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# Processes in the development of mathematics in kindergarten children from Title 1 schools



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### ABSTRACT

This study examined how well nonverbal IQ (or fluid intelligence), vocabulary, phonological awareness (PA), rapid automatized naming (RAN), and phonological short-term memory (STM) predicted mathematics outcomes. The 208 participating kindergartners were administered tests of fluid intelligence, vocabulary, PA, RAN, STM, and numeracy in the fall of kindergarten, whereas tests of numeracy and applied problems were administered in the spring of kindergarten. Fall numeracy scores accounted for substantial variation in spring outcomes ( $R^2$  values = .49 and .32 for numeracy and applied problems, respectively), which underscores the importance of preschool math instruction and screening for mathematics learning difficulties on entry into kindergarten. Fluid intelligence and PA significantly predicted unique variation in spring numeracy scores ( $\Delta R^2 = .05$ ) after controlling for autoregressive effects and classroom nesting. Fluid intelligence, PA, and STM significantly predicted unique variation in spring applied problems scores ( $\Delta R^2 = .14$ ) after controlling for autoregressive effects and classroom nesting. Although the contributions of fluid intelligence, PA, and STM toward math outcomes were reliable and arguably important, they were small.

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## Introduction

Early math achievement is critical for placing children on a positive educational trajectory. Children who start behind in mathematics tend to stay behind (Duncan et al., 2007; Jordan, Kaplan, Ramineni, & Locuniak, 2009; Stock, Desoete, & Roeyers, 2010; Toll, Van der Ven, Kroesbergen, & Van Luit, 2011). In particular, early difficulties with whole numbers interfere with learning fractions, which subsequently impedes algebraic learning (National Mathematics Advisory Panel, 2008). Indeed, one of the strongest predictors of later school achievement is early math achievement (Duncan et al., 2007), predicting children's reading achievement better than early literacy skills (Duncan & Magnuson, 2011; Duncan et al., 2007; Koponen, Salmi, Eklund, & Aro, 2013) and predicting math achievement through age 15 years (Watts, Duncan, Siegler, & Davis-Kean, 2014). Furthermore, evidence of widespread differences in early math achievement (Geary, 2006; Mullis, Martin, & Arora, 2012; National Research Council [NRC], 2009; Sarama & Clements, 2009) and that children from low-income and minority backgrounds persistently score below their middle-income peers (Geary, 1993; Griffin, Case, & Siegler, 1994; Lee, 2002; National Assessment of Educational Progress, 2007, 2013; NRC, 2009; Sarama & Clements, 2009; Saxe, Guberman, & Gearhart, 1987; Siegler, 1993) has led to attempts to improve math education in the United States. For example, comprehensive math standards that begin in kindergarten, called the *Common Core State Standards for Mathematics*, were recently adopted by 42 states and the District of Columbia (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Given the importance of early math achievement and the persistent achievement gap between children from low-income and minority backgrounds and their majority peers, it is important to advance the field's understanding of cognitive and linguistic processes underlying its early development in these children. This knowledge can be used to inform instructional math programs and screenings for mathematics learning difficulties. There is growing consensus concerning which cognitive and linguistic processes are important to early math development (and math disabilities) (e.g., Fletcher, Lyon, Fuchs, & Barnes, 2006; Geary, 1994). Although intelligence is known to be related to the development of cognitive, linguistic, and mathematics skills (Geary, 1993, 2007; Noël, 2009; Primi, Ferrão, & Almeida, 2010; Stock et al., 2010), recent research suggests that vocabulary is involved in solving many different types of math problems (Foster, Sevcik, Ronski, & Morris, 2014; Hooper, Roberts, Sideris, Burchinal, & Zeisel, 2010; LeFevre et al., 2010; Praet, Titeca, Ceulemans, & Desoete, 2013). Evidence also indicates that phonological processing abilities (PPAs) are related to children's early math achievement (Baddeley, 1986; Bull & Johnston, 1997; Clarke & Shinn, 2004; Dehaene, 1992; Dehaene, Piazza, Pinel, & Cohen, 2003; Geary, Hoard, Byrd-Craven, Nugent, & Numtee, 2007; Hecht, Torgesen, Wagner, & Rashotte, 2001; Vukovic, 2012). In the following section, we review these predictors.

### *Predictors of early math achievement*

#### *Vocabulary*

Research demonstrates that vocabulary competencies predict later numeracy scores (Praet et al., 2013; Purpura, Hume, Sims, & Lonigan, 2011). In particular, receptive vocabulary is thought to be related to children's ability to acquire vocabulary in the number system (LeFevre et al., 2010), whereas expressive vocabulary helps children to express relationships inherent in mathematical problems (Rothman & Cohen, 1989). Receptive vocabulary refers to the understanding of words (e.g., "big," "more," "three") and word classes (or parts of speech). Expressive vocabulary, however, refers to the bank of words used to communicate when speaking or writing. Given that vocabulary is essential for learning through classroom instruction, children entering into formal education (i.e., kindergarten) with poor vocabulary are likely at a disadvantage when it comes to mathematical and other areas of learning. Children rely on their vocabulary knowledge to help them understand spoken math statements (e.g., "three plus two equals five") and to help them understand written math statements (e.g.,  $3 + 2 = 5$ ). With regard to written math statements, children must clearly understand the meaning of Arabic numerals (e.g., 3), operational symbols (e.g., +), and concepts embedded within the

statement (e.g., equality—that the quantities on the left and right sides of the “equals” sign must be the same; Munro, 1979).

#### *Phonological processing abilities*

Wagner and Torgesen (1987) outlined three PPAs and described their relations with reading acquisition. All three PPAs have also been linked to the development of basic mathematics skills and conceptualized within the weak phonological representation hypothesis (see Simmons & Singleton, 2008). This hypothesis posits that poorly specified phonological representations can result in poor performance on math tasks that involve the retention, retrieval, or manipulation of phonological codes (e.g., retrieving number words, solving arithmetic facts, counting speed; Hecht et al., 2001; Robinson, Menchetti, & Torgesen, 2002; Simmons & Singleton, 2008). Phonological awareness (PA) refers to the sensitivity to and ability to manipulate the sound structure of one's oral language. With regard to mathematics, PA helps children to manipulate individual words in the number sequence (Krajewski & Schneider, 2009) and encode verbal information (i.e., phonological representations for mathematical terms and operators) when solving arithmetic problems (Hecht et al., 2001; Robinson et al., 2002; Simmons & Singleton, 2008).

Rapid automatized naming (RAN) refers to the efficiency of retrieving phonological codes from long-term memory. RAN may be important for solving arithmetic problems through an influence on children's ability to quickly retrieve arithmetic facts and when linking the appropriate fact or facts to a particular problem in an effort toward arriving at a solution (Geary, 1993; Hecht et al., 2001; Kaye, 1986). For instance, RAN is involved when children need to retrieve phonological name codes for numbers from long-term memory to solve arithmetic problems (Dehaene, 1992; Geary, 1993; Kaye, 1986). Quick and efficient retrieval of phonological name codes decreases the likelihood that a child will forget the association between an arithmetic problem and its solution. In other words, cognitive resources are freed as RAN improves and one can attend to other aspects of problem solving, which are needed when learning more complex mathematics skills (Bull & Johnston, 1997; Geary, 1993; Hecht et al., 2001; Kaye, 1986).

Phonological short-term memory (STM) is the non-executive component of working memory that involves the coding and rehearsing of verbal information in a sound-based representation system for temporary storage (Baddeley, 1986; Torgesen, 1996). This limited capacity processing resource helps a child to actively maintain the math task in mind as he or she decides on a course of action for solving it. For instance, consider the following verbally presented math problem: “There are five red balloons. Three of them popped. How many balloons are left?” In this problem, the child needs to hold the problem in memory while deciding on a course of action (i.e., add or subtract the balloons). The child then solves the problem by retrieving a solution from long-term memory or by using a count-based strategy (cf. Fuson, 1988, or Geary, 1993). While executing a course of action (e.g., counting backward from 5), the child needs to actively maintain the original problem in mind. If not, the child may forget the association between the arithmetic problem and its solution. Thus, when a child counts backward to solve  $5 - 3 = \_$  (i.e., 5, 4, 3, 2 to reach the solution, 2), he or she must keep the original problem in mind in order to both remember how far to count backward and connect the result of the counting strategy to the original problem context. In short, limited processing resources such as short-term storage for phonological information could interfere with learning arithmetic facts, could interfere with mathematical development, and could limit one's ability to solve math problems.

#### *Studies of the prediction of mathematics*

##### *Phonological processing abilities and vocabulary*

PA, RAN, STM, and vocabulary have all been found to be predictive of early math achievement, depending on the mathematical task and the age of the population assessed (Baddeley, 1986; Bull & Johnston, 1997; Clarke & Shinn, 2004; Foster et al., 2014; Geary et al., 2007; Hecht et al., 2001; Krajewski & Schneider, 2009). A seminal study by Bryant, MacLean, Bradley, and Crossland (1990) accounted for the influences of socioeconomic status (SES), receptive vocabulary, and general intelligence in its examination of the relation between PA and arithmetic achievement. Although SES, general intelligence, and PA measured at 4 and 5 years of age were significant predictors of arithmetic

achievement at 6 years of age, vocabulary was not. Another seminal study (Hecht et al., 2001) accounted for the influences of vocabulary and the autoregressor effect of prior math ability in its examination of the predictive relations of PPAs with arithmetic achievement. Vocabulary and prior math ability significantly predicted arithmetic achievement at each time period under study (i.e., second to fifth grades, second to third grades, third to fourth grades, and fourth to fifth grades). The study by Hecht and colleagues (2001) also showed that RAN and STM significantly predicted arithmetic achievement from second to third grades after accounting for prior math ability. However, of the three PPAs, only PA significantly predicted arithmetic achievement after accounting for prior math ability during the remaining time periods. Collectively, findings from these studies indicate that, from among PPAs, PA tends to be the strongest and most robust predictor of math achievement. They also suggest that RAN and STM may be more involved in early math development when children are learning arithmetic facts rather than in later math development. With regard to the mixed findings concerning the relation between vocabulary and arithmetic achievement, it is unclear whether this discrepancy is due in part to the different measures used in these studies. Hecht and colleagues (2001) employed an expressive measure of vocabulary, whereas Bryant and colleagues (1990) employed a receptive measure of vocabulary.

More recent studies have directly modeled the interrelated nature of PPAs and other linguistic skills in their predictions of math achievement in young children. For instance, in a sample of preschool and kindergarten children, LeFevre and colleagues (2010) modeled a factor indexed by PA, vocabulary, and RAN and found the factor to account for substantial variance in math achievement measured 2 years later. In a more recent study of monolingual and bilingual kindergarten students, Kleemans, Segers, and Verhoeven (2011) found that PA and grammatical ability were *directly* related to concurrent math achievement, whereas fluid intelligence and working memory were *indirectly* related to math achievement through PA and grammatical ability. A study of Finnish students reported that PA, RAN, and STM assessed in kindergarten were predictive of math achievement assessed in third grade (Koponen et al., 2013). Finally, Vukovic (2012) found that early numerical skills and PA influenced growth in mathematics from kindergarten to third grade. In summary, research indicates that PPAs and vocabulary are predictive of early math achievement and math development. Moreover, their relations with mathematics appear to be in part independent of the influence of intelligence.

#### *Early mathematics knowledge*

Of course, the most palpable predictor of later math achievement is prior math achievement (Bodovski & Youn, 2011). Research has shown that knowledge of mathematics in preschool correlates .46 with 10th-grade math achievement (Stevenson & Newman, 1986) and successfully predicts math achievement through age 15 years even after accounting for early reading, cognitive skills, and family and child characteristics (Watts et al., 2014). For many topics and abilities, initial knowledge predicts learning and later knowledge (Bransford, Brown, & Cocking, 2008; Jimerson, Egeland, & Teo, 1999; Thomson, Rowe, Underwood, & Peck, 2005; Wright, 1994). However, the effect of early knowledge of mathematics is unusually strong and notably persistent (Duncan, Claessens, & Engel, 2004). Furthermore, the rate of growth of mathematical skills is faster among those with higher, rather than lower, initial mathematical skills (Aunola, Leskinen, Lerkkanen, & Nurmi, 2004). Across six studies, the strongest predictor of later achievement was school entry math skills (Duncan et al., 2007). Moreover, this result has been replicated within the context of holding constant children's preschool cognitive ability, behavior, and other important background characteristics (Romano, Babchishin, Pagani, & Kohen, 2010).

#### *The current study*

Empirical research has provided ample evidence to conclude that intelligence, PPAs, and (to a lesser extent) vocabulary are related to early math achievement. However, the exact nature of the interrelations among intelligence, PA, RAN, STM, vocabulary, and domains of math achievement are not clear. Studies that have simultaneously included all of these relevant predictors within a comprehensive framework for understanding early math achievement are scarce, and the inclusion of intelligence as a predictor is inconsistent. A comprehensive investigation that includes intelligence and each

linguistic skill that is potentially implicated in math achievement is necessary to clarify prior research findings because findings of unique predictive relations are conditional on the variables included in a model as well as the variables that are absent from a model.

In addition to a dearth of studies that have examined early math achievement using a comprehensive framework, even fewer studies have modeled autoregressor effects. Most of the studies discussed above have either modeled concurrent relations (e.g., Kleemans et al., 2011) or modeled longitudinal relations without accounting for initial levels of math achievement (i.e., the autoregressor) (e.g., Bryant et al., 1990; Kleemans et al., 2011; Koponen et al., 2013; LeFevre et al., 2010). The danger in not accounting for autoregressor effects is that the predictive association of PPAs, vocabulary, and intelligence could be spurious tertiary variables associated with prior levels of math achievement (Wagner & Torgesen, 1987). By accounting for autoregressor effects, the current study aimed to extend extant research by providing a more rigorous evaluation of the predictive utility of cognitive and linguistic skills. Thus, the first aim of the current study was to examine the extent to which fluid intelligence, PA, RAN, STM, and vocabulary uniquely predict children's math outcomes at the end of kindergarten while accounting for the influences of the autoregressor.

The predictive value of fluid intelligence, PA, RAN, STM, and vocabulary also may differ according to the type of mathematics skills being predicted. Therefore, two different math outcomes were included in the current study to permit comparison of the predictive value of each variable as it relates to separate math domains (i.e., numeracy and applied problems). The first domain, numeracy, is composed of several early number competencies that include the ability to discern the value of small quantities immediately, make judgments about numbers and their magnitudes (e.g., 4 is closer to 3 than 6), recognize numerals, understand the concepts underlying the procedures for counting (e.g., ordinality, cardinality; see Gelman & Gallistel, 1978), count objects, compose numbers (e.g., 3 and 2 makes 5), and decompose numbers (e.g., 5 take away 2 is 3). In contrast, applied problems require children to listen to a verbally presented math problem, recognize the arithmetic operation to be followed, perform the relevant calculation(s), and contextualize the answer in units of the original applied problem.

Because numeracy and applied problems represent potentially separate math domains, children may rely on different cognitive and linguistic skills when solving items from each domain. For example, whereas fluid intelligence is expected to be related to both math domains, PA and RAN may be more strongly related to numeracy achievement than to applied problems achievement. This is because many early number competencies require the efficient retrieval of well-specified phonological codes for number words (Logie & Baddeley, 1987). Phonological STM, however, may be more strongly related to applied problems achievement because phonological codes for numbers and the other relevant verbal information must be maintained in temporary phonological storage while the child uses calculation procedures to solve related problems (Bull & Johnston, 1997). Vocabulary also may be more strongly related to applied problems achievement because children interpret verbally presented math problems within their language structures (Munro, 1979) and any problems with comprehending the verbal information may interfere with or delay subsequent mathematical processing. Thus, the second aim of the current study was to investigate whether patterns of relations among hypothesized predictors and math achievement vary as a function of math domain (i.e., numeracy and applied problems).

## Method

### Participants

Participants included three annual cohorts of kindergarten children who participated in a randomized controlled evaluation of two computerized tutoring programs: Earobics Step 1 and Building Blocks. The current sample consisted of 208 monolingual English children (106 female students and 102 male students) who attended 37 classrooms housed in nine Title 1 schools. The percentage of students eligible for free or reduced price lunch at each school ranged from 70% to 98% ( $M = 92.2\%$ ,  $SD = 8.6$ ). Participants ranged in age from 5.03 to 6.70 years ( $M = 5.62$  years,  $SD = 0.32$ ), and most represented ethnic minorities: 62% African American, 32% Hispanic/Latino, 3% mixed ethnicity, 2%

Caucasian, and 1% other. At the beginning of kindergarten, most participants achieved low average or below average scores on norm-referenced standardized tests of verbal ability ( $M = 84$ ,  $SD = 14$ ) and nonverbal ability ( $M = 78$ ,  $SD = 10$ ), as estimated by the Expressive One-Word Picture Vocabulary Test (EOWPVT; Brownell, 2000) and a composite of the Copying and Pattern Analysis subtests of the Stanford–Binet Intelligence Scale–Fourth Edition (SB-4; Thorndike, Hagen, & Sattler, 1986). Note that although participants' achievement on measures of nonverbal ability confirmed their risk status, their standardized scores of math achievement on the Applied Problems subtest of the Woodcock–Johnson III Tests of Achievement (WJ-III; Woodcock, McGrew, & Mather, 2001) was well within the range of typical achievement at the end of kindergarten ( $M = 99.01$ ,  $SD = 12.76$ ).

### *Procedures*

After obtaining active parental consent and child assent, children were tested individually at their school during the school day. Completion of the test battery required approximately 2 h. However, testing was typically spread across two testing sessions that occurred within a given week. Children were assessed at the beginning of the school year (i.e., September) and again at the end of the school year (i.e., April). Measures from the larger assessment battery that were selected for inclusion in the current study included tests of fluid intelligence, vocabulary, PA, RAN, STM, and numeracy from the beginning of the school year and tests of numeracy and applied problems from the end of the school year. The test of applied problems was not administered at the beginning of the school year.

### *Measurement instruments*

#### *Intelligence*

Fluid intelligence was assessed using the Copying and Pattern Analysis subtests of the SB-4. The Copying subtest requires examinees to either reproduce block models (Items 1–12) or draw geometric designs, such as lines, rectangles, and arcs, that are shown on cards (Items 13–28). Items 1 through 6 of the Pattern Analysis subtest require examinees to complete puzzles. Items 7 through 42 require replication of visual patterns through manipulation of shaded blocks. Standardized administration and scoring procedures were followed. It is important to highlight that these subtests index fluid (or nonverbal) intelligence within the context of understanding spatial and geometric relations. Internal consistency was calculated for the SB-4 nonverbal composite, which was restricted to the highest item administered (see Table 1 in Results for reports of reliability indices).

#### *Vocabulary*

Children's vocabulary was estimated using the EOWPVT, which presents examinees with colored line drawings that depict an action, an object, a category, or a concept. Children were asked by an examiner to verbally respond to prompts such as “What is this?”, “What is she doing?”, and “What are these?” Standardized administration and scoring procedures were followed.

#### *Phonological awareness*

PA was assessed with an elision multiple choice task and an elision free response task (Anthony, Lonigan, Driscoll, Phillips, & Burgess, 2003; Lonigan, Burgess, Anthony, & Barker, 1998; Lonigan, Wagner, Torgesen, & Rashotte, 2002). Previous research with these same measures has found them to demonstrate good convergent validity with measures of the same construct and good discriminant validity with measures of different but related constructs (Anthony, Williams, McDonald, & Francis, 2007; Anthony et al., 2006; Dunlap et al., 2015). The elision tasks assessed children's ability to identify a word from an array of four pictures or to produce a target word that resulted from deletion of part of a stimulus word (e.g., *farm* without /f/). Both elision tasks spanned three levels of linguistic complexity such that each task included some items that required deletion of syllables, deletion of onsets, and deletion of phonemes that were not onsets. Children were administered all 11 multiple choice trials and all 29 free response trials.

### *Rapid automatized naming*

Efficiency of phonological access to lexical storage was assessed with RAN for objects. Rapid Object Naming was chosen rather than Rapid Letter Naming because of our sample's low achievement. Specifically, we did not want children's letter knowledge (or lack thereof) to confound measurement of efficiency of access to well-specified phonological codes. Rapid Object Naming required children to name pictures of four common objects as quickly as they could. The stimulus card consisted of seven rows of four pictures. The pictures illustrated single-syllable words common in young children's vocabularies (i.e., car, ball, dog, and tree). Each row presented the same four pictures in a random order, and the first row served as practice for the remainder of the task. The number of errors made during naming of all 28 pictures and the time elapsed (minutes and seconds) were recorded. Each trial was scored as the number of pictures correctly named per minute. Children were administered two trials that were usually, although not always, administered on the same day.

### *Phonological short-term memory*

The non-executive component of working memory, phonological STM, was assessed with the Memory for Words subtest of the Preschool Comprehensive Test of Phonological and Print Processing (PCTOPPP; Lonigan et al., 2002). The Memory for Words task measured children's ability to reproduce a list of one-syllable words in the same order that they were presented by an examiner. The measure consisted of 21 items. Items were divided into seven groups of 3 items each, according to the number of words to be recalled on a given item. The first group of items required repetition of one word, the second group of items required repetition of two words, and so on. Standardized administration and scoring procedures were followed.

### *Numeracy*

Children's math achievement at the beginning and end of the school year was assessed with the Research-based Early Math Assessment (REMA; Clements, Sarama, & Liu, 2008). Items from the number concepts strand were administered, but items from the geometry strand were not administered. Core mathematics skills within the number concepts strand include verbal counting, object counting, number recognition and subitizing, number comparison, number sequencing, numeral recognition, number composition and decomposition, and adding and subtracting. General concepts and processes, such as part-whole thinking, and the corresponding processes of composition and decomposition, classification, and seriation were woven throughout the core areas enumerated above. Standardized administration and scoring procedures were followed.

### *Applied problems*

Children's math achievement at the end of the school year also was assessed using the Applied Problems subtest of the WJ-III. The Applied Problems subtest requires children to analyze and solve verbally presented math problems. For example, when looking at a stimulus page that depicts five ducks, two of which are swimming, children are asked, "How many ducks are in the water?"

## **Results**

### *Pre-analysis data inspections*

All bivariate relations were linear and in the expected direction. Descriptive analyses revealed no floor or ceiling effects and found minimal evidence of non-normality (see Table 1). Because students had participated in supplemental individualized computer-administered training in either PA or mathematics as part of the larger project, all variables were examined for mean differences between experimental groups. Multilevel analyses of covariance (ANCOVAs) accounting for classroom nesting found that the groups did not differ on any of the pretest or posttest variables (see Table 2). Moreover, a two-group confirmatory factor analytic model that constrained the 28 pairs of like covariances to equality across the groups characterized these data extremely well ( $\chi^2 = 31.82$ ,  $p = .06$ , comparative fit index [CFI] = .98, Tucker–Lewis index [TLI] = .96, root mean square error of approximation

**Table 1**  
Descriptive statistics.

| Variable           | <i>M</i> | <i>SD</i> | Min  | Max   | Skew  | Kurtosis | Reliability      |
|--------------------|----------|-----------|------|-------|-------|----------|------------------|
| Fluid intelligence | 78.40    | 9.91      | 60.0 | 109.0 | 0.61  | −0.29    | .90 <sup>a</sup> |
| PA                 | 17.01    | 8.06      | 4.0  | 41.0  | 0.75  | −0.13    | .90 <sup>a</sup> |
| RAN                | 39.74    | 10.23     | 21.5 | 87.0  | 1.35  | 3.60     | .66 <sup>b</sup> |
| STM                | 9.85     | 2.30      | 0.0  | 16.0  | −1.21 | 2.95     | .73 <sup>a</sup> |
| Vocabulary         | 45.40    | 12.14     | 15.0 | 86.0  | 0.37  | 0.52     | .83 <sup>a</sup> |
| Numeracy–pretest   | 13.51    | 5.74      | 2.0  | 31.0  | 0.29  | −0.02    | .86 <sup>a</sup> |
| Numeracy–posttest  | 20.34    | 6.67      | 3.0  | 37.0  | 0.11  | −0.13    | .87 <sup>a</sup> |
| Applied problems   | 18.46    | 3.68      | 2.0  | 27.0  | −0.88 | 1.92     | .87 <sup>a</sup> |

Note: *N* = 208. Min, minimum; Max, maximum; Skew, skewness; PA, phonological awareness; RAN, rapid automatized naming; STM, short-term memory. All values are reported in raw score units except fluid intelligence and RAN. Fluid intelligence is reported in standard score units, whereas RAN is reported in number of seconds to name all 28 objects.

<sup>a</sup> Cronbach's alpha for the current sample.

<sup>b</sup> Pearson's *r*.

**Table 2**  
Tests of mean differences between experimental groups.

| Variable           | PA group |           | Math group |           | <i>df</i> | <i>F</i> | <i>p</i> |
|--------------------|----------|-----------|------------|-----------|-----------|----------|----------|
|                    | <i>M</i> | <i>SD</i> | <i>M</i>   | <i>SD</i> |           |          |          |
| Fluid intelligence | 7.45     | 3.50      | 7.77       | 3.49      | 1, 170    | 1.79     | .38      |
| PA                 | 17.25    | 10.33     | 16.77      | 10.33     | 1, 170    | 0.18     | .67      |
| RAN                | 39.30    | 13.14     | 40.19      | 13.14     | 1, 170    | 0.39     | .53      |
| STM                | 9.58     | 3.14      | 10.12      | 3.13      | 1, 170    | 3.07     | .08      |
| Vocabulary         | 43.97    | 16.32     | 46.77      | 16.25     | 1, 170    | 2.89     | .09      |
| Numeracy–pretest   | 13.87    | 7.53      | 13.08      | 5.91      | 1, 168    | 1.00     | .32      |
| Numeracy–posttest  | 19.74    | 8.52      | 20.93      | 8.52      | 1, 170    | 1.66     | .20      |
| Applied problems   | 18.12    | 5.22      | 18.82      | 5.18      | 1, 170    | 2.11     | .15      |

Note: PA, phonological awareness; RAN, rapid automatized naming; STM, short-term memory. All values are reported in raw score units except RAN, for which times (in seconds) to name all 28 objects are reported.

[RMSEA] = .06). Thus, preanalysis data inspections ruled out both mean differences between experimental groups and covariance differences between groups. Finally, potential moderation effects were directly ruled out as well by testing the interaction between group and each independent variable in separate multilevel ANCOVA models used to predict end-of-year numeracy scores ( $F_s = 0.21$ – $3.07$ ,  $p_s > .08$ ) and again when used to predict end-of-year applied problems scores ( $F_s = 0.04$ – $2.72$ ,  $p_s > .10$ ). There was no evidence of intervention-related moderation of the prediction of either math outcome. The thorough investigation of groups' equivalence strongly supported pooling these data across experimental conditions. Therefore, all subsequent analyses were performed using the sample as a whole without including experimental group in the models.

Age was included in initial correlational analyses (see Table 3) because raw scores were being evaluated. However, age was not reliably associated with any of the independent variables or dependent variables and, therefore, was ignored in subsequent analyses. Vocabulary, PA, RAN, and fluid intelligence (but not STM) were significantly correlated with children's numeracy at posttest. All predictors were correlated with applied problems at posttest. Because STM was not correlated with numeracy posttest scores, it was not included in subsequent analyses used to predict children's numeracy scores at posttest. Finally, the two math outcomes were only moderately correlated ( $r = .61$ ,  $p < .001$ ). This finding suggests that although there is overlap in skills assessed by these measures, they may index different mathematical skill sets, which may explain why STM was correlated with one math outcome but not the other.

### Overview of data analysis plan

Raw scores were used in all statistical analyses, which were performed using SAS 9.3 (SAS Institute, 2002–2006). To account for the multilevel structure of the data (i.e., children nested within

**Table 3**  
Correlations.

| Variable              | 2    | 3      | 4       | 5     | 6      | 7       | 8       | 9       |
|-----------------------|------|--------|---------|-------|--------|---------|---------|---------|
| 1. Age                | -.08 | .08    | -.02    | .03   | .12    | .13     | .11     | .01     |
| 2. Fluid intelligence | –    | .32*** | -.19**  | .13   | .32*** | .35***  | .34**   | .37***  |
| 3. PA                 | –    | –      | -.29*** | .22** | .53*** | .51***  | .49***  | .51***  |
| 4. RAN                | –    | –      | –       | -.11  | -.15*  | -.31*** | -.34*** | -.28*** |
| 5. STM                | –    | –      | –       | –     | .22**  | .10     | .10     | .29***  |
| 6. Vocabulary         | –    | –      | –       | –     | –      | .39***  | .32***  | .43***  |
| 7. Numeracy–pretest   | –    | –      | –       | –     | –      | –       | .70***  | .56***  |
| 8. Numeracy–posttest  | –    | –      | –       | –     | –      | –       | –       | .61***  |
| 9. Applied problems   | –    | –      | –       | –     | –      | –       | –       | –       |

Note: PA, phonological awareness; RAN, rapid automatized naming; STM, short-term memory.

\*  $p < .05$ .

\*\*  $p < .01$ .

\*\*\*  $p < .001$ .

classrooms), analyses were performed using “Proc Mixed” with students’ classroom identified as the Level 2 random intercept. Proc Mixed provides hypothesis tests for the variance and covariance components of random variability at Level 1 (child level) and Level 2 (classroom level) of the model. Thus, the fixed effects of hypothesized Level 1 predictors (i.e., intelligence, vocabulary, PPAs, and autoregressor) on children’s math outcomes (i.e., numeracy or applied problems) was investigated while accounting for variability at the classroom level (Level 2 random intercept). It is noteworthy that all predictors were grand mean centered at Wave 1, which was approximately 1 month after children entered kindergarten.

In an effort to clarify conflicting findings of prior research and to fully understand the roles of cognitive and linguistic processes in early math development, three multilevel regression models were evaluated for both math outcomes. First, the “autoregressor model” included a random intercept to reflect classroom nesting and a fixed effect of pretest numeracy scores. Second, the “processes model” included a random intercept and fixed effects of pretest scores obtained on tests of PA, RAN, STM, fluid intelligence, and vocabulary. Third, the “full model” included a random intercept and fixed effects of pretest scores obtained on tests of PA, RAN, STM, fluid intelligence, vocabulary, and numeracy. By accounting for prior levels of math achievement, relations between the remaining predictors and end-of-year math achievement were not confounded with the level of mathematics skills with which students entered kindergarten (Hecht et al., 2001). Finally, we examined the standardized estimates and the amount of change in each model’s pseudo- $R^2$  (referred to as  $R^2$  throughout this article)<sup>1</sup> to decompose the predictive variance into that which was shared or unique.

### Predictors of numeracy achievement

Evaluation of the Level 2 variance in the autoregressor model of the prediction of numeracy scores suggested that classrooms did not reliably differ in their average numeracy scores at posttest ( $B = 1.04$ ,  $p = .23$ ). There was only a small amount of variance in posttest numeracy scores that was attributable to classroom nesting ( $\rho = .045$ ). Furthermore, numeracy scores at pretest significantly predicted 49% of the variability in numeracy scores at posttest (see Table 4). The processes-only model revealed that the cognitive and linguistic processes accounted for 32% of the variance in numeracy scores at posttest. Fluid intelligence, PA, and RAN significantly predicted unique variation in numeracy posttest scores, whereas vocabulary did not. Thus, more variation in numeracy posttest scores was accounted for by the autoregressor than by the group of predictors. In the full model, numeracy posttest scores were simultaneously regressed on the autoregressor and all remaining cognitive and linguistic predictors. The full model explained 54% of the variance in numeracy posttest scores, which was a small

<sup>1</sup> Note that although  $R^2$  is reported throughout the study,  $R^2$  within multilevel models yields pseudo- $R^2$  because the estimation of variance components changes depending on the variables in the model (see Singer & Willett, 2003).

**Table 4**  
Level 1 results of the prediction of kindergarten math achievement.

| Model               | Independent variable(s) | Numeracy       |                |                |                | Applied problems |       |       |                |
|---------------------|-------------------------|----------------|----------------|----------------|----------------|------------------|-------|-------|----------------|
|                     |                         | B (SE)         | t              | p              | R <sup>2</sup> | B (SE)           | t     | p     | R <sup>2</sup> |
| Autoregressor model |                         |                |                |                | .49            |                  |       |       | .32            |
|                     | Numeracy–pretest        | 4.95 (0.35)    | 14.20          | <.001          |                | 2.21 (0.22)      | 10.22 | <.001 |                |
| Processes model     |                         |                |                |                | .32            |                  |       |       | .38            |
|                     | Fluid intelligence      | 1.71 (0.53)    | 3.22           | .002           |                | 0.96 (0.28)      | 3.46  | <.001 |                |
|                     | PA                      | 2.36 (0.48)    | 4.90           | <.001          |                | 1.15 (0.25)      | 4.35  | <.001 |                |
|                     | RAN                     | –1.33 (0.42)   | –3.13          | .002           |                | –0.42 (0.22)     | –1.90 | .058  |                |
|                     | Vocabulary              | 0.39 (0.47)    | 0.83           | .41            |                | 0.52 (0.21)      | 2.53  | .01   |                |
|                     | STM                     | – <sup>a</sup> | – <sup>a</sup> | – <sup>a</sup> |                | 0.58 (0.25)      | 2.33  | .02   |                |
| Full model          |                         |                |                |                | .54            |                  |       |       | .46            |
|                     | Numeracy–pretest        | 3.98 (0.41)    | 9.70           | <.001          |                | 1.42 (0.24)      | 5.94  | <.001 |                |
|                     | Fluid intelligence      | 0.88 (0.45)    | 1.96           | .05            |                | 0.63 (0.27)      | 2.37  | .02   |                |
|                     | PA                      | 1.09 (0.42)    | 2.58           | .01            |                | 0.68 (0.24)      | 2.75  | .01   |                |
|                     | RAN                     | –0.67 (0.36)   | –1.87          | .06            |                | –0.18 (0.20)     | –0.86 | .39   |                |
|                     | Vocabulary              | –0.11 (0.40)   | –0.29          | .77            |                | 0.41 (0.23)      | 1.77  | .08   |                |
|                     | STM                     | – <sup>a</sup> | – <sup>a</sup> | – <sup>a</sup> |                | 0.59 (0.19)      | 3.10  | .002  |                |

Note:  $N = 208$ . PA, phonological awareness; RAN, rapid automatized naming; STM, short-term memory.

<sup>a</sup> STM was not included in analysis of numeracy because preanalysis data inspection found that it was not reliably associated with numeracy.

improvement over the autoregressor model (i.e.,  $\Delta R^2 = .05$ ). In the full model, the autoregressor, fluid intelligence, and PA uniquely predicted numeracy posttest scores, whereas RAN and vocabulary did not.

#### Predictors of applied problems achievement

Evaluation of the Level 2 variance in models of the prediction of applied problems posttest scores suggested that classrooms did reliably differ in their average applied problems scores at posttest ( $B = 1.49$ ,  $p = .02$ ). However, only a small portion of variance in this outcome was attributable to classroom nesting ( $\rho = .16$ ). Because the Applied Problems subtest was not administered at the beginning of the school year, the numeracy pretest was used as the autoregressor. The autoregressor model demonstrated that numeracy scores at pretest significantly predicted applied problems scores at posttest, accounting for 32% of its variability. The processes-only model revealed that cognitive and linguistic processes accounted for 38% of the variance in applied problems posttest scores. In this model, all cognitive and linguistic variables except RAN significantly predicted unique variation in the applied problems outcome (see Table 4). In the full model, the autoregressor, fluid intelligence, PA, and STM significantly predicted unique variance in applied problems, whereas RAN and vocabulary did not. Together, the six predictors in the full model explained 46% of the variance in performance on the applied problems outcome, which reflects a sizable increment in variance explained over the autoregressor-only model (i.e.,  $\Delta R^2 = .14$ ). Of the predictors, the autoregressor, IQ, PA, and STM accounted for significant unique variation in applied problems scores at posttest.

#### Discussion

The current study sought to advance the field's understanding of cognitive and linguistic processes that are important for kindergarten math achievement. We built on prior research by comprehensively including many theoretically and empirically informed predictors of mathematics, by using a longitudinal framework, by including an autoregressor, by investigating the relations of hypothesized predictors with two different math outcomes, and by accounting for classroom nesting. This comprehensive framework yielded somewhat different accounts of the relations among cognitive processes, linguistic skills, and math achievement than those reported in prior research.

When predicting numeracy achievement, correlational results indicated that children who entered kindergarten with relatively higher levels of fluid intelligence, PA, RAN, and vocabulary evidenced relatively higher levels of numeracy at the end of kindergarten. Subsequently, multiple regression analysis demonstrated that the predictive associations of PA and RAN with numeracy were independent of fluid intelligence. However, PA and fluid intelligence were the only significant unique predictors of numeracy achievement after controlling for prior numeracy skills.

When predicting applied problems achievement, correlational results indicated that children who entered kindergarten with relatively higher levels of fluid intelligence, PA, RAN, STM, and vocabulary displayed evidence of higher levels of achievement for solving applied problems at the end of kindergarten. Subsequent multiple regression analysis demonstrated that the predictive associations of PA, vocabulary, and STM with applied problems were independent of fluid intelligence. However, fluid intelligence, PA, and STM were the only significant unique predictors of applied problems achievement after controlling for prior numeracy skills.

With regard to the autoregressor, beginning-of-year numeracy scores accounted for substantial amounts of variation in end-of-year math outcomes. Specifically, numeracy scores in the fall of kindergarten accounted for 49% and 32% of the variance in numeracy and applied problems scores obtained in the spring of kindergarten, respectively. Strong autoregressor effects are consistent with prior research in mathematics (Jordan et al., 2009; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005). Collectively, these findings suggest that it may be beneficial for systematic mathematics instruction to begin prior to kindergarten. Indeed, research has demonstrated that early mathematics skills are malleable and that high-risk students benefit from early math instruction (see Clements & Sarama, 2008; Clements, Sarama, Spitler, Lange, & Wolfe, 2011; Griffin et al., 1994; Starkey, Klein, & Wakeley, 2004). Thus, engaging preschool-age children in numeracy activities and games should become the norm because these experiences may serve as a foundation from which young children develop knowledge of the symbolic number system (Geary, 1995; Levine, Jordan, & Huttenlocher, 1992).

The strong autoregressor effects also suggest that it may be beneficial to screen for mathematics learning difficulties on entry into kindergarten. Other studies have demonstrated that kindergarten numeracy is predictive of math achievement at the end of second and third grades over and above the effects of supporting cognitive competencies (Jordan et al., 2009; Locuniak & Jordan, 2008; Mazzocco & Thompson, 2005). Moreover, Jordan and colleagues (2009) demonstrated that numeracy at the beginning of kindergarten predicted numeracy growth as well as children's math achievement trajectories through third grade as measured by norm-referenced measures. Given the strong relationship between numeracy and math achievement trajectories, screening for mathematics learning difficulties should begin when most children enter into formal schooling during kindergarten. In doing so, identified children can be provided with the opportunity to participate in prevention services before the onset of severe deficits (Fuchs et al., 2007; Stock et al., 2010).

From among the cognitive and linguistic predictors, the strong relation between fluid intelligence and math achievement is especially interesting and noteworthy given that our measures of fluid intelligence arguably assessed many mathematically based competencies (i.e., patterning, geometry, mental rotation, and symbolic problem solving). Our findings that fluid intelligence predicted numeracy and applied problems achievement suggest that spatial and geometric skills contribute to children's mathematical development over and above early numeracy skills. Specifically, spatial and geometric properties that children understand at school entry may support learning of numeracy skills that develop during the school year (e.g., mathematical operations). The current results similarly support the argument that fluid intelligence or spatial and geometric competencies support the development of applied problems.

As hypothesized according to prior research (Hecht et al., 2001; Kleemans et al., 2011), PA uniquely predicted both numeracy and applied problems achievement. However, it is important to keep in mind that PA uniquely accounted for only approximately 1.25% and 5% of the variance in end-of-kindergarten numeracy and applied problems, respectively, once accounting for prior math achievement, IQ, and other linguistic competencies. These findings are consistent with the results of many studies (e.g., Bryant et al., 1990; Foster et al., 2014; Koponen et al., 2013; LeFevre et al., 2010; Wise et al., 2008).

The current study's confirmation of unique effects of PA in the development of both numeracy skills and applied problems suggests that PA is involved in a wide array of mathematics skills beyond simple arithmetic. That is, numeracy tasks and applied problems both appear to rely on the manipulation, retrieval, and retention of phonological codes. Indeed, speech sound processes are used when counting and when solving arithmetic problems (Geary, 1993; Logie & Baddeley, 1987). When solving arithmetic problems, children first translate terms and operators into a speech-based code (Campbell, 1998; Dehaene, 1992). Children then retrieve a phonologically stored solution from long-term memory or process phonological information when using counting-based strategies to calculate (e.g., *sum* or *counting all* strategy and *counting up* or *min* strategy; Geary, Hoard, & Hamson, 1999; Siegler & Shipley, 1995; for a review, see Siegler & Robinson, 1982). Alternatively, the association of PA with math achievement could reflect a common reliance on working memory resources. Working memory is defined as "the ability to hold and preserve available information and [simultaneously] conduct manipulations on this information" (Shaul & Schwartz, 2014, p. 752). Elision tasks, like those used in the current study, require a child to keep an auditory stimulus (e.g., "batman") in short-term storage while processing auditory instructions (i.e., delete the word part "man") and executing the processes needed to arrive at a solution (i.e., "bat"). In accord with Baddeley's multicomponent model (Baddeley, 1986; Baddeley & Hitch, 1974), the central executive system interacts with the phonological loop during online processing of such tasks. Specifically, the phonological loop holds verbal information (e.g., "Say batman without 'man'," "How much is two plus three?") in mind via rehearsal processes of (sub)articulation, whereas the central executive coordinates activities involved (e.g., elision, addition) in arriving at a solution. In short, PA elision tasks and solving math problems similarly draw from working memory resources.

The current study also provides a fine-grained analysis of which cognitive and linguistic processes predict two somewhat different math outcomes. For example, after accounting for classroom nesting and prior math ability, fluid intelligence and PA predicted both numeracy and applied problems outcomes, whereas STM predicted only the applied problems outcome. These findings bolster evidence (e.g., De Smedt, Taylor, Archibald, & Ansari, 2010) that the quality of long-term phonological representations and/or working memory resources, rather than the size of short-term phonological storage, is important for early development in numeracy skills. Alternatively, STM might not have been associated with numeracy in the current study because children used manipulatives when solving test items on the REMA. The use of manipulatives likely reduces the amount of phonological information that children must keep in temporary storage, mitigating the need for taxing STM. In contrast, results pertaining to applied problems suggest that solving such problems draws on both STM capacity and the quality of phonological representations (and/or working memory resources). Increased short-term memory capacity probably helps children to maintain the entire verbally presented mathematical problem in temporary storage (Baddeley, 1986; Raghobar, Barnes, & Hecht, 2010). In addition, STM capacity may be especially important when children need to rule out extraneous information, decide on arithmetic strategies, and execute calculation procedures. It also is likely that short-term storage is taxed when children are solving verbally presented math problems because they are holding information in mind while processing additional information to arrive at a solution (Swanson, 2004). Thus, solving applied problems may be associated with an increased verbal load compared with solving numeracy problems.

With regard to RAN, correlational results indicated that RAN was significantly correlated with posttest math achievement, such that more efficient access and retrieval of phonological information was associated with higher numeracy scores and higher applied problems scores at the end of kindergarten. However, RAN's predictive associations with numeracy and solving applied problems lessen when fluid intelligence and other PPAs were included in the statistical model, and they essentially disappeared when pretest math achievement was statistically controlled. It is important to note that our findings differ from those reported in other studies, and these differences may be explained by methodological differences among studies. For example, that Koponen and colleagues (2013) found RAN to be predictive of later math achievement but we did not can probably be explained by the current study's inclusion of fluid intelligence and prior levels of math achievement in the analyses. Similarly, the study by Hecht and colleagues (2001), which found RAN to be predictive of math achievement, did not statistically control for individual differences in fluid intelligence even though

it did control for differences in vocabulary and prior math achievement. Another methodological difference between the current study and that of Hecht and colleagues is that the latter study included a latent RAN variable composed of six indicators, three of which were RAN for digits. Other research (e.g., Clarke & Shinn, 2004) has shown that similar number fluency measures are indeed predictive of young children's achievement with solving applied problems. Therefore, it is possible that a domain-specific relation exists between RAN tests and math achievement such that RAN for numbers is important but RAN for objects is not. Finally, the nature of the math outcomes used in the current study may also help to explain the weak relations identified between RAN and math achievement. Specifically, the current study's math outcomes had scores that indexed accuracy of children's responses without regard to how long it took children to complete the test items. One would expect math measures that consider both accuracy and latency in their scoring to be more closely associated with RAN.

With regard to vocabulary, its predictive relations with math achievement were weaker than expected. That vocabulary was not predictive of numeracy achievement is consistent with other research and suggests that early numerical skills may develop relatively independent of general language ability (e.g., Feigenson, Dehaene, & Spelke, 2004; Geary, 2007; Gelman & Butterworth, 2005). For applied problems achievement, vocabulary significantly predicted unique variation in scores within the processes model. This finding is consistent with that reported by LeFevre and colleagues (2010), suggesting that vocabulary is related to math achievement on measures that require language-mediated responses. However, after controlling for prior numeracy skills within the full model, vocabulary was not a significant predictor of unique variation for applied problems. This collective pattern of results suggests that children largely draw on their early number competencies (i.e., autoregressor) and, to a lesser extent, draw on their early geometric and spatial skills (i.e., fluid intelligence), long-term phonological representations, and short-term storage resources when solving numeracy and applied problems.

Another interesting finding from the current study was that at the end of the year, children's classrooms differed in their average scores on applied problems but not in their average scores on numeracy. The absence of kindergarten classroom effects for numeracy may be attributed to the preverbal or innate nature of early number competencies (Feigenson et al., 2004). Arguably, two core systems underlie early number competencies. The first is an approximate magnitude system that enables inexact estimation of relatively large quantities, magnitudes, or size (Geary, 2007). The second system provides for precise representation of small numbers of individual objects (sets of three or four items) and represents information about their continuous or non-numerical properties (see Feigenson et al., 2004). Both systems become integrated with the symbolic verbal number system and provide a foundation for the development of more sophisticated mathematics skills (Feigenson et al., 2004; Geary, 2006, 2007; Spelke, 2000). Because these systems are preverbal, classrooms of children with minimal formal education would not necessarily be expected to significantly differ with respect to corresponding mathematics skills. In contrast, classroom variation in competence with applied problems may be due to classroom instructional practices that target children's knowledge of the verbal number system (Case & Griffin, 1990; Geary, 1995; Levine et al., 1992). For instance, it may be that classrooms that practiced solving story problems or classrooms that used a particular mathematics curriculum were those that, on average, scored better for applied problems.

Finally, it is important to note that the results of the processes model and the full model are conditional on the variables included in the analyses. When the effects of the autoregressor are not taken into consideration as in the processes model, there is a risk of overestimating the true relations between the predictors and the math outcomes. In contrast, when the effects of the other variables are conditional on the autoregressor as in the full model, there is a risk of underestimating the true relations between the predictors and math outcomes. This is because some predictive variance in the math outcomes is shared by the autoregressor and the other predictors. This shared variance is typically ascribed to the autoregressor even though cognitive and linguistic processes may have had a role in the development of early numeracy. Despite this caution in the interpretation of the current findings, the strong autoregressor effects suggest that cognitive and linguistic processes might not be as uniquely influential in the development of mathematics as authors of previous studies have

asserted. The magnitude of their true influence probably lies somewhere between the magnitudes found in the full and processes-only models.

### *Limitations and future studies*

First, the current findings should not be generalized beyond the population of ethnic minority kindergarten children in Title 1 schools in the United States. For example, the relations between cognitive and linguistic skills and mathematical outcomes may differ in other populations of children whose profile of achievement is more characteristic of typical development and learning in kindergarten. Second, the current study did not include a pretest specific to applied problems. This may have influenced the difference in the variance accounted for by the numeracy autoregressor in end-of-year numeracy ( $R^2 = .49$ ) compared with applied problems ( $R^2 = .32$ ). As a consequence, variables such as vocabulary and STM may be accounting for more variance in applied problems, a measure that has a relatively high verbal load, than expected had an autoregressor specific to applied problems been employed in predictive analyses. Thus, using the numeracy pretest as the autoregressor in analyses concerned with the prediction of applied problems may have underestimated the true autoregressive relation for this outcome. As a result, this study may have overestimated relations of the other predictors with applied problems. Although this may be a limitation, it should be noted that the numeracy pretest scores correlated similarly with applied problems posttest scores ( $r = .56$ ) and numeracy posttest scores ( $r = .70$ ). In addition, the correlation between numeracy pretest and applied problems scores is consistent with other reports (e.g., [Jordan et al., 2009](#)), suggesting that the current findings are generalizable. As it may be, future studies could benefit from including a true autoregressor for applied problems. Third, as illustrated in our findings, the relations between cognitive processes and mathematics development is complex. It is unclear how specific spatial and geometric skills as indexed by our measure of general fluid intelligence are related to domains of mathematics development. Therefore, additional research is needed to disentangle the relations among specific spatial and geometric skills, general nonverbal ability, and specific mathematical competencies. Fourth, to rule out the possibility that null effects concerning RAN were a consequence of how we measured RAN, how we measured math achievement, or both, future work should include RAN for digits, RAN for objects, and math outcomes that do and do not include a timed component. Such an assessment battery is needed to disentangle the relations between RAN and mathematics and to investigate the potential for a domain-specific effect (e.g., RAN numbers) or a method-specific effect (i.e., timed tasks). Finally, we did not include measures that directly assessed working memory or executive functioning. Working memory is one of several cognitive processes that are involved in the control and coordination of information and is included under [Brocki and Bohlin's \(2004\)](#) definition of executive functions. As it may be, future studies would benefit from including measures that explicitly assess the central executive, phonological loop, and visual–spatial sketchpad within a comprehensive framework that considers the effects of cognitive and linguistic predictors, including prior mathematical ability, on math achievement in kindergartners as in the current study. Doing so could help to explicate how the brain processes mathematical information during the first year of children's formal schooling and the unique roles of correlated abilities such as working memory, phonological loop functioning, PA, and vocabulary.

### *Conclusion*

Early math achievement is critical for placing children on a positive educational trajectory. After controlling for autoregressive effects and classroom nesting, fluid intelligence and PA significantly predicted posttest numeracy achievement, but these predictors accounted for a small increment in predictive accuracy over the autoregressor-only model. Similarly, fluid intelligence, PA, and STM significantly predicted competence with applied problems after controlling for autoregressive effects and classroom nesting. The current findings extend the extant literature and provide further evidence that PA is linked to the development of a wider array of math domains than just arithmetic achievement. They also indicate that fluid intelligence and/or early spatial and geometric understanding are important to the development of both numeracy and applied problems. In addition, a more

fine-grained view of development was provided and suggests that children rely on the quality of long-term phonological representations rather than their short-term storage when solving numeracy-related problems. In contrast, solving applied problems appears to rely on both the quality of children's phonological representations and their short-term storage capacity. Although the relations of fluid intelligence, PA, STM, and math outcomes are reliable and arguably important, they were small in comparison with children's pretest scores. Thus, the current findings underscore the importance for providing planned math instruction in preschool and screening for mathematics learning difficulties at kindergarten entry. Engaging children in early number activities can provide them with needed experiences for developing early number competencies, whereas screening for mathematics learning difficulties in kindergarten permits identified children to participate in prevention services before the onset of severe deficits.

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