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EARLY NUMERICAL COMPETENCE AND NUMBER LINE TASK PERFORMANCE IN KINDERGARTENERS

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ABSTRACT

This work aims to evaluate the relationship between early numerical competence in kindergarteners and their numerical representations as measured by the number line task (NLT). Thirty-four 5-year-old children participated in the study. Children's early performance on symbolic and non-symbolic numerical tasks was considered to determine which was a better predictor of NLT performance. Children completed an early number competence standardized test comprising symbolic semantic tasks (Arabic digit comparison and Arabic digit linear order), lexical tasks (numbers recognition and numbers reading), and non-symbolic semantic tasks (dots comparison and picture linear order), and the NLT 0–100. The relationship between early number competence (both symbolic and non-symbolic) and performance on the NLT was analyzed using a regression model in which the predictors were identified through a forward selection based on the use of the index BIC (Bayesian Information Criterion). Results show that symbolic number knowledge tasks (Arabic linear digits order and Arabic digits comparison) are the best predictors of performance on the NLT. This suggests that knowledge of the semantics of Arabic digits is more important than non-symbolic quantity knowledge in predicting number line task accuracy among young children. This finding brings additional evidence to the debate on the relationship between non-symbolic numeral knowledge and symbolic number processing.

KEYWORDS

Early numerical competence, number line task, kindergarteners, symbolic number knowledge

1. INTRODUCTION

In numerical cognition research, it is widely believed that numeracy is founded upon an early non-symbolic system of numerical representation in which children implicitly discriminate between smaller and larger collections of objects, and represent and manipulate numerical information without using symbols (see Carey, 2001; Feigenson et al., 2004). Dehaene (1997) calls the cognitive foundation of mathematics number sense, and argues that it is represented by a mental number line, an analog magnitude representation system that serves as a core representation of quantities.

Non-symbolic numerical magnitude knowledge provides potentially useful referents for learning the magnitudes of numbers expressed symbolically. Following this approach, children acquire a symbolic system to represent numbers by repeatedly linking a quantity with its associated numeral, which is then mapped onto a pre-existing, approximate non-symbolic number system, or ANS (Barth et al., 2005; Mundy & Gilmore, 2009; see Carey, 2004 for an alternative view). This mapping idea is mainly supported by studies investigating children with number processing difficulties or developmental dyscalculia who show problems with discriminating both non-symbolic and symbolic numerosities (e.g., Landerl et al., 2009; Mazzocco et al., 2011; Mussolin et al., 2010). Additional empirical evidence supporting the mapping idea comes from studies investigating the distance effect, a phenomenon by which more distant magnitudes (e.g., 2 and 7) are easier to differentiate than neighboring numbers (e.g., 8 and 9) (for a review see Gallistel & Gelman, 2005). This effect has been observed both in symbolic and non-symbolic comparison tasks (Holloway & Ansari, 2009; Rousselle & Noël, 2007).

While the idea that early non-symbolic numerical magnitude knowledge can be the foundation for successive symbolic numerical knowledge is well established, the nature of this relationship is still unclear (see, for instance, Carey, 2001; Siegler, 2016). A number of developmental studies suggest that the link between non-symbolic and symbolic numeral knowledge is weak; some have found that non-symbolic magnitude discrimination is weakly correlated with overall math achievement (see Chen & Li, 2014 and Fazio, et al., 2014 for meta-analysis), while others have found no evidence that training children in non-symbolic numerical magnitude discrimination helps develop knowledge of symbolic magnitudes (Dewind & Brannon, 2012; Park & Brannon, 2014; Wilson, et al., 2006).

The number line task (NLT) proposed by Siegler and Opfer (2003) is commonly used to investigate children's numerical representation ability. The NLT reflects how children represent numerical magnitudes on a mental number line, asking them to translate between numerical and spatial representations without assuming knowledge of specific measurement units; in its classic version, a written or spoken number is presented to a child and he/she is asked to locate its position on a physical number line representing a certain numerical range.

The number line task has been used to measure how children represent numbers and how this representation changes over time. In Geary's words, target number placements on the external number line "that conform to the natural logarithm of the numbers, may reflect dependence on the core system that represents approximate magnitudes (Feigenson et al., 2004; Gallistel & Gelman, 1992), whereas linear placements indicate the child is learning the mathematical number line" (Geary, 2011, p. 4). Berteletti et al. (2010) observed that a transition from approximately logarithmic to approximately linear distributions in children's representations of whole number magnitudes in the NLT takes place between ages 3 and 6.

Following this line of reasoning, NLT performance at a young age should be strongly influenced by non-symbolic knowledge. It is not clear, however, how representations on the number line task relate to general non-symbolic representations.

There are few studies directly investigating the relationship between NLT and early symbolic and non-symbolic numeral knowledge, and the studies that do exist have obtained mixed results. Sasanguie et al. (2013) tested NLT and both symbolic and non-symbolic tasks in the same sample of 6- to 8-year-old children. They found a significant correlation between NLT performance and a symbolic comparison task, but no correlation with non-symbolic tasks. Bertelletti et al. (2010) investigated the NLT response pattern (logarithmic vs. linear) in 3- to 6-year-old children and found that a better knowledge of both symbolic and non-symbolic magnitudes was correlated with a more linear response pattern on the NLT.

In the present study, we examined the relationship between kindergarteners' NLT performance and their symbolic and non-symbolic numeral knowledge. Children were tested on several tasks: symbolic semantic tasks (an Arabic digit comparison task and an Arabic digits linear order task), lexical tasks (numbers recognition task and Arabic numbers reading task), and non-symbolic semantic tasks (dot comparison and a task in which different objects had to be ordered from smaller to bigger) to study which of them is predictive of NLT performance.

This work aims to clarify the nature of numerical representation systems involved in solving the number line task in young children, and to acquire additional evidence on the relationship between non-symbolic and symbolic number processing at an early stage of numerical acquisition. To our knowledge, this is the first work investigating predictors of children's NLT performance exploring both symbolic and non-symbolic abilities. If young children, to carry out the NLT task, draw on their pre-existing non-symbolic representations in addition to their symbolic number knowledge, we would find that both symbolic and non-symbolic abilities predict NLT performance. If the two kinds of knowledge are not strictly linked, as the findings of Sasanguie et al. (2013) suggest for older children, we would expect only one of them to predict good performance on the NLT.

To rule out possible interference and to explore other possible factors influencing NLT performance, we controlled for lexical symbolic numerical knowledge. We expected that semantic number processing would be more predictive of NLT performance than lexical knowledge.

2. METHOD

2.1 Participants

Thirty-four typically developing children were recruited from a public kindergarten in the town of Cagliari, Italy. The participants' average age was 69.24 months (SD = 3.32), range: 63-74 months (16 males and 18 females).

Children with clinically developmental delays (as identified by local health services) were excluded. Both the school and the children's parents agreed to let the students take part in the research study, and signed informed consent forms.

2.2 Procedure

The tests were administered in the month of March, from Monday to Friday, from 8:30 a.m. to 11:45 a.m. in one of the school's classrooms. The children were tested individually by an experienced psychologist. Each session lasted about 20 minutes.

Children's early number skills were evaluated through some of the tasks on the Battery for Numerical Intelligence from 4 to 6 years of age (BIN 4–6) (Molin et al., 2007), a standardized early math test providing norms for Italian children from 4 to 6 years of age. Two tasks assessing lexical competence with numbers (knowledge of numerical symbols) were proposed: a number recognition (NR) task (children were asked to select among three written Arabic numbers in the 1–9 range after hearing one spoken by the researcher) and an Arabic number reading (ANR) task (children were asked to read the Arabic digits in the 1–9 range). In addition, their semantic knowledge of numbers was evaluated though an Arabic digit comparison (ADC) task (children were asked to determine which of two Arabic digits was bigger, range 1–9). Children's ability to order Arabic digits was evaluated using the Arabic digits linear order (ADO) task (children were asked to order by magnitude the numbers written on five cards, range 1–5). To assess non-symbolic numerical knowledge, children were given a dots comparison (DC) task (children were shown two sets of dots with different magnitude and were asked, "Which is more?") and a task in which different objects cards (balloons of various size) had to be ordered from the smaller to the bigger (PO).

After this, the number line task NLT 0–100 was administered (Siegler & Opfer, 2003) using the software PyNLT (Massidda et al., 2015). Participants were shown on a laptop PC screen (14 inches) a 25-cm. horizontal line, marked 0 on the left end and 100 on the right end. In each trial, children were shown an Arabic digit (stimuli: 2 – 3 – 4 – 6 – 18 – 25 – 48 – 67 – 71 – 86, corresponding to Sets A and B for the interval 0–100 in Siegler & Opfer, 2003), and were asked to click with the mouse on the number line where they thought the target number should be placed. Each stimulus appeared twice.

Finally, we assessed children's IQ using the Colored Progressive Matrices test in the Italian standardization (Belacchi et al., 2008) to control for the influence of general intelligence on NLT performance. For all symbolic and non-symbolic numerical competence tasks, the number of correct answers was used as an index. The accuracy of a child's NLT estimates was assessed by calculating for each child her/his percentage of absolute error (PAE, Siegler & Booth, 2004) as follows: |[(Estimate – Estimated Quantity) / Scale of Estimate] *100|. For example, if a child was asked to locate "18" on the 0–100 number line, and placed the mark at the location that corresponded to "10," the percentage of absolute error would be 8% |[(10-18)/100]*100|.

We conducted a preliminary analysis of children's NLT estimation patterns by fitting linear and logarithmic functions to the group means and to each individual child (Siegler & Opfer, 2003). Thirty of the 34 participants were identified as having a logarithmic estimation pattern; the other four showed a linear pattern.

Descriptive statistics of the different experimental measure were calculated: means, standard deviations, and minimum and maximum scores (see Table 1).

Table 1. Descriptive statistics of children's performance in symbolic and non-symbolic tasks (n = 34)

Task type	Task name	Index	Mean	sd	Min	Max
Symbolic task	Number line task (NLT)	Percentage of Absolute Error (PAE)	21.75	9.67	21.75	9.67
	Lexical task	number recognition (NR)	8.82	0.58	7.00	9.00
		number reading (ANR)	8.88	0.41	7.00	9.00
	Semantic task	comparison of Arabic digits (ADC)	10.68	0.81	8.00	11.00
	Linear order task	Arabic digits linear order (ADO)	4.15	1.88	0.00	5.00
Non-symbolic task	Semantic task	Dots comparison (DC)	9.79	0.41	9.00	10.00
	Linear order task	Objects linear order (PO)	5.41	2.09	1.00	7.00

We examined the relations between symbolic and non-symbolic tasks and NLT performance using the correlation index Pearson's r (see Table 2). Our analysis shows a significant negative correlation between NLT performance and the comparison of Arabic digits (ADC), r (34) = -.443, p < .01, and a significant negative correlation between NLT performance and the Arabic digits linear order task (ADO), r (34) = -.601, p < .001; in other words, the better the child is at deciding which Arabic digit is larger or smaller and at ordering digits by magnitude, the more accurate were her/his estimations on the NLT. Finally, a strong correlation between the two lexical symbolic tasks (number reading and number recognition) was found, r (34) = .681, p < .001. No other significant correlations were found.

Table 2. Spearman's *rho* (top-right half) and Pearson's *r* (low-left half) correlation between symbolic and non-symbolic early numerical competence tasks. PAE: percentage of absolute error; NR: numbers recognition; ANR: numbers reading; ADC: comparison of Arabic digits; ADO: Arabic digits linear order; DC: dots comparison; PO: objects linear order

	PAE	NR	ANR	ADC	ADO	DC	PO
PAE		-0.333	-0.251	-0.331	-0.451 **	-0.019	0.101
NR	-0.131		0.645 ***	0.103	0.136	-0.158	-0.034
ANR	-0.075	0.681 ***		-0.143	0.152	-0.158	-0.161
ADC	-0.443 **	0.004	-0.119		0.207	0.112	0.002
ADO	-0.601 ***	0.137	0.260	0.213		0.157	0.205
DC	-0.150	-0.158	-0.149	-0.024	0.158		0.182
PO	-0.012	-0.139	-0.154	-0.152	0.108	0.172	

*** *p* < .001; ** *p* < .01; * *p* < .05.

Moreover, we examined the relationship between gender, age, and IQ and performance to exclude the possibility that correlations were due to differences in gender, age, or general ability. To do this, we performed a multiple regression analysis with the NLT percentage of absolute error (PAE) as dependent variable and gender, age, and IQ as predictors. These three predictors together explain only 2% of variance; thus we conclude that these variables did not influence NLT performance.

Finally, the relationship between NLT performance (with the PAE as dependent variable) and both early symbolic number competence (semantic, linear order, and lexical tasks) and early non-symbolic number competence (dot comparison and objects linear order) as predictors was examined adapting a regression model in which the predictors were identified through a forward selection based on the use of the Bayesian Information Criterion (BIC) index (Schwarz, 1978), which selected the best model from a Bayesian perspective.

Our results showed that in the best final selected model, NLT accuracy was predicted by the Arabic digits linear order (ADO) task with β = -.53, p <.001, and by the comparison of Arabic digits (ADC) task with β = -.33, p = .019 (see Figure 1), which together explained 46% of the variance in performance on the NLT, while the other investigated skills were not predictive of NLT performance.

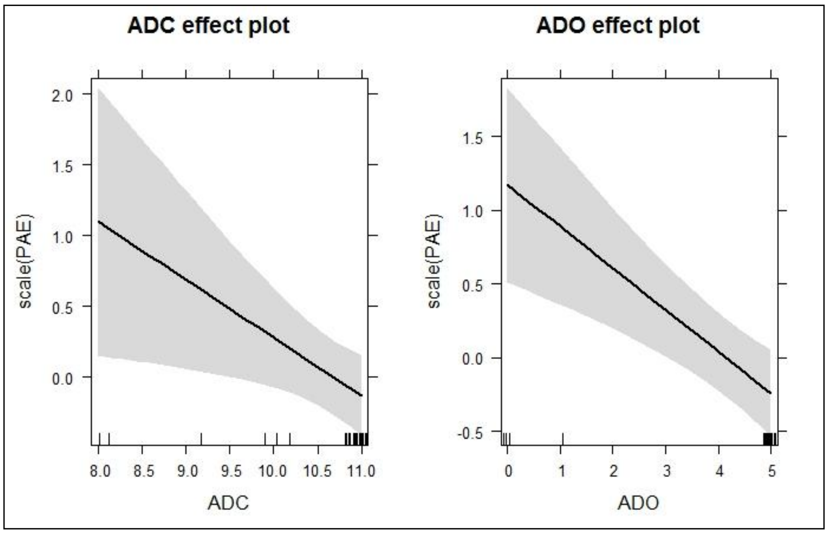

Figure 1. Effects reported in multiple regression with the dependent variable PAE in the vertical axes and the predictors Arabic Digits Comparison (ADC) and Arabic Digits Order (ADO) in the horizontal axes. The gray shadows represent the 95% confidence intervals

3. DISCUSSION AND CONCLUSIONS

The NLT is extensively used in research on numerical cognition because children's performance on this task has been identified as a predictor of future mathematics skills (e.g. Booth & Siegler, 2006; Booth & Siegler, 2008; Geary et al., 2008; Geary, 2011; Siegler, 2016). Moreover, several authors (Siegler & Opfer, 2003; Boot & Siegler, 2006; Siegler & Booth, 2004; Siegler, 2016) have considered the change over time in pattern performance on the NLT (linear vs. logarithmic) as an indicator of the shape of the mental number line, the analogical core representation of quantities hypothesized by Dehaene (1997).

In our work, we explored the ability to represent numbers on a physical number line in 5-year-old children, with the aim of determining how NLT estimation accuracy is related to both general non-symbolic representation of quantities and to symbolic number knowledge. We decided to explore the issue in kindergarteners since in children of this age, the symbolic system is starting to form but is still closely related to the early non-symbolic system.

To study the issue, the 0–100 NLT was proposed to kindergarteners. Moreover, symbolic semantic tasks (comparison of Arabic digits and linear ordering of Arabic digits) and non-symbolic tasks (dot comparison and linear object ordering) were administered to the participants. To control for the influence of numerical symbol knowledge, two lexical tasks were also administered (a number recognition task and number reading).

The results of our analysis show that kindergarten children's performance on the NLT depends exclusively on symbolic knowledge; together, performance on the comparison of Arabic digits task and the Arabic digits linear order task explained 46% of the variance in NLT performance, while non-symbolic numeral knowledge tasks did not predict NLT performance.

Although our study is the first, to our knowledge, to investigate NLT predictors among children in this age range, other studies on younger children have reported correlations between symbolic and non-symbolic tasks and NLT. For example, Berteletti et al. (2010) found that in 3- to 6-year-old children, both symbolic and non-symbolic magnitude knowledge were correlated with a linear NLT response pattern. Our results are not in line with this finding. In the Berteletti et al. study, however, the type of number line representation (logarithmic vs. linear) was considered in the correlation analysis, whereas we used PAE scores. This may explain the difference between our findings and those of Berteletti et al.

Our results are instead in line with the findings of Sasanguie et al. (2013) that in older children there is a correlation between NLT accuracy (PAE) and performance on a symbolic comparison task but not between NLT accuracy and performance on non-symbolic tasks such as magnitude comparison.

Our data, gathered from young children who have not yet begun formal schooling, show that the NLT is solved mainly based on symbolic abilities and suggest that, at least at this age, NLT performance is loosely dependent on the core system that represents approximate magnitudes, despite the fact that children were tested on an unfamiliar number range and that the vast majority of them (30 out of 34) showed a logarithmic pattern of response.

Moreover, we found that number lexical knowledge did not predict NLT performance. Thus, it seems that the ability to identify the numerical symbol corresponding to a spoken numeral or to correctly read an Arabic digit are not crucial to the NLT, whereas semantic knowledge about the meaning of a numeral is necessary.

A further interesting finding emerging from our analysis is that the two non-symbolic quantity knowledge tasks are not correlated with any of the tests exploring symbolic number knowledge. This result seems in line with studies that have considered the influence of symbolic and non- symbolic quantitative tasks on math skills in kindergarteners. For instance, in a study on 5-year-old children, Sasanguie et al. (2013) found no correlation between symbolic and non-symbolic comparison tasks. Sasanguie et al. (2012) investigated the effects of a dot comparison task and a symbolic numbers comparison task on math performance in kindergarteners, first graders, second graders, and sixth graders. Only the comparison of symbolic numbers predicted math performance in 5-year-olds. Another study by Sasanguie et al. (2014) found no association between the accuracy of the performance on a dots comparison task in 5-year-old children and the performance in a symbolic comparison task presented to the same children six months later.

The present study covered a little investigated area of numerical development yielding to new and interesting results. Its exploratory nature and relatively small sample size do not permit us to draw firm conclusions on the relationship between non-symbolic and symbolic number processing, or on the early math knowledge underlying NLT performance, but our findings suggest that symbolic knowledge about quantities and numerals may play a stronger than expected role in the stage of development in which children are not yet very familiar with culturally determined numeric symbols. Further investigation on these topics should be undertaken. In particular, a better knowledge of early magnitude estimation ability and its underlying mechanism will be critical for better understanding important stages in numerical development, such as learning to connect symbolic numbers to their non-symbolic referents or accurately representing the magnitude of rational numbers, both of which are considered crucial in recent theories on numerical development (e.g., Siegler, 2016).

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