Computation Error Analysis:

Students with Mathematics Difficulty Compared to Typically Achieving Students

Gena Nelson

American Institutes for Research

Sarah R. Powell

University of Texas at Austin

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Author Note

Gena Nelson, American Institutes for Research; Sarah R. Powell, Department of Special Education, University of Texas at Austin.

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Correspondence concerning this article should be addressed to Gena Nelson, 1050 Thomas Jefferson St. NW, Washington, DC 20007. Email: gnelson@air.org

Abstract

Though proficiency with computation is highly emphasized in national mathematics standards, students with mathematics difficulty (MD) continue to struggle with computation. To learn more about the differences in computation error patterns between typically achieving students and students with MD, we assessed 478 3rd-grade students on a measure of mathematics computation. Results indicated that using the wrong operation was the most common identifiable error for all students. Students with MD had similar accuracy rates for item categories (e.g., addition items) compared to typically achieving students, but students with MD consistently had more variability in incorrect item responses. This study has implications for efficacious computation instruction for students in the elementary grades.

Keywords: computation, error analysis, mathematics difficulty

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National mathematics standards emphasize that, by the end of second grade, students should demonstrate fluency with addition and subtraction facts and understand the algorithms for adding and subtracting multi-digit numbers (National Governors Association Center for Best Practices & Council of Chief State School Officers, [NGA& CCSSO] 2010). Fluency with facts related to multiplication and division should be established by the end of third grade, and students should understand and apply algorithms for solving multi-digit multiplication and division by the end of fourth grade. Even with this emphasis on developing strong computation skill during the elementary grades, students continue to struggle with whole-number computation beyond elementary grades and into secondary school (Calhoon, Emerson, Flores, & Houchins, 2007). With this in mind, we conducted a study to investigate differences in the frequency and types of computation errors among third-grade students identified as at-risk for mathematics difficulty (MD) and not at risk for MD.

In this introduction, we review the importance of students developing computation skills, describe previous research on computation errors, discuss the limitations of this previous research, and provide the purpose and research questions guiding this study.

Importance of Computation

Computation is defined as mathematics problems in the four primary operations (i.e., addition, subtraction, multiplication, and division) that involve an understanding of place value and algorithms. To effectively perform different computations, students must have a strong understanding of mathematics facts and the place-value system (Russell, 2000). Beyond the elementary grades, mastery of computation provides the foundation for more complex mathematics skills, such as problem solving and algebra (Fuchs et al., 2014; Siegler et al., 2012). Because of the role of computation within higher-level mathematics, computational fluency was named goal for all students, as outlined on national sets of mathematics standards (e.g., National Council of Teachers of Mathematics, [NCTM] 2000).

Because developing efficiency with computation is important for higher-level mathematics, it is important to understand why students have difficulty with computation. Typically, students with MD have demonstrated lower computation performance than students without MD (Fuchs et al., 2008). One difficulty with computation or students with MD stems from the learning and remembering of arithmetic facts (Chong & Siegel, 2008; Geary, 1993; Geary & Hoard, 2001; Gersten, Jordan, & Flojo, 2005; Jordan, Hanich, & Kaplan, 2003a). As Mabbott and Bisanz (2008) noted, fluency with arithmetic facts allowed for more fluid computation procedures and less taxation on working memory for students with MD. Beyond facts, students with MD have also demonstrated difficulty performing computation procedures (Miller & Milam, 1987; Zhang, Ding, Barrett, Xin, & Liu, 2014). Results of teacher-reported student computation difficulties indicated that, through high school, students with MD made regrouping errors, disregarded decimals while performing computation, and misread operation signs within computation problems (Bryant, Bryant, & Hammill, 2000).

With regard to errors, students have added both factors in a multiplication problem (e.g., $3 \times 4 = 7$) or committed errors that suggested students were unaware of the correct procedures or symbol means ($5 \times 2 = 52$; Siegler, 1988; Zhang et al., 2014). In other instances, students with MD executed errors unrelated to the specific operation featured in the problem (e.g., subtraction in multi-step division; Bryant et al., 2000; Miller, & Milam, 1987). Because students with MD struggle with many aspects of computation and because computational skills are directly related to mathematics performance in later grades (Geary, 2011; Primi, Ferrão, & Almeida, 2010), there is a need to provide focused instruction on computation.

Current intervention efforts in the elementary grades focus on individualizing intervention efforts for students with MD who demonstrate low or no response to intervention (Powell & Stecker, 2014). Suggestions for intensification of intervention include breaking down problems into smaller steps, using worked examples, and using manipulatives. To inform instruction for students without MD, it is necessary to conduct an examination of the types of computation errors students frequently commit. This type of analysis could change classroom practice. To inform intensive intervention for students with MD, in which teachers may want to use worked examples and break down problems into smaller steps, an understanding of error patterns for students with MD could be used to design and inform instructional intervention practices. This is especially true if error patterns were substantively different for students with and without MD. Researchers have collected empirical data about error patterns, and we discuss this collection of research in the next section.

Research on Specific Computation Errors

As described by Ashlock (2010), teachers can improve mathematics instruction when using student work as a diagnostic. When students solve problems incorrectly, an in-depth analysis of misconceptions and error patterns can help teachers understand if errors are idiosyncratic or whether errors stem from misunderstandings in conceptual or procedural learning. Ashlock notes that error analyses provide an initial snapshot of student understanding; teachers should also employ open-ended assessments and interviews to get at the root of error patterns. In this section, we describe the research base related to computation error patterns to understand which errors emerge as persistent.

McLoed and Armstrong (1982) evaluated teachers' responses to a survey regarding observations of students with learning disabilities (LD) in sixth through 12th grade. Teachers frequently observed student deficits in computation of multiplication and division of whole numbers and computation involving fractions, decimals, and percentages. Bryant et al. (2000) also gathered teacher-reported data regarding computation difficulties. Teachers reported that behaviors within multi-step problems and regrouping errors explained the most variance between students with and without LD. Based on the idea that many students with LD struggle with many aspects of computation, researchers have introduced scoring techniques to identify specific errors in computation with the goal of providing teachers with valuable information to address computation deficits with instruction. Two common scoring techniques to identify specific computation errors include *individual error analysis* and the *error ratio analysis technique*.

Miller and Milam (1987) used individual error analysis to examine the types of errors committed by students with LD on one multiplication item and on one division item. The authors reported a wide variety of errors. Students responded with 96 different answers to the multiplication item and 93 different answers to the division item. Results indicated that many student mistakes did not reflect errors of multiplication and division. In other words, 42% of the mistakes on the division problem were made in the subtraction and multiplication used within a division problem. Räsänen and Ahonen (1995) also analyzed computation errors of individual participants in their study; the authors, however, only administered items with whole number addition, subtraction, and multiplication (i.e., division and items with fractions were not included). Räsänen and Ahonen compared the computation performance of typically achieving students and students with MD in a secondary analysis of student data. Students in third through sixth grade with MD made significantly more computation errors than typically achieving students in all categories of errors coded by the authors including but not limited to, wrong operation, algorithm errors, and rule errors in addition and subtraction, and random errors.

In another study, Cox (1975) examined individual errors on a systematic level (errors that students committed at least three times on the same problem type) across second through sixth grade for addition, subtraction, multiplication, and division. Authors identified the types and frequency of systematic errors across the sample. For example, some of the most common systematic errors with addition were: adding each digit separately (e.g., for 34 + 2 student adds 3 + 4 + 2), does not add the "regrouped 10" number (e.g., 48 + 3 = 41), and subtracts instead of adds. Cox reported that of the students who committed systematic errors, 23% of those students were still committing systematic errors one year later. Cox examined the individual errors that students committed and determined that multiplication items resulted in 67 different systematic errors, which was the largest number of different errors across the four operations.

Cawley, Parmar, Yan, and Miller (1996) introduced an error ratio analysis technique as a class-wide or group scoring procedure for computation. Error ratio analysis was considered more time efficient than individual item error analysis and provided more instructional information about students than the total correct scoring method. To perform error ratio analysis, first, the number of incorrect answers per problem type is counted (e.g., number of wrong answers for all single-digit by single-digit multiplication). Then, the number of different types of errors made per problem type is counted (e.g., calculation, regrouping). The error ratio was the ratio of the number of different incorrect responses to the total number of incorrect responses. Larger ratios of approximately 70:100 or greater, symbolized a larger number of different error types within a classroom compared to the actual number of errors; a smaller ratio, less than 30:100, represented only a few different errors (Cawley et al., 1996; Miller & Carr, 1997).

Cawley et al. (1996) compared typically achieving students and students with LD regarding response variability on 92 computation items (i.e., addition, subtraction, multiplication, and division). Cawley et al. concluded that items with higher ratios were placed later in the curriculum and represented multi-step problems that required students to perform many steps to reach the correct answer; therefore, leading to more variability in student responses. Items with lower ratios were likely due to simple procedural errors, such as performing the wrong operation. Cawley et al. also reported that many errors in multiplication and division were actually the result of incorrect addition or subtraction in one of the earlier steps of solving the item.

Though most studies that have conducted error analysis have examined computation with whole numbers, Malone and Fuchs (2016) examined individual responses on items specific to fractions as a method for assessing understanding of part-whole concepts. Participants in this study were identified as at-risk in mathematics by performing below the 35th percentile on a mathematics screening measure. Participants were administered items that required them to correctly order and estimate fractions according to value, demonstrate part-whole understanding in word problems, and verbally explain why one fraction was greater than or less than another fraction. Results indicated that when ordering fractions, one of the most common errors students committed incorrectly applying whole number values to fractions (i.e., they incorrectly ordered fractions based on either the numerator or denominator). Another common error was that students treated fractions with the smallest denominator as the largest fraction, regardless of the numerator. This indicated that students may have been aware that the larger denominator represented larger fraction parts. Finally, the authors reported that students at-risk for MD had difficulty verbally explain their procedures for comparing fraction values.

Limitations of Previous Studies

In the present study, we aim to address the limitations of previous studies on computation error analysis in several ways. First, much of the research on error analysis was conducted several decades ago (e.g., Cawley et al., 1996; Cox, 1975; Miller & Milam, 1987). Based on a search of the literature, even the most recent error analyses were conducted more than 20 years ago, which is prior to national initiatives to raise standards for student learning, such as the Common Core State Standards (NGA & CCSSO, 2010) or the NCTM content and process standards (2000) and Focal Points (2014). We would anticipate that instructional practices related to computation may have changed since the inception of these different sets of standards with higher expectations related to computation. A primary goal of this study was to provide current estimates of students' computation abilities and specific information regarding types of errors that students commit, especially as it relates to students with MD.

Second, ambiguous results from studies are difficult to generalize to all students and specific subgroups of students (such as students with LD or MD) for several reasons. For example, studies analyzed only one item type (e.g., multiplication), compiled results from several item types and generalized to all computation, included multiple grade levels with students of varying computation abilities, or reported data on a small number of items. We examined individual errors committed by third-grade students on a large set of addition, subtraction, multiplication, and division items with whole and rational numbers to increase generalization of errors to other similar computation problems. We focused on third-grade students because the national standards and recommendations for instruction (NCTM, 2000; NGA & CCSSO, 2010) emphasize that by the end of third grade, students should be able to solve problems involving the four operations, understand the properties of the four operations, and understand and use place value to solve problems involving the four operations.

Purposes and Research Questions of the Present Study

The purposes of this study were to investigate the frequency and types of computation errors among third-grade students and determine differences, if any, according to students' risk of MD considering the frequency with which types of errors were committed. This information could inform classroom instruction and intervention development related to typically achieving students and students with MD. Specifically, we asked the following research questions:

1. What are the most common errors (e.g., miscalculation, wrong operation) on individual computation items and within categories of computation items for third graders (e.g., all addition items)?
2. Which items were most difficult for third graders to solve correctly, and which items and item categories resulted in the largest error ratios?
3. Were error ratios by item category different for students with MD versus typically achieving students?

Method

Participants and Setting

Participants ($N = 478$) were sampled from third-grade classrooms in a school district in a state in the Southwest. Demographics were available for 60% of participants. Approximately 51.4% of participants were female, and the sample identified as Hispanic (47.2%), Caucasian (25.9 %%), African American (11.7%%), Asian (6.7%), American Indian (5.7%), and other (2.8%). The average age of participants was 8 years, 8 months. Overall, 5.7% of the sample had been retained one grade level, 4.8% received special education services, and 44.8% were English Learners.

Measures

We measured computational knowledge with the Math Computation subtest of the *Wide Range Achievement Test 4* (WRAT4; Wilkinson & Robertson, 2006). The examiner read directions aloud and students worked independently. Students had 15 min to answer 40 written computation problems of increasing difficulty. According to the WRAT4 administration manual, students answering four or fewer written problems correctly are administered 15 oral arithmetic problems. In the current study, no students required administration of the oral arithmetic section of Math Computation. Students received one point for each correct answer. Because no students required administration of the 15 oral arithmetic problems, every student was awarded 15 additional points to their raw score. The maximum possible score is 55. As reported by Wilkinson and Robertson (2006), median reliability for students in third grade is .88. Cronbach's alpha for this sample was .82.

Procedures

The WRAT4 Math Computation was administered within a battery of mathematics assessments for a larger study during the second week of September; whole-class screening was conducted in one 45-min session with 15 min dedicated to the WRAT4. Examiners were graduate research assistants from a local university working on Master's or doctoral degrees in education-related fields. All examiners participated in a 2-hour training to review the screening protocols. During administration, all examiners read directly from a screening script.

For scoring, teams of research assistants entered student responses on 100% of the screening protocols for each measure on an item-by-item basis into two separate databases. We compared the databases and rectified all discrepancies to reflect the student's original response, ensuring data entry was 100% accurate.

Coding responses and errors. Student responses to each item on WRAT4 Math Computation were scored as correct or incorrect, and when student responses were incorrect, we recorded the error, when identifiable (e.g., wrong operation, regrouping, miscalculation). To code the errors, first, the authors created a list and description of potential errors from previous research on error analysis (e.g., Cox, 1975; Räsänen & Ahonen, 1995). This list was then expanded on for each item by the authors calculating what the answer would be for each problem if the student committed one of the errors. The list and answers for each error for each problem were agreed upon by both authors. For example, for an item "3 + 2" the authors pre-determined that any answer of "1" would be coded as "wrong operation- subtraction." From there, errors were coded systematically. Each error only received one code. When errors were not initially identifiable from the pre-determined list, authors worked together to determine what error a student may have committed to receive the incorrect answer. In instances where authors were not equally confident that a student may have committed a specific error (rather than guessing), or if authors could not determine the steps a student took to obtain an answer, the error was coded as "unknown." Initially, the first author coded all incorrect student responses according to the error that the student committed. Then, the second author coded errors for 30% of students for each item. Inter-rater reliability was calculated as total [agreements ÷ (agreements + disagreements) × 100]. The inter-rater agreement for this sample was 96%. Discrepancies between the pre-determined list of errors and the database were discussed and resolved. Please note that due to test copyright and security, any examples provided in tables or text are not representative of the WRAT4 Math Computation, and not all subtype information for individual items is presented in the tables.

Data Analysis

Regarding common error types, each student's response to every item on the WRAT4 Math Computation was coded for the type of error that was committed. The frequencies of error types were calculated for each item and similar items (e.g., all addition items). The item difficulty index was calculated as the proportion of students who answered an item correctly; the calculation of item difficulty included "blank" responses as "incorrect" responses. The error ratio represents the ratio of the number of different incorrect responses to the total number of incorrect responses (Miller & Carr, 1997) and was calculated as the number of different incorrect responses (n) divided by the total incorrect answers (k) or $[(n \div k) \times 100]$. When examining differences between students identified typically achieving or as having MD, students were categorized based on their performance on the WRAT4 Math Computation assessment. Students were identified as MD if they scored at or below the 25th percentile on the WRAT4 Math Computation, similar to other research related to students with or at-risk for MD (Hecht & Vagi, 2010; Jordan, Hanich, & Kaplan, 2003b; Locuniak & Jordan, 2008). Students were identified as typically achieving if they scored at or above the 35th percentile on the WRAT4 Math Computation; we intentionally selected the 35th percentile to allow for a gap between students we classified as typically achieving and students with MD (Vukovic & Siegel, 2010). When students have scores that fall near a cut score it is more difficult to discriminate between groups of students (i.e., students classified as scoring ≤ 25th percentile may not really be that different from students who score at the 26th percentile).

Of the 478 students who participated in this study, 120 students (25.1%) were identified as students with MD (i.e., scoring below the 25th percentile on the WRAT4 Math Computation) and 312 students (65.3%) were identified as typically achieving (i.e., scoring above the 35th percentile on the WRAT4 Math Computation). Overall, 46 fell in the "buffer zone" between the

25th and 35th percentiles and, therefore, were not included when examining the differences between MD and typically achieving students in error ratio analysis and percent correct by item category.

Results

The average score for all participants on the WRAT4 Math Computation was 25.33 (*SD* = 3.40). Originally, our intent was to use the first page of the WRAT4 Math Computation (items 1 through 25) to evaluate error patterns because the majority of items on the second page of the WRAT4 Math Computation (items 26 through 40) represented mathematics content typically introduced after third grade. For example, most of the items on the second page require operations with fractions and percentages. After an examination of the items on the first page of the WRAT4 Math Computation, we determined that students in this study consistently attempted items 1 through 21. Fewer than 72 students (15%) attempted items 22 through 25, and fewer than 90 students (19%) attempted items 26 through 40. Only one student responded correctly to items 22, 23, 24, and 25; therefore, the results of this study focus only on items 1 through 21.

Common Errors

We examined common error patterns for items 1 through 21, and Table 1 displays the common errors across items. Table 1 also provides criteria for the error as well as an example of the error. We discuss the results for common errors within specific item categories (e.g., subtraction with single digits) within the text.

Common errors across all items. On items 1 through 21, students committed 2,427 total errors, with 650 unique errors across items. Of these errors, the authors were able to identify 1,724 errors (71.0%); the remaining 29.0% of errors were not identifiable and were coded as unknown. The most common set of errors across all items was use of the wrong operation (20.0%), meaning that the student performed the incorrect operation in the item prompt (e.g., the student added instead of multiplied). The error of wrong operation-addition (i.e., the student added instead of subtracted on a subtraction problem) was the most common error of all the wrong operations (addition, subtraction, multiplication). Students also committed errors that suggested they had not been taught the correct algorithm or had not yet mastered the procedures for performing addition and subtraction. For example, across six items students added all of the digits as separate integers (e.g., $\frac{1}{3} + \frac{1}{3} = 1 + 3 + 1 + 3$; $22 + 6 = 2 + 2 + 6$) resulting in 227 (9.4%) of the total errors. In two other items, students performed subtraction with multi-digit numbers but committed a procedural error when they always subtracted the smaller integer from the larger integer, regardless of the integer's placement in the item. This resulted in nearly 200 (7.8%) of the total errors committed.

Other common errors included simple miscalculation errors with addition and subtraction, mistakes with regrouping, not completing all of the steps to obtain the correct answer, and reporting a close multiplication fact as the answer (i.e., student performed multiplication on the multiplication item but reported an incorrect fact in the same fact family; e.g., reported $3 \times 4 = 9$ due to reported the close fact of 3×3). Curiously, students also copied part or all of the item prompt and used this as their answer, which suggests that students may not have understood the steps to solve the item prompt. This type of error only accounted for 4% of the total errors committed and the error was recorded in 20 of the 21 items; however, there was not a pattern across problems or within students that would suggest this error was due to the students not hearing the directions correctly.

Common errors across specific items. Regarding single-digit addition and subtraction items, results indicate that students committed the most errors in both item categories of miscalculation and wrong operation. The proportion of errors due to miscalculation was nearly identical across single-digit addition items (32.7%) and single-digit subtraction items (31.8%); however, more errors in single-digit addition items were due to students committing the wrong operation (45.2%) compared to wrong operations in single-digit subtraction items (22.7%).

A somewhat similar pattern emerged with the multi-digit addition and multi-digit subtraction items. The second most common error in multi-digit addition was miscalculation (22.7%) and wrong operation in multi-digit subtraction (18.1%). Though, in both categories students more frequently committed other errors. Students displayed difficulty with regrouping procedures (30%) in multi-digit addition. With multi-digit subtraction, students commonly disregarded digit placement and subtracted the smaller integer from the larger integer (31.2%).

Other than the many unknown errors (41.8%) that students committed on single- and multi-digit multiplication items, the most frequent identifiable error was reporting a close multiplication fact as the correct answer (20.0%). The next most common error was coded as incomplete procedure (7.4%), and included instances where a student performed part of the multiplication problem but not all steps were completed so the recorded answer was incorrect. Regarding single- and multi-digit division items, students continued to display difficulty with items by performing the wrong operation. Students more frequently performed addition instead of division (31.9%) compared to performing subtraction instead of division (22.3%).

The final category of items we examined included addition with fractions. Students frequently treated the numerator and denominator of the fraction as separate integers (54.4% of errors), which resulted in students adding all of the digits in the problem to reach the solution.

Item Difficulty and Error Ratio Analysis

Results of the item difficulty and error ratio analysis by item on the WRAT4 Math Computation (items 1 through 21) are presented in Table 2, with results for item difficulty and error analysis according to item categories, and separated by typically achieving versus MD, are highlighted in Table 3. First, we calculated item difficulty (i.e., percentage of students who correctly answered the item) and error ratio at the individual item level. The calculation of item difficulty included “blank” responses as “incorrect” responses. Then, we analyzed trends across subtypes of items (e.g., all addition items).

Item difficulty. In this section, reported percentages represent item difficulty, or the percentage of students who answered the item correctly. Generally, item difficulty increased (i.e., fewer students answered the item correct) with each successive item on the WRAT4 Math Computation. More than 50% of students who attempted the items with single- and multi-digit computation with and without regrouping (addition and subtraction) and single-digit multiplication solved the items correctly; however, there was an obvious decrease in the number of students who attempted and solved correctly items that introduced single-digit division, subtraction with regrouping, addition with three- and four-digit numbers, and fractions.

Regarding items presented as whole numbers only, items that required the use of addition ($n = 6$) were answered correctly more often (72%) than items that required subtraction ($n = 6$; 62%). Fewer items ($n = 3$) required multiplication or division, and students had more difficulty (33% answered correctly) with those items than both addition and subtraction (72% and 62%, respectively). Regarding the subtype of whole-number addition, subtraction, multiplication, and division items, difficulty varied according to whether the item required regrouping and by the number of digits in the item. Students had less difficulty with items that did not require regrouping or were math facts ($n = 11$; 76% of students answered correctly) than items that required regrouping procedures (*n* = 4; 18% of students answered correctly). Furthermore, items that required whole-number operations where both operands were only one digit (e.g., 1 + 4) were easier (*n* = 8; 78%) for students to solve than items (*n* = 7) where at least one operand was multi-digit (e.g., 12 × 6; 40%). Although some students attempted the few items (*n*= 3) that contained fractions, students answered these items correct less often (2%) than items that contained only whole numbers (*n* = 18; 57%).

Error ratio. In this section, we converted the error ratio to a percentage for ease of interpretation (e.g., a ratio of 15:100 is represented as 15% and is the equivalent of the ratio 15 different incorrect answers to 100 total incorrect responses). Regarding the ratio of the number of *different* incorrect responses to the total number of incorrect responses, the types of items that had the largest average error ratio (i.e., more incorrect responses per total incorrect responses) values included multi-digit items (38%), multi-digit items that also required regrouping (38%), and items that required the use of addition to solve correctly (36%). On average, one-digit by one-digit operations had lower error ratios (26%), as did the three items that contained fractions (17%). Lower error ratio values represented items that had few different incorrect responses compared to the total number of incorrect responses. With regard to items that contained fractions, the majority of students (56%) who committed an error, committed the same error of adding all the digits as part of the fraction to arrive at the solution. The fact that most students committed the same error accounts for the low error ratio.

Particular items that resulted in larger error ratios, and therefore, much more variability in student response, included an item that required to students to fill in the missing number in a sequence (error ratio = 83%), and items with multi-digit addition. The specific multi-digit items that resulted in the largest error ratios included an item that required students to add three two-digit numbers without regrouping and another addition item that introduced numbers with three and four digits. These items produced 40 and 66 different incorrect responses, respectively.

Typically achieving students versus students with MD. Table 3 reports results for the percent correct (i.e., item difficulty) and the error ratio of item categories, but results are displayed by student risk status (typically achieving versus MD). Interestingly, there were only minor differences in the percent correct by item category for students with MD versus typically achieving students. Across item categories, students with MD always answered items correct less often on average, with most item categories having a relatively little difference in average correct response (i.e., approximately a 0 to 3% difference). The largest difference in average percent correct was with the three multiplication and division items (a 17.1% difference). Interestingly, there were more noticeable differences with regard to the error ratios by item category for students with MD versus typically achieving. Error ratios by category were always larger for students with MD, with large differences. This suggests that on average, students with MD consistently reported many different incorrect answers for an item, compared to typically achieving students. Error ratios for students with MD were higher than 52:100 the majority of the time, while error ratios for typically achieving students were lower than 43:100 the majority of the time.

Discussion

We assessed the computation skill of 478 third-grade students on the WRAT4 Math Computation. Based on computation scores, we selected students as with or at-risk for MD when the standard score of the WRAT4 Math Computation was at or below the 25th percentile (n = 120). We categorized students scoring at or above the 35th percentile (n = 312) as typically achieving. Of the 21 WRAT4 Math Computation items included in this analysis, 12 of the items were listed as first- or second-grade standards. Because participants were in in third grade, we were surprised that very few students demonstrated proficiency with these items. In summary, the average score for students with MD was 20.96 (*SD* = 2.19); typically achieving students scored 27.25 (*SD* = 2.11).

Common Errors

Regarding common errors, our results were consistent with previous research. For example, we know from previous research that students who are low achieving or identified as MD generally have difficulty with retrieving basic facts and performing correct calculations (Geary, 1993; Geary & Hoard, 2001; Jordan, Hanich, & Kaplan, 2003b). We also understood that students often performed the wrong operation (e.g., performed addition when the item signaled subtraction; Cox, 1975) and committed procedural errors (Cawley et al., 1996).

With our first research question, we investigated the common errors on individual computation items and within categories of computation items for all students. The 478 third-grade students in this analysis committed 2,427 errors, and we were able to identify 71% of the error patterns. Of the identifiable error patterns, the most consistent error was that of using the wrong operation. When students made this mistake, students often added when the operation of the problem signaled for subtraction. Making this error may be due to the way that operations are practiced in the classroom. First, all mathematics curricula introduce addition before subtraction, so students have received more exposure to addition than any other operation. In introducing addition first, students receive exposure to the plus sign (+), yet students do not always connect the plus sign to the operation of addition because every problem practiced with the teacher is addition. In this way, students become conditioned to not paying attention to the operational symbols of mathematics. This trend has also been noted with student interpretation of the equal sign (McNeil & Alibali, 2005). Second, because of the lack of attention to operational symbols, students may become conditioned to computation based on the initial item. On the WRAT4 Math Computation, the first item is an addition item. Therefore, we hypothesize that some students may assume subsequent problems require additions, when, in fact, the second item requires subtraction.

That 20% of the errors were related to using the wrong operation indicates that teachers must provide more focused instruction on the symbols of mathematics and help students become aware of the meaning of a symbol before starting to perform a computation (Powell, 2015). Teachers should provide more opportunities for students to discern among symbols and decide which operation needs to be performed. By teaching students to briefly assess any problem before working the problem, students might not make this very common mistake as often.

The next most prevalent error pattern also involved addition, but this error involved students adding all digits as separate integers. This error was related to problems with a plus sign, but the complexity of the multi-digit whole numbers (e.g., 22) or rational numbers (e.g.,$\frac{1}{3}$) contributed to student errors. These results are similar to the findings of Malone and Fuchs (2016) as students in this study also misapplied whole number concepts to fractions (i.e., students treated numerators and denominators as separate numbers). As teachers introduce computation beyond single-digit numerals, it may be important to discuss the differences between single-digit computation and computation with multi-digit whole numbers and fractions. Because students have only received practice adding or subtracting single digits, it is understandable that students would transfer this skill to more complex problems if appropriate instruction is not provided.

The other more common error pattern was related to subtraction. In multi-digit subtraction, students subtracted the integer that was less from the greater integer, regardless of integer placement. This result was not too surprising as many students, not only those with MD, struggle with subtraction and previous research indicates that this is a common error (Miller & Milan, 1987). Similar to the previous error, this is a creative error to work around not knowing what to do when the problem is not presented with the greater integer minus the integer that is less. When teachers introduce single-digit subtraction (e.g., 7 – 2), they may instruct students to "always take away the smaller number from the greater number." Although this may appear to be a helpful recommendation for students, it creates confusion when students are taught multi-digit subtraction where students must observe place value (Karp, Bush, & Dougherty, 2014). If students have been taught this technique or produced this conclusion independently, the procedure does not work for more complex problems (e.g., 32 – 17). For this reason, teachers must provide explicit instruction on common misconceptions related to this error.

Other errors included calculation mistakes (e.g., $2 + 7 = 8$; $3 \times 4 = 9$), mistakes with regrouping, only completing part of a problem, or rewriting part or all of the item prompt as the answer. Each of these mistakes could be lessened with multiple practice opportunities practice on the concepts and procedures for operations and explicit use of worked examples and non-examples (Booth, Lange, Koedinger, & Newton, 2013; Renkl, 2017). For example, by the end of second grade, all students should have developed fluency with addition and subtraction mathematics facts so that $2 + 7 = 8$ is not a mistake that students make; teachers may consider providing students with regular and daily opportunities to become fluent in grade level skills (Gersten et al., 2009). Additionally, helping students understand and practice the concepts of regrouping, rather than merely memorizing regrouping procedures, may help with the error patterns of some students. Competence with mathematics skills such as computation relies on students linking procedural fluency and conceptual understanding (Rittle-Johnson, Siegler, & Alibali, 2001).

Error Ratios

A consistent finding across this study and previous research is that some items produced many different incorrect responses (Miller & Milam, 1987); however, our study extends the work on error analysis by investigating the error ratios for a larger set of computation items and exploring whether error ratios were different for typically achieving students and students with MD. Although we knew that accuracy rates for students with MD would be lower than that of typically achieving students, research has not examined if error types and error ratios are different between these two groups of students. If the error ratios are different, that may suggest that instruction to remediate misconceptions and computation mistakes may also have to be designed differently. To our knowledge, this is the first study that examines and compares error ratios for students with and without MD with this level of detail.

Students made the most errors with multi-digit items including those that involved regrouping. Typically, multi-digit addition and subtraction is introduced in first or second grade with second grade as the school year in which multi-digit addition and subtraction skill should become firm. From the error ratios, it is obvious that many students still have difficulty with this task and require more opportunities for learning and practice. Given that multiplication and division is a primary focus within third grade (i.e., the grade level of the students in our sample), we expected students to have difficulty with multiplication and division computation problems. It is surprising, therefore, that the largest error ratios related to multi-digit computation problems with addition as the operation. It may be important to provide students with more addition and subtraction practice before introducing multiplication and division, especially because students need to understand the relationship between addition and multiplication.

Students also had difficulty with solving for an unknown addend. This skill is introduced in first-grade standards, but it is likely that students do not receive much practice on determining addends. Besides not receiving enough practice opportunities, students may not understand the equal sign as a balance between two sides of the equation (Powell, 2015). Teachers must give students opportunities to solve problems where the sum, difference, product, or quotient is not always the answer. This pre-algebraic reasoning would likely help improve understanding of operations within computation but also lay foundational knowledge related to equation solving.

In terms of typically achieving students and students with MD, the error ratios were greater for students with MD (but students with MD and typically achieving students did have similar rates of attempting items within each category), and the differences in error ratios were stark. For example, for problems without regrouping, which should be easier for all students, students with MD made errors over 60% of the time compared to typically achieving students (36% error ratio). The error ratios provide evidence that students with MD provide many different answers for a single item. For teachers, the multitude of errors may make it difficult to pinpoint areas for computation instruction. In these cases, the most judicious instructional pathway may be focusing on the concepts and procedures and ensuring students understand both while providing many practice opportunities with immediate feedback. Teachers may also focus on the most prevalent errors and engaging students in activities that alleviate such error patterns.

Limitations

We have limitations worth declaration. First, we used one assessment given at a single time point. This computation assessment is widely used in both the classroom and research (e.g., Fuchs et al., 2014; Swanson, 2011) and is highly reliable (Wilkinson & Robertson, 2006). We wanted to collect error pattern information for groups of students on a wide range of computation problems, and the WRAT4 Math Computation provided us with such an instrument. Yet, most types of problems (e.g., subtraction with regrouping) only had one or two items for each student to solve. Furthermore, some item categories had low percentages of attempted items for both students with MD and those who were typically achieving (e.g., multiplication and division items were attempted approximately 50% of the time). Future research should provide more of each item type to learn whether errors are idiosyncratic or perpetual. If there are more opportunities for students to respond to each type of item, larger proportions of students might also attempt certain problem types. Low attempt rates for certain item categories may create less variability in error ratios, which has implications for how researchers and practitioners apply the results of studies or practices that use error ratio analysis.

Other limitations of this study are the small subset of items per item category and the need for a larger sample in order to identify more defined error patterns. For example, only one item was attempted by 100% of participants, and a larger sample of students would allow for more responses per item. Items that are not attempted by participants do not add value to understanding error patterns. Furthermore, each item category had only a small number of items, which makes it difficult to draw conclusions and generalize results for specific item categories. A larger sample of students and items would provide more detailed information for teachers in order to design and customize instruction.

Implications for Practitioners

The results of this study and the error ratio analysis technique for computation have implications for practitioners. When conducting error ratio analysis, teachers must review student work in a step-by-step fashion in order to determine if, and where, students made mistakes in the solution (Ashlock, 2010). This allows teachers to determine if groups of students have conceptual understanding or procedural fluency deficits in particular areas of mathematics in order to target whole group core instruction or intervention content for specific subgroups of students. For example, when presented with the item 3×4, an incorrect student response of 16 may indicate a procedural fluency deficit or difficulty with retrieval; however, an incorrect response of 7 may indicate a conceptual misunderstanding of multiplication and its meaning. This is useful information for teachers because matching interventions to specific student deficits in a customized manner results in greater student growth during the intervention (Burns et al., 2015). Previous research shows that interventions such as cover-copy-compare (Codding, Chan-Iannetta, Palmer, & Lukito, 2009), incremental rehearsal (Burns, 2005), and taped problems (McCallum, Skinner, Turner, & Lee, 2006; Poncy, Skinner, & Jaspers, 2007) are generally effective in addressing computation deficits (Codding, Hilt-Panahon, Panahon, & Benson, 2009), as are interventions that incorporate the concrete-representational-abstract framework for teaching computation (Flores, Hinton, & Strozier, 2014; Mancl, Miller, & Kennedy, 2012).

Despite the potential benefits of error ratio analysis for practitioners in designing instruction and intervention for groups of students who commit computation errors, it is also important to note the amount of time needed to conduct error ratio analysis as well as the content expertise required to understand what error and when in the process a student commits an error. In this study, we conducted error ratio analysis for nearly 500 students on a set of 21 items. Creating a systematic process of reviewing the student answers and errors committed was invaluable and allowed us to efficiently determine what error patterns existed in this sample. Teachers may employ a similar pattern by examining student responses for each item, before proceeding to the next item for each student. Furthermore, to save on time and resources, teachers may consider targeting the computation items that most students answered incorrectly first.

Conclusion

Computation is foundational to all domains of mathematics; therefore, it is necessary for students to establish strong computational skill across the elementary grades. Despite the importance of computation, since the inception of national initiatives such as the CCSS and the NCTM Focal Points, to our knowledge, an analysis of computation errors has not been conducted. Furthermore, most of the research on computation error analysis is more than 20 years old and many studies only examined a handful of items or a particular type of computation items. This limitation makes it difficult to generalize to all types of computation problems and particularly to students who are expected to meet higher standards due to national initiatives.

In this analysis of the computational error patterns of typically achieving third-grade students and students with MD, we learned that, collectively, students made hundreds of different errors. The sheer number of errors may overwhelm teachers who want to design effective computation instruction, so we categorized errors according to type of error. Students had more difficulty with multi-digit computation than single-digit computation and made errors related to using the wrong operation for solution, regrouping, and adding all numerals involved in the problem. Students with MD made significantly more errors than typically achieving students. To alleviate computational difficulties, teachers must focus on correcting errors or teaching about errors before students have the opportunity to learn incorrect computational concepts and procedures.

References

Ashlock, R. B. (2010). *Error patterns in computation: Using error patterns to help each student learn* (10th ed.). Boston, MA: Allyn & Bacon.

Booth, J. L., Lange, K. E., Koedinger, K. R., & Newton, K. J. (2013). Using example problems to improve student learning in algebra: Differentiating between correct and incorrect examples. *Learning and Instruction*, *25*, 24-34. doi: 10.1016/j.learninstruc.2012.11.002

Bryant, D. P., Bryant, B. R., & Hammill, D. D. (2000). Characteristic behaviors of students with LD who have teacher-identified math weaknesses. *Journal of Learning Disabilities, 33*, 168–199.

Burns, M. K. (2005). Using incremental rehearsal to increase fluency of single-digit multiplication facts with children identified as learning disabled in mathematics computation. *Education and Treatment of Children*, 237–249.

Burns, M. K., Walick, C., Simonson, G. R., Dominguez, L., Harelstad, L., Kincaid, A., & Nelson, G. S. (2015). Using a conceptual understanding and procedural fluency heuristic to target math interventions with students in early elementary. *Learning Disabilities Research and Practice*, *30*, 52–60. doi:10.1111/ldrp.12056

Calhoon, M. B., Emerson, R. W., Flores, M., & Houchins, D. E. (2007). Computational fluency performance profile of high school students with mathematics disabilities. *Remedial and Special Education, 28*, 292–303. doi:10.1177/07419325070280050401

Cawley, J. F., Parmar, R. S., Yan, W. F., & Miller, J. H. (1996). Arithmetic computation abilities of students with learning disabilities: Implications for instruction. *Learning Disabilities Research & Practice, 11*, 230–237.

Chong, S. L., & Siegel, L. S. (2008). Stability of computational deficits in math learning disability from second through fifth grades. *Developmental Neuropsychology, 33*, 300–317. doi:10.1080/87565640801982387

Codding, R. S., Chan-Iannetta, L., Palmer, M., & Lukito, G. (2009). Examining a classwide application of cover-copy-compare with and without goal setting to enhance mathematics fluency. *School Psychology Quarterly*, *24*, 173–185. doi:10.1037/a0017192

Codding, R. S., Hilt-Panahon, A., Panahon, C. J., & Benson, J. L. (2009). Addressing mathematics computation problems: A review of simple and moderate intensity interventions. *Education and Treatment of Children, 32*, 279–312. doi:10.1353/etc.0.0053

Cox, L. S. (1975). Systematic errors in the four vertical algorithms in normal and handicapped populations. *Journal for Research in Mathematics Education, 6*, 202–220. doi:10.2307/748696

Flores, M. M., Hinton, V., & Strozier, S. D. (2014). Teaching subtraction and multiplication with regrouping using the concrete-representational-abstract sequence and strategic instruction model. *Learning Disabilities Research & Practice*, *29*, 75-88. doi: 10.1111/ldrp.12032

Fuchs, L. S., Powell, S. R., Cirino, P. T. Schumacher, R. F., Marrin, S., Hamlett, C. L., …Changas, P. C. (2014). Does calculation or word-problem instruction provide a stronger route to prealgebraic knowledge? *Journal of Educational Psychology, 106*, 990–1006. doi:10.1037/a0036793

Geary, D. C. (1993). Mathematical disabilities: Cognition, neuropsychological and genetic components. *Psychological Bulletin, 114*, 345–362. doi:10.1037/0033-2909.114.2.345

Geary, D. C. (2011). Cognitive predictors of achievement growth in mathematics: A 5-year longitudinal study. *Developmental Psychology*, *47*, 1539–1552. doi:10.1037/a0025510

Geary, D. C., & Hoard, M. K. (2001). Numerical and arithmetical deficits in learning-disabled children: Relation to dyscalculia and dyslexia. *Aphasiology, 15*, 635–647. doi:10.1080/02687040143000113

Gersten, R., Beckmann, S., Clarke, B., Foegen, A., Marsh, L., Star, J. R., & Witzel, B. (2009). *Assisting students struggling with mathematics: Response to Intervention (RtI) for elementary and middle schools* (NCEE 2009-4060). Washington, DC: National Center for Education Evaluation and Regional Assistance, Institute of Education Sciences, U.S. Department of Education. Retrieved from http://ies. ed.gov/ncee/wwc/publications/practiceguides/.

Gersten, R., Jordan, N. C., & Flojo, J. R. (2005). Early identification and interventions for students with mathematics difficulties. *Journal of Learning Disabilities*, *38*, 293–304. doi:10.1177/00222194050380040301

Hecht, S. A., & Vagi, K. J. (2010). Sources of group and individual differences in emerging fraction skills. *Journal of Educational Psychology, 102*, 843-859. doi:10.1037/a0019824

Jordan, N. C., Hanich, L. B., & Kaplan, D. (2003a). A longitudinal study of mathematical competencies in children with specific mathematics difficulties versus children with co-morbid mathematics and reading difficulties. *Child Development, 74*, 834–850. doi:10.1111/1467-8624.00571

Jordan, N. C., Hanich, L. C., & Kaplan, D. (2003b). Arithmetic fact master in young children: A longitudinal investigation. *Journal of Experimental Child Psychology, 85*, 103–119. doi:10.1016/S0022-0965(03)00032-8

Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics, 21*, 18–25.

Locuniak, M. N., & Jordan, N. C. (2008). Using kindergarten number sense to predict calculation fluency in second grade. *Journal of Learning Disabilities*, *41*, 451–459. doi:10.1177/0022219408321126

Mabbott, D. J., & Bisanz, J. (2008). Computational skills, working memory, and conceptual knowledge in older children with mathematics learning disabilities. *Journal of Learning Disabilities, 41*, 15–28. doi:10.1177/0022219407311003

Malone, A. S., & Fuchs, L. S. (2016). Error patterns in ordering fractions among at-risk fourth-grade students. *Journal of Learning Disabilities*. Advance online publication. doi: 10.1177/0022219416629647

Mancl, D. B., Miller, S. P., & Kennedy, M. (2012). Using the concrete-representational-abstract sequence with integrated strategy instruction to teach subtraction with regrouping to students with learning disabilities. *Learning Disabilities Research & Practice*, *27*, 152-166. doi: 10.1111/j.1540-5826.2012.00363.x

McCallum, E., Skinner, C. H., Turner, H., & Lee, S. (2006). The taped-problems intervention: Increasing multiplication fact fluency using a low-tech, classwide, time-delay intervention. *School Psychology Review*, *35*, 419–434.

McLeod, T. M., & Armstrong, S. W. (1982). Learning disabilities in mathematics – skill deficits and remedial approaches at the intermediate and secondary level. *Learning Disability Quarterly, 5*, 305–311. doi:10.2307/1510297

McNeil, N. M., & Alibali, M. W. (2005). Why won't you change your mind? Knowledge of operational patterns hinders learning and performance on equations. *Child Development, 76*, 883–899. doi:10.1111/j.1467-8624.2005.00884.x

Miller, J. H., & Carr, S. C. (1997). Error ratio analysis: Alternate mathematics assessment for general and special educators. *Diagnostique, 23*, 225–231. doi:10.1177/153450849702300105

Miller, J. H., & Milam, C. P. (1987). Multiplication and division errors committed by learning disabled students. *Learning Disabilities Research, 2*, 119–122. doi:10.1177/153450849702300105

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA, Author.

National Governors Association Center for Best Practices & Council of Chief State School Officers (2010). *Common Core State Standards mathematics.* Washington, DC: Authors.

Poncy, B. C., Skinner, C. H., & Jaspers, K. E. (2007). Evaluating and comparing interventions designed to enhance math fact accuracy and fluency: Cover, copy, and compare versus taped problems. *Journal of Behavioral Education*, *16*, 27–37. doi: 10.1007/s10864-006-9025-7

Powell, S. R. (2015). The influence of symbols and equations on understanding mathematical equivalence. *Intervention in School and Clinic, 50*, 266–272. doi:10.1177/1053451214560891

Powell, S. R., & Stecker, P. M. (2014). Using data-based individualization to intensify mathematics intervention for students with disabilities. *Teaching Exceptional Children*, *46*, 31-37. doi: 10.1177/0040059914523735

Primi, R., Ferrão, M. E., & Almeida, L. S. (2010). Fluid intelligence as a predictor of learning: A longitudinal multilevel approach applied to math. *Learning and Individual Differences*, *20*, 446–451. doi:10.1016/j.lindif.2010.05.001

Räsänen, P & Ahonen, T. (1995). Arithmetic disabilities with and without reading difficulties: A comparison of arithmetic errors. *Developmental Neuropsychology, 11*, 275–295. doi:10.1080/87565649509540620

Renkl, A. (2017). Learning from worked-examples in mathematics: students relate procedures to principles. *ZDM*, 1-14. doi: 10.1007/s11858-017-0859-3

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology*, *93*, 346–362. doi: 10.1037/0022-0663.93.2.346

Siegler, R. S., Duncan, G. J., Davis-Kean, P. E., Duckworth, K., Claessens, A., Engel, M., …Chen, M. (2012). Early predictors of high school mathematics achievement. *Psychological Science, 23*, 691–697. doi:10.1177/0956797612440101

Swanson, H. L. (2011). Working memory, attention, and mathematical problem solving: A longitudinal study of elementary school children. *Journal of Educational Psychology, 103*, 821–837. doi:10.1037/a0025114

Vukovic, R. K., & Siegel, L. S. (2010). Academic and cognitive characteristics of persistent mathematics difficulty from first through fourth grade. *Learning Disabilities Research and Practice, 25,* 25-38. doi:10.1111/j.1540-5826.2009.00298.x

Wilkinson, G. S., & Robertson, G. J. (2006). *Wide Range Achievement Test: Fourth Edition.* Lutz, FL: PAR.

Zhang, D., Ding, Y., Barrett, D. E., Xin, Y. P., & Liu, R. (2014). A comparison of strategic development for multiplication problem solving in low-, average-, and high-achieving students. *European Journal of Psychology of Education, 29*, 195–214. doi:10.1007/s10212-0194-1

Table 1
Common Identifiable Errors Across Items

Error	Errors (n)	Items with error (n)	Criteria	Example (student action in bold)
Total errors	2427	21		
Unknown	705	21	Answer was incorrect; error was idiosyncratic	32 + 40 = 953 (action unknown)
Wrong operation-addition	246	11	Performed addition instead of the signaled operation	7 – 3 = 10 (**7 + 3**)
Added all digits	227	6	Added all digits as separate numbers	½ + ½ = 6 (**1 + 2 + 1 + 2**) 21 + 13 = 7 (**2 + 1 + 1 + 3**)
Miscalculation (+, –)	202	14	Close miscalculation or counting error	9 + 6 = 14
Subtracted smaller integer	189	2	Subtracted the smaller integer from the larger integer; ignored regrouping procedures	$\begin{array}{r} 721 \\ -\,358 \\ \hline 437 \end{array}$ (**7 – 3**; **5 – 2**; **8 – 1**)
Regrouping	172	7	Regrouping error	$\begin{array}{r} 39 \\ +\,45 \\ \hline 74 \end{array}$ (**did not regroup the 10**)
Procedural error	254	9	Performed some steps to complete the problem but did not perform all steps	$\begin{array}{r} 12 \\ 30 \\ +\,21 \\ \hline 42 \end{array}$ (**12 + 30**)
Wrong operation-subtraction	212	7	Performed subtraction instead of the signaled operation	6 + 4 = 6 (**6 – 4**)
Copied prompt	97	20	Copied all or part of the item prompt	$\frac{16}{2} = \boldsymbol{\frac{16}{2}}$

Close fact (×,÷)	77	4	Close multiplication fact	3 × 5 = 12 (**3 × 4**)
Wrong operation-multiplication	28	2	Performed multiplication instead of the signaled operation	3 + 3 = 9 (**3 × 3**)
Did not understand prompt	18	6	Error that was identifiable but clear student did not understand the prompt	2 × 3 = 23 **(put numbers together)**

Note. This table represents errors for the full sample ($N = 478$ students). The examples in this table are a similar format to the actual errors students committed, but the items represented here are not identical to the items on the WRAT4 Math Computation.

Table 2

Error Ratio Analysis and Percent Correct by Item

Item	Type	Correct	Correct	Attempt	Incorrect	Different Incorrect	Error Ratio (%)
1	Addition	472	98.3%	100%	6	2	33.3%
2	Subtraction	445	92.7%	98.5%	26	6	23.1%
3	Number line	443	92.3%	95.2%	12	10	83.3%
4	Subtraction	440	91.7%	97.1%	24	8	33.3%
5	Addition	429	89.4%	96.2%	41	10	24.4%
6	Addition	409	85.2%	97.5%	57	10	17.5%
7	Subtraction	408	85.0%	93.3%	38	11	29.0%
8	Addition	360	75.0%	89.1%	66	40	60.6%
9	Subtraction	318	66.3%	89.7%	111	29	26.1%
10	Multiplication	270	56.3%	75.5%	91	27	29.7%
11	Addition	311	64.8%	91.6%	127	37	29.1%
12	Division	141	29.4%	55.9%	126	19	15.1%
13	Subtraction	132	27.5%	82.6%	263	44	16.7%
14	Addition	103	21.5%	60.3%	185	66	36.7%
15	Multiplication	66	13.8%	45.8%	153	64	41.8%
16	Subtraction	53	11.0%	64.4%	255	97	38.0%
17	Rounding	76	15.8%	51.0%	168	35	20.8%
18	Division	12	2.5%	41.0%	184	26	14.1%
19	Addition	9	1.9%	50.6%	233	31	13.3%
20	Addition	12	2.5%	38.7%	173	40	23.1%
21	Algebra	43	9.0%	27.5%	88	38	43.2%

Note. This table represents errors for the full sample ($N = 478$ students). For clarity in interpreting the magnitude of the error ratio, the error ratio is presented as a percentage (e.g., a ratio of 15:100 would be represented in the table as 15% and means 15 different incorrect answers to 100 total incorrect responses). Larger percentages represent items with more variability in the number of different incorrect responses; smaller percentages represent items with less variability in the number of different incorrect responses.

Table 3

Error Ratio Analysis and Percent Correct by Item Category for Students with MD and Typically Achieving Students

		Percent correct and attempted				Error ratio			
		TA		MD		TA		MD	
Category	Items (*n*)	Correct (attempt) %		Correct (attempt) %		*M*	(*SD*)	*M*	(*SD*)
Items with Whole Numbers									
Whole numbers	18	58.5	79.1	56.8	76.9	39.9	(17.4)	56.6	(22.5)
Addition	6	73.0	63.1	74.4	88.3	42.7	(15.5)	62.3	(30.2)
Subtraction	6	64.2	87.9	61.8	86.6	31.1	(9.8)	53.3	(22.4)
Multiplication/division	3	37.2	55.1	20.1	51.2	26.6	(30.9)	48.8	(23.8)
No regrouping necessary	8	86.4	95.1	86.8	95.0	36.1	(13.2)	60.5	(27.8)
Regrouping necessary	4	33.0	75.6	30.7	72.4	38.4	(17.0)	52.3	(23.8)
Single-digits only	8	79.9	89.7	78.6	88.3	33.7	(11.5)	58.0	(25.6)
Multi-digit	7	42.0	73.1	38.9	69.0	41.3	(15.8)	55.3	(23.9)
Items with Fractions									
Fractions	3	2.6	43.7	3.0	42.3	19.4	(7.5)	27.4	(7.2)

Note. TA = Typically achieving; MD = Mathematics difficulty. This table represents errors for only students who scored below the 25th percentile on the WRAT4 Math Computation (MD; *n* = 120) and students who scored above the 35th percentile on the WRAT4 Math Computation (TA; *n* = 312). Items presented with fractions is not presented with subcategories due to the small number of items with fractions. Regrouping and regrouping categories reflect whole number computation items only presented as addition and subtraction problems.