EXPLORING THE STRUCTURE OF EQUIVALENCE ITEMS IN AN ASSESSMENT OF ELEMENTARY GRADES

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This study is focused on the structure of equivalence problem to probe the evolution from operational to relational view of students' understanding of equals sign. We propose a modified construct map which incorporates the intermediate levels in such a transition which were previously ignored. Our findings suggest that the structure of number sentences (place value and the position of answer box) has undeniably significant role in developing students' conception of equivalence. In addition, the designed and validated example presented here could potentially serve as a tool for better assessment of understanding of equivalence.

Keywords: Number Concepts and Operations, Algebra and Algebraic Thinking, Elementary School Education, Assessment and Evaluation.

Background / Overview

There is a consistent and increasing focus by educational researchers on the development of elementary grade algebraic reasoning, specifically in regard to the use of open number sentences (e.g. 4 + ☐ = 5 + 7) (Falkner, Levi, & Carpenter, 1999; McNeil & Alibali, 2004; Molina & Ambrose, 2008; McNeil, Fyfe, Petersen, Dunwiddie & Berletic-Shipley, 2011). Such open number sentences often include expressions on both sides of the equation and are often introduced as arithmetical equations where students are tasked to find the unknown or missing number in place of a blank or empty box. This affords students the opportunity to explore the underlying structure of an arithmetical equation and improve their understanding of the meaning of symbols and operations. Various researchers (Mc Neil & Alibali, 2004; McNeil et al., 2006; Sherman & Bisanz, 2009; Powell, 2014) suggest that many elementary students are introduced to only traditional arithmetic equations (i.e., a + b = c). These studies suggest that such *operations equals answer* type equations encourage the operational view and may hinder students' development of a relational view of the equals sign.

In an operational view of the equal signs students carry the notion that the equals sign means *makes*, *produces the answer*, *find the total*, or as an indication *to do something* such as computation (Behr, Erlwanger & Nichols, 1976; Kieran, 1981; Seo & Ginsburg, 2003; Knuth, Stephens, McNeil & Alibali, 2006; McNeil et al., 2006; Jacobs et al, 2007). Students holding a relational view consider the equals sign as a mathematical symbol which represents the sameness of the expressions or quantities on either side of an equation (Kieran, 1981; Baroody & Ginsburg, 1983; Falkner et al., 1999; Alibali, Knuth, Hattikudur, McNeil, & Stephens, 2007; Blanton, Levi, Crites, & Dougherty, 2011). A vast majority of research suggests that a relational understanding of equivalence is a first step towards early algebraization (Falkner et al., 1999; Carpenter & Levi, 2000; Blanton & Kaput, 2005; Jacobs et al., 2007; Byrd, McNeil, Chesney, & Matthews, 2015). Students holding a more operational view tend not to develop the conceptual understanding of arithmetic and other more advanced mathematics such as Algebra (Kiren, 1981; Knuth et al., 2006; Jones, Inglis, Gilmore, & Dowens, 2012; Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). Thus, the traditional way of introducing equal sign ($a + b = c$) tends to focus predominately on step-by-step computation to find the answer rather. By contrast, explorations of the underlying structure or number relations between and within the expressions appears to require alternative forms of equations.

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The research to date generally interprets the structure of equations as open number sentences ($a + b = c + □$) vs traditional arithmetic equations ($a + b = □$). However, it is unclear whether and to what degree other mathematical structure within such equations interacts with children's conceptions of equivalence. Namely, place-value is an aspect of mathematical structure that children engage concurrently with their developing conceptions of equivalence. The aim of this study is to investigate the role of number structure, specifically place-value with whole numbers, in students' conception of mathematical equivalence. To facilitate this purpose, we examined third grade students' responses to a conceptions of equivalence assessment using psychometric analysis.

Theoretical Framework

Numeric Structure in Number Sentences

Traditionally, most studies on equivalence define two basic categories of students' conceptions of the equals sign: operational and relational. More recently, Rittle-Johnson et al. (2011) elaborated on this dichotomy (operational vs relational) and expanded it into four levels ranging from rigid operational to comparative relational. Students at Level 1, the rigid operational level, are expected to successfully solve the traditional format (i.e., a + b = c). Students at Level 2, the flexible operational level, maintain an operational view of the equal sign with some flexibility to correctly solve and accept the atypical or "backwards" equations (i.e., c = a + b and a = a) as valid. At Level 3, the basic relational level, children successfully solve, evaluate, and encode equation structures with operations on both sides of the equal sign (such as a + b = c + d or a + b - c = d + e). Finally, children identified at Level 4, the comparative relational level, show a more nuanced understanding of the equals sign. These students can correctly solve and evaluate equations by comparing the expressions on both sides of equal sign. Students at this level use compensatory strategies. For example, in solving 37 + 24 = 36 + □, such students may recognize that 36 is 1 less than 37 and use this knowledge to determine that the unknown number must be one more than 24 (Carpenter et al. 2003).

More recently, Singh & Kosko (2015, 2016a) observed other possible levels in the continuum of conception of equivalence. Therefore, we argue that further modifications to the field's models for the ways students consider equivalence are needed. Specifically, students who can successfully solve $a + b = c + d$ types of equations are currently evaluated as holding a *basic relational* conception of the equals sign. However, Rittle-Johnson et al (2011) suggest that the construct is continuous, which allows for the possibility of other sub-constructs between consecutive levels.

Singh and Kosko (2015) conducted a teaching experiment with a variety of equivalence problems and found that some students demonstrated a *pseudo-relational conception* (PRC) of equivalence. Specifically, student with a PRC can solve problems of the form $a + b = c + □$ when the numbers involved allow them to regroup 10's and 1's in an obvious manner. For example, in problems like 34 + 25 = 50 + □**,** such students first regroup 10's from 34 and 25 (30 + 20 = 50), which is visually available on right hand side. Thus, these students are then able to add the ones (4 + 5 = 9) to find the solution. At first glance, it may seem like these students hold a relational view of equivalence. However, when closely comparing their strategies in other equations of the same structure but different numeric structure, it was apparent that the place-value structure of such equations allowed students to use different strategies than working with other $a + b = c + □$ types of equations. For example, in solving 15 + 24 = 20 + □, a student successful with the prior example failed to provide the correct solution when using their regrouping of 10's and 1's strategy. This suggests that the mathematical structure numbers in an equation, such as that of place-value, plays an important role in students' conceptions of equivalence. This was verified in another study by Singh & Kosko (2016a) in which the authors found that some students can successfully solve problems like 4 + 5 + 8 = □ + 8 by finding the answer 9 (i.e., 4 + 5), whereas the same students demonstrated a different conception

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in solving very similar problem like 7 + 6 + 4 = 7 + □☐. These students used an *adding all then subtracting* strategy to solve the problem 7 + 6 + 4 = 7 + □☐. Specifically, students first added 7 + 6 + 4 and then the 7 on the right-hand side to get 24. They then subtracted 7 to obtain the answer 17. Singh and Kosko (2016a) suggest that the use of this strategy may have more to do with the position of the missing value box than students' operational or relational view of the equal sign.

Conceptual Model for Conception of Equivalence

To define the way an equivalence problem is presented to students, different researchers have used different terminologies, or the same terminology with different meanings. Molina & Ambrose (2008) used the term structure in reference to the structure of mathematics operations. Later, Molina, Castro, & Castro (2009) used the term structure in the same sense as used by Kieran (1989), describing the surface structure of arithmetic and algebraic expressions. Recently. Stephens et al. (2013) used the term "equation structure" as opposed to focusing on probing their computational fluency. This study uses a broad definition of structure of number sentences which includes an emphasis on place value and the position of the missing value box(es). Thus, the definition differs significantly from the meaning of structure used in prior studies.

Using our definition of structure of number sentence, and based both on our previous findings (Singh & Kosko, 2015; Singh & Kosko, 2016a), and ongoing work with elementary students, we suggest six levels of conception of equivalence along a continuum (Figure 1). The construct map in Figure 1 includes levels from basic operational (least sophisticated) to full relational (most sophisticated). A student at the basic operational level can successfully solve traditional number sentences (a + b = c) with various positions of unknown or box such as 6 + 7 = ☐ or 5 + ☐ = 9, while students at the flexible operational level can successfully solve less typical number sentences (e.g. 19 = ☐ + 3; 24 = 10 + ☐; ☐ = 5 + 7). However, both types of conceptions include student strategies that rely on an operational view of equals.

We argue that the transition from flexible operational to basic relational is not always smooth and is accompanied by the existence of *pseudo-relational level*. Students at this level are able to solve number sentences which have operations on both sides (such as a + b = c + d), but only in cases where the number sentences can be solved by using regrouping of ones and tens addends.

Similar to the pseudo-relational level, students at the basic relational level can successfully solve number sentences which have operations on both sides (a + b = c + d) with the position of the box directly after or before the equals sign. Such students can confirm the sameness of expressions on both sides of the equals sign through computation.

Prior to a full relational conception, we suggest some students demonstrate what we describe as an advanced basic relational level. This level is characterized by students who can successfully solving number sentences with operations on both sides (a + b = c + d), but such students may use either computation or compensatory strategies. Finally, at the full relational level, students can successfully solve number sentences which have operations on both sides (such as a + b = c + d) and two unknowns either both on same side or one on each side of equal sign by relying predominately on compensatory strategies.

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Direction of Increasing sophistication in "Mathematical Equivalence"

Students	Responses to Items
Full Relational	Anticipates successful use of compensatory strategies to solve item types, e.g. □ + 28 = 46 + ○;
Advance Basic Relational (□ at the end of RHS of =)	Anticipates confirmation of the sameness of expressions on both sides of equal sign either using computation or compensatory strategies in items e.g. 15 + 27 = 18 + □
Basic Relational	Anticipates confirmation of the sameness of expressions on both sides of equal sign with computation in items e.g. 13 + □ = 24 + 8; 27+ 16 = □ + 35.
Pseudo-relational (with regrouping	Anticipates to solve specific items e.g. 22 + 15 = 30 + □ by regrouping of ones and tens of addends. Students get first addend in right side of equal sign by regrouping tens of both addends of left side and find the unknown by simply regrouping ones.
Flexible-Operational	Anticipate to solve atypical number sentences such as operations on right e.g. 19 = □ + 3; 24 = 10 + □; □ = 5 + 7 or 12 = □.
Basic Operational	Only successfully solve traditional number sentences (a + b = c) with various positions of unknown or box such as 6 + 7 = □ or 5 + □ = 9.

Direction of Decreasing Sophistication in "Mathematical Equivalence"

Figure 1. Construct map for knowledge of equivalence.

The different levels of students' conception of equivalence are included in the construct map shown in figure 1. As discussed above, prior studies (Singh & Kosko, 2015,2016a) describe students' transition from operational to relational conception of equivalence is not always smooth. Rather, students' holding an operational view may do things that resemble, but do not comprise, a relational view of equivalence (i.e. *pseudo-relational*) when solving some specific types of equations. The construct map in figure 1 illustrates that as students move in a continuum they should engage in different levels of conception of equivalence.

Methods

Sample

Data were collected in Fall 2016 from 157 third grade students (49.7% male; 50.3% female) in a suburban school district in a Midwestern U.S. state. Students were enrolled in one of eight classrooms across four schools in the district. The district includes a predominately white student population (74%), with a significant portion of economically disadvantaged students (40%).

Test Development and Item Design:

Our previous work on equivalence indicates some observable gaps in Rittle-Johnson's established framework (Singh & Kosko, 2015, 2016a). In order to address these gaps, we designed a new assessment utilizing the aforementioned construct map. An initial version of the assessment was piloted with fourth and fifth grade students (n = 157) with 33 items. The overall reliability (Cronbach's alpha = 0.92) of the initial test was sufficient. However, the item discrimination for several items was not sufficient. Also, the infit mean square statistics for more than a quarter of items indicated that the initial construct map lacked unidimentionality. To improve these shortcomings in item design we revised some items for better fit. First, items with insufficient fit statistics were removed or revised. Many of these items illustrated what may be different constructs related to but not identifiable as conception of equivalence. Next, true/false items were removed since these items were found to have significant structural differences than missing-value addends, and such items also tended to have too low of difficulty to provide sufficient information for the assessment. The revised instrument included 22 items across six sub-constructs along a continuum (thus, these subconstructs are theorized to be hierarchical). Figure 1 shows the revised construct map with example items for the six subconstructs.

The revised instrument was used in the analysis of Fall 2016 data. Raw data was inputted into digital files before dichotomously coding student responses (0 = incorrect answer; 1 = correct answer). This allowed for examining raw response distributions, as well as analyzing the dichotomous data via a Rasch model. The significant feature of the Rasch model is its ability to transform ordinal data into equal-interval scales (Bond and Fox, 2015). The item difficulties in the Rasch model can be determined, by the process of item calibration, independently of the distribution of persons' abilities in the data and the measurement of person's traits (i.e. abilities) is independent of test items used to measure that trait. Another useful feature of the Rasch Model is that it facilitates the process of constructing measurement variables. In other words, the model is derived independently of data, tests are then constructed to fit the model, and then the data are used to see if they conform to the requirements of the model.

Results

The equivalence assessment was found to have sufficient internal reliability (α = 0.92). Crocker & Algina (1986) suggest that a Cronbach's alpha of 0.90 or higher is sufficient for cognitive assessments. Test statistics for the Rasch model indicate sufficient item reliability (0.92) and person reliability (0.89). To examine the unidimentionality of the assessment, infit and outfit statistics were calculated. The item difficulties range from -3.14 to + 2.12, which is considered as a good practice to have a range of difficulty among items in an assessment. In this assessment, we hypothesized that the item difficulty should increase from lower (i.e. operational) to higher (i.e. advance relational) levels and item difficulty appeared to increase as expected. The infit statistics are weighted and provide more weight to the performance of person whose ability is closer to the item difficulty level whereas outfit statistics is not weighted and as a result more sensitive to outlying scores. This is the reason that investigators give more attention to any small irregularities in infit scores than large outfit scores (Bond and Fox, 2015) The average item mean-square infit statistics is 0.90 and average mean-square outfit statistics is 0.77, which is considered sufficient. Contrasting the overall sufficient item fit statistics, item 8 (12 = ☐) was found to have a relatively high mean square infit statistic (1.45), which indicates more randomness than expected (figure 2). We had observed similar results for such items with our pilot of grade 4 and 5 students, and data from grade 3 students indicates that this particular item format (a =□ ☐) may need further study in regards to the concept of equivalence. We decided to remove this item from our final equivalence assessment, given the continued poor fit across samples. All remaining items appeared to have sufficient fit (Bond & fox 2015), with point-biserial statistics ranging between 0.65 to 0.79. To examine how items hypothesized to target specific levels along the continuum in the construct map, a Wright map was constructed (figure 2).

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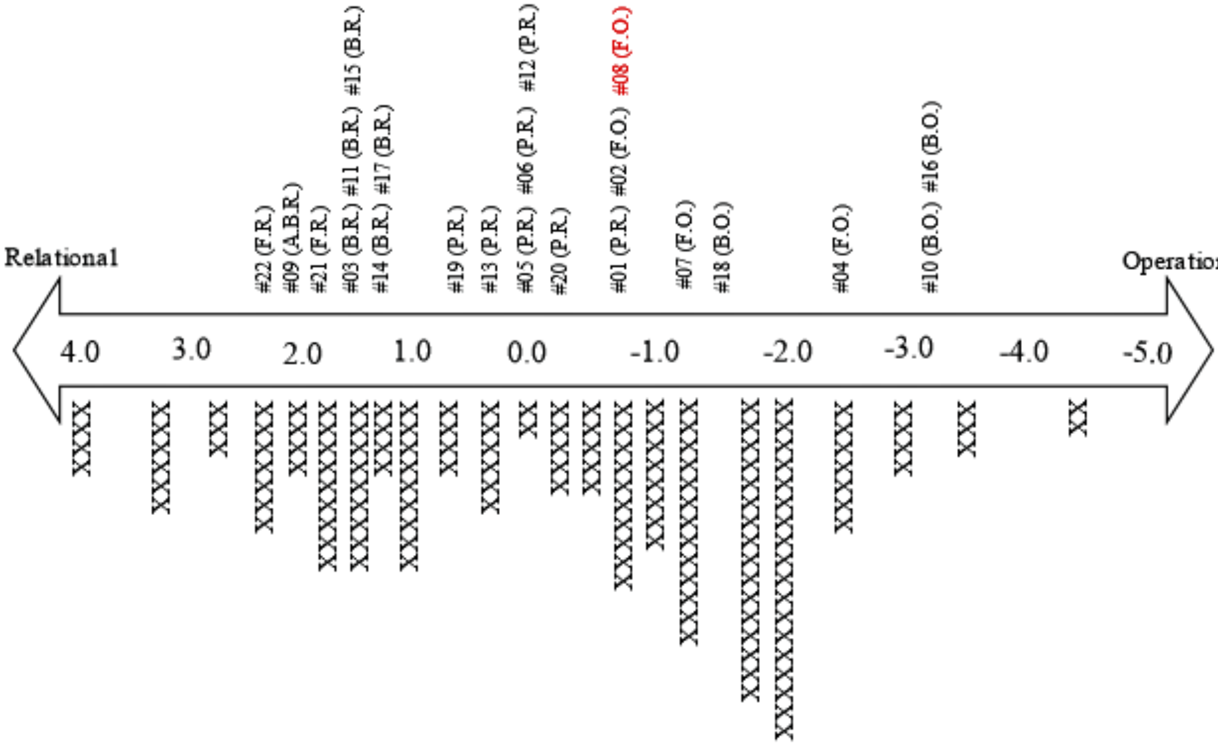

Figure 2. Wright map for equivalence assessment.

The Wright map shown in figure 2 was produced by ConstructMap 4.6.0 (Kennedy et al., 2010). Targeted levels of sophistication (see Figure 1) are abbreviated after each item number (*B.O.=Basic Operational; F.O.=Flexible Operational; P.R.=Pseudo-Relational; B.R.=Basic Relational; A.B.R.=Advanced Basic Relational; and F.R. Full Relational*). The location of items in the Wright map (Figure 2) aligns well with the hypothesized level of sophistication on the construct map (Figure 1). However, certain items (i.e., 8, 2, 18, & 21) visually appear at the same level as items at lower or higher hypothesized levels. Examination of the items' delta statistics and associated confidence intervals indicates that those items hypothesized as more relational than operational do indeed have higher delta statistics.

Item 21 (☐ + 28 = 46 + ○) had a lower delta statistic than predicted, and therefore appears lower on the Wright map than expected. After a close inspection of students' raw responses, it was observed that some students put a zero in the circular blank position "○." However, the the instructions for this item stated that students should "find a number bigger than 10 to write in the ☐…[and] any other number to write in the ○ that makes the problem true". By allowing for the possibility of students to use zero, it may have reduced the difficulty of the item. Specifically, the item was meant to engage students in composing and decomposing number in relation to equivalence. Thus, instructions for these items may need revision in future assessments.

Discussion and Conclusion

Through our findings, we establish that the structure of number sentences, particularly in regard to the role of place value, has a significant role in students' conception of equivalence. The results of our statistical analysis indicate that students may rely on more visually obvious aspects of place value in solving equations. This may appear to be relational at face value, but is not as sophisticated a

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conception of equivalence as other similarly formatted number sentences. Motivated by our findings from previous work (Singh & Kosko, 2015, 2016a) we incorporated new intermediate levels in Rittle-Johnson and colleagues' (2011) framework of students' conception of equivalence. Our suggested, modified construct map appropriately incorporates students' transition from basic operational to full relational understanding by considering these additional transitional stages. Furthermore, the designed and validated assessment described here can serve as a tool for researchers and practitioners interested in students' conceptions of equivalence.

Our results provide useful guidelines for instructors and curriculum designers. Specifically, our findings suggest more attention be paid to the role of place value in the teaching and learning of equivalence. Furthermore, there is a need for more careful examination of students' understanding of equivalence in regard to various mathematical concepts. Future research is needed to confirm and extend the findings presented here. For example, prior research has found relationships between students' conception of equivalence and multiplicative reasoning (Singh & Kosko, 2016b). Given the connection identified here regarding place value, a better understanding of how equivalence interrelates with children's developing number sense is highly needed. Additionally, findings presented here provide evidence that the structure of items similar to a = □ is in need of further study, given that understanding the *Reflexive Property of Equivalence* is crucial for students' success in future advanced mathematic. We expect that by conducting such research, the field may better understand why such items do not consistently align students' conceptions of equivalence.

This study found that place value appears to be inherently tied with conception of equivalence. These results are highly significant and need additional study. There appear to be other connections between various concepts and equivalence research (Singh & Kosko, 2016b), potentially indicating that students' unit coordination may relate to their coordination between expressions. Such interrelationships warrant detailed investigation.

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