-grade students from two public elementary schools in a rural Midwestern school district where 32% of students were classified as English-language learners (ELLs) and 75% qualified for free or reduced-price lunch. Based on their pretest scores, 17 of the 32 fifth graders who returned permission slips were selected for this analysis as they had not reached the ceiling in terms of their order and value integer mental models (formal mental model) (Bofferding, 2014). It was important not to include students with formal mental models so that we could compare their growth to the second graders' growth. Because we had already worked with second graders at the schools where we recruited the fifth graders, we recruited second graders from one classroom at a different school. The public elementary school was in a small city within a Midwestern school district where 14% of students were classified as ELLs and 57% qualified for free or reduced-price lunch. Overall, 16 second-grade students from the recruited class participated.

Design and Materials

Students from both grades completed a pretest, participated in two small-group sessions, engaged in a whole-class lesson, and finally took a posttest.

Pretest and posttest. Test items were designed to evaluate students' knowledge related to integers and were identical on both pretest and posttest. Both tests took approximately 30 minutes to finish for students in both grades. Questions focused on ordering integers, integer value comparisons, integer addition, directed magnitude comparisons, and transfer problems with addition. After both tests, we interviewed some of the students to learn more about their answers and strategies and to clarify students' insights.

Small-group sessions. During the sessions, students analyzed sets of contrasting integer addition problems in groups of two to four. They analyzed worked examples of correct and incorrect solutions to integer addition problems within four different illustration contexts including (a) a gingerbread boy starting at zero and moving on a number path situated on a hill (translation model [see Wessman-Enzinger & Mooney, 2014]), (b) an ant starting at the first number in the addition problem and moving below and above ground next to a vertical number line, (c) a chip model (counterbalance model [see Wessman-Enzinger & Mooney, 2014]), and (d) a folding number line (see Tsang, Blair, Bofferding, & Schwartz, 2015). Each session took approximately 20 minutes.

In their first session, students analyzed integer addition problems with two positive numbers in comparison with adding a positive number to a negative number (e.g., 3 + 5 = 8 versus -3 + 5 = 2). This contrast can help emphasize that regardless of the starting number, adding a positive number always corresponds to a movement towards the positive direction (or up). At the end of the session, students solved eight integer addition problems, four where they added two positive numbers and four where they added positive numbers to negative numbers.

In the second session, students compared addition problems with two negative numbers to addition problems with adding a negative number to a positive number (e.g., -2 + -5 = -7 versus 2 + -5 = -3). This comparison can help students to realize that adding a negative number to either a positive or a negative number always results in a movement in the negative direction (or down). At the end of the session, students solved eight integer addition problems, four where they added two negative numbers and four where they added negative numbers to positive numbers. The collective

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16 problems solved at the end of their both sessions composed what we call a midtest and helped us understand how they would answer the problems immediately after making comparisons with them.

Whole-class instruction. The whole-class lesson, led by one of the researchers and given separately to fifth and second graders, had two parts: interactive instruction and a game. Instruction began by helping students think about moving on an integer number path to solving integer addition problems. Therefore, adding a positive number represented moving up on the number path (going more in the positive direction), and adding a negative number indicated a downward movement on the number path (moving more in the negative direction). Students helped explain how they would move the gingerbread boy along the number path to solve example integer addition problems. As students' understanding built upon the directional movements of adding two integers, the researcher-teacher presented an additive inverse problem (e.g., -2 + 2) to introduce the "zero pair" concept. As demonstrated, zero pairs occurred when the two movements added on the number path ended at zero. Students later engaged in a game where they had to make zero pairs using positive and negative one cards.

Analysis

To address the first research question about students' performance, we ran a repeated measures ANOVA grouped by grade level on the 17 arithmetic items presented to students on the pretest and posttest. Next, in order to determine students' progress before, during, and after the study more thoroughly, we determined students' average percent correct on six common problems (-9 + 2, -8 + 8, 6 + -8, 9 + -9, -1 + -7, and -2 + -2) that were given to students on the pretest, midtest, and posttest. Students' responses to items of a similar problem type (e.g., -9 + 2 and -8 + 8) provided some insight into their interpretations of the minus signs and helped paint a picture of their conceptual change. We sought further clarification of the meanings students attributed to the minus signs through the interview data.

Results

Overview of Students' Arithmetic Gains

The results of the repeated measures ANOVA indicate a significant main effect of test, $F(1, 31) = 62.32$, $p<.001$. Students from both grades gained significantly from pretest to posttest. Table 1 shows that even though the fifth-grade students' average score on the pretest arithmetic items was higher than second graders, they scored a lower average on the posttest arithmetic items than second-grade students. In fact, based on the repeated measures ANOVA, the difference between the grade level gains is significant, with second graders making greater gains on average than the fifth graders, $F(1, 31) = 4.51$, $p<.001$.

Table 1: Students' Mean Scores (And Average Percent Correct) On Integer Addition Problems

Grade Level /Tests	Pre-test ($M \pm SD$) (average percent correct) (17 items)	Mid-test ($M \pm SD$) (average percent correct) (12 items)	Post-test ($M \pm SD$) (average percent correct) (17 items)
Second Grade (N = 16)	5.1 ± 6.2 30%	7 ± 3.8 58%	13.5 ± 2.8 79%
Fifth Grade (n = 17)	7 ± 3.9 41%	6.9 ± 2.8 58%	11.8 ± 3.8 69%

Students' Interpretation of the Minus Sign

We explored how student's interpretation of the minus sign might support the significant difference between the two grade level gains. We, therefore, investigated students' responses to six integer addition problems common to the three tests. Students' percentage correct on each problem was calculated for each group (see Table 2). On average, fifth-grade students performed better than second graders on four of the six common integer addition problems (67%) on the pretest, while second graders scored higher on all posttest common items (with one tie).

Table 2: Students' Average Percent Correct on Common Integer Addition Problems

Common integer addition		-9 + 2	-8 + 8	9 + -9	6 + -8	-1 + -7	-2 + -2
Second Grade	Pretest	38%	38%	19%	19%	31%	25%
	Mid test	44%	75%	44%	56%	81%	75%
	Post test	75%	94%	94%	69%	88%	69%
Fifth Grade	Pretest	12%	29%	41%	29%	71%	59%
	Mid test	35%	41%	47%	29%	88%	82%
	Post test	65%	82%	82%	65%	88%	47%

Note: Shaded cells indicate questions where that grade level did better than the other.

Fifth graders' pretest. Fifth graders frequently gave answers consistent with a strategy of adding a negative sign to the sum of two integers' absolute values. If students only used this strategy on the problems where it would lead to correct answers, we would expect them to use it roughly 29% of the time. Among all of the 17 integer addition problems, they had these types of answers for 46% of the problems on average. Because of this, not only did fifth graders have a higher average performance on -1 + -7 and -2 + -2 compared to second graders on the pretest, but their average scores were the highest on these compared to the other problems (9 + -9, 6 + -8, -9 + 2, and -8 + 8) where that strategy would not work. A fifth-grader, Jennifer, in describing her answer said, "They're both negative[s], so it is going to be [a] negative."

Another consistent strategy applied by fifth graders was treating the negative sign as a subtraction sign, the binary meaning of the minus sign. Across the problems fifth graders provided answers consistent with subtracting one number from the other and getting positive answers for 19% of the 17 problems on average. In six cases, students provided answers illustrating the common misconception that you always subtract a smaller number from a bigger number (e.g., solving 4 + -5 = 1). Interpreting the minus sign as a subtraction sign could help them in solving 5 + -2 = 3, which 7 fifth graders did. However, only one of them also solved 7 + -3 = 4, because they were more inclined to add the numbers. Additionally, their tendency to subtract in the treatment of the negative numbers could not help them to correctly answer problems such as -9 + 2, for which counting up or down from the negative requires unary meaning of the minus sign.

Second graders' pretest. Unlike the fifth graders, second graders did not often provide answers consistent with adding a negative sign to the sum of two integers' absolute values in the problems.

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Their answers only reflected this strategy on 9% of the 17 problems on average. Also, their answers were consistent with subtracting to get a positive number only 9% of the time on average as well, which seemed more likely when the negative was first, given their better performance on -8 + 8 versus 9 + -9. When solving 6 + -8, lower number of possible answers on using subtraction operation with integers (e.g., 6 + -8 = 2 or -2 or 0) suggest they applied the binary meaning of the minus sign less often. When solving 6 + -8, Sandra who answered two demonstrated a conception of starting with the larger absolute value, "I did eight, seven, six, five, four, three, two and that is how I got it, and negative, if negative plus a positive number, you go backward to the negative." However, Edward, a second grader used the binary meaning to get a negative answer, reasoning, "Because six minus eight is negative two [be]cause it has to go below zero," also demonstrating an awareness of the unary meaning given to his answer.

Starting at the negative number and counting up or down the other number (e.g., -9 + 2 counting up -8 and -7 or counting down -10 and -11) leverages the unary meaning of the minus sign. Second-grade students' slightly higher average scores on -9 + 2 are consistent with them interpreting negative numbers as corresponding to a point on the number line, the unary meaning of the minus sign. Tara explained her solution for -9 + 2, "So I counted on my fingers negative eight, negative seven, and it is negative seven." Timmy said, "If it is a negative number plus [a] regular number, it would be less of the negatives. So, negative nine plus two equals negative seven."

Midtest. Students' gains for the six problems on the midtest show their development in interpreting multiple meanings of the minus sign. Second graders still averaged higher for -9 + 2 and -8 + 8 compared to fifth graders. Interestingly, even though fifth graders started higher in 9 + -9 and 6 + -8 on the pretest, second graders had a higher average correct for 6 + -8 on the midtest. Fifth-grade students' responses appeared to be influenced by their prior knowledge and treating the minus sign with a symmetric meaning (answering -14). In fact, fifth-grade students' consistent responses over 12 integer addition problems illustrate they applied the symmetric meaning on average for 33% of problems; whereas, second graders did so for 26% on average.

Second graders' posttest**.** Both grades' highest average was in -8 + 8 and 9 + -9, which demonstrates the significance of the zero pairs introduction in the lesson. Tara said, "I know that negative nine plus nine equals zero because negative means below zero and it is nine below zero, so it would be just zero. [Be]cause it is nine more than the negative." Timmy described a zero pair problem, "If you have a negative number plus a normal number, it would be zero. If you have normal nine plus a negative nine, it would be zero."

Surprisingly, students' average scores were lower on -2 + -2 after the midtest. Since zero was the most usual incorrect answer, one explanation could be that they identified the two same numbers and a negative sign in the problem, associated it with the zero pairs problem, and answered zero. Another second-grade student, Anthony, said, "So, I looked at it and I thought it's zero for a second. But I thought that would be negative two plus two. So, I thought negative two plus negative two would be negative four." Based on their responses, they used the symmetric meaning 15% of the time on average.

Second graders' responses to 6 + -8 did not result in -14 anymore, which reveals that they abandoned the symmetric meaning of the minus sign for this problem and treated the negative sign in a way that could provide the correct answer (unary meaning starting at -8 or binary meaning instead). Tara's unary interpretation is exemplified, "So, I know that since that is a lower number [referring to 6] than this one [referring to 8], then the answer would be still in the negatives. So, I started at negative eight and went up six." Another student, Kathryn, who interpreted the negative sign with a binary meaning said, "I did six and took away eight." The researcher asked, "So, when you ran out of six, what happened after that?" She continued, "I got into the negatives." Also, students explained their responses with zero pairs. Anthony mentioned zero pairs when solving 6 + -8, "I noticed six

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plus eight is 14 but six plus negative eight, when we did the zero problems [zero pairs] if it was six plus negative six, he [gingerbread man] would be going up and then go down. So, I noticed that so I did it." Further, when they answered -9 + 2 = -11, it was not necessarily because they focused on a symmetric meaning of the minus sign. Anthony started at -9 and answered -11 stating, "So, it would be nine plus whatever. It is still be going up but if you do [plus] a negative, it would have to go down." This student started to understand integer addition as involving directional movements that can be both upward or downward but was still trying to make sense of how the direction interacted with the numbers.

Fifth graders' posttest. Fifth graders scored slightly lower on -8 + 8 and 9 + -9, often answering -16 or -18 respectively, either due to overuse of the symmetric meaning or using the unary meaning and moving in the wrong direction. Further, their responses to the problem 6 + -8 involved -14, unlike second graders. Similar to second graders, they scored lower on -2 + -2, often associated with overgeneralizing the zero pairs concept. Though both grades averaged the same in -1 + -7, fifth graders did not improve from the midtest. Overall, fifth graders' responses highlighted the symmetric meaning for an average of 30% of 17 integer problems. This compared to 15% average among second graders provides initial evidence that fifth-grade students' stronger prior conception limited their ability to make use of the contrasting cases and change their thinking based on the instruction compared to the second graders.

Discussion

Our results provide insights into students' interpretations of minus signs and solution strategies for integer addition problems before and after making comparisons between them. Similar to the results of the fifth graders presented here, Murray (1985) found that adding two negatives was easy for upper elementary students compared to other integer addition problems. The results here suggest that this may be due to older students' tendency to focus on the symmetric meaning of the minus sign. Given this, one interpretation of our data is that if students naturally focus on the symmetric meaning and this interpretation persists without challenge (as could be the case with the fifth graders who relied on the symmetric interpretation heavily), they have more difficulty shifting away from it. Therefore, the second graders may have been more open to changing their conceptions about numbers, minus signs, and operations because they had not had much prior opportunity to think about them and establish strong initial conceptions. The data shows fifth graders' stronger prior conception about the negative sign influenced their future reasoning; however, second graders changed their interpretation of the minus sign throughout the study. Their willingness to apply the new information in their solution strategies helped them to improve higher.

Students experience a barrier to learning negative numbers when they have time to establish strong preconceptions without the opportunity to address them, a problem exacerbated by the location of integer standards in upper grades. In order to overcome students' difficulties in conceptualizing the negative numbers, we should provide opportunities for them to develop their integer number sense knowledge from at least second grade. Contrasting problem types (Rittle-Johnson & Star, 2009) using different conceptual models (Tsang, Blair, Bofferding, & Schwartz, 2015; Wessman-Enzinger & Mooney, 2014) helped students to improve their understanding. By capturing students' reasoning, discoveries, and interpretation through these activities, we can provide additional tasks more in-line with their needs to facilitate their learning and exploration process (Behrend & Mohs, 2005/6) throughout the following grade levels.

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