Braithwaite, D., Tian, J., & Siegler, R. S. (in press). Do children understand fraction addition? *Developmental Science.* Anticipated publication date: 2018.

Do Children Understand Fraction Addition?

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This work was supported in part by the Institute of Education Sciences, U.S. Department of Education, through Grants R305A150262, R324C100004:84.324C, Subaward 23149, and R305B100001 to Carnegie Mellon University, and by the Advanced Technology Center and Siegler Center for Innovative Learning, Beijing Normal University.

Research Highlights

- Many children do not accurately estimate the magnitudes of fraction sums.
- Estimates of fraction sums are often smaller than estimates of one of the addends.
- Estimates of fraction sums are very inaccurate on at least three different tasks.
- Fraction sums are estimated less accurately than decimal or whole number sums.

Abstract

Many children fail to master fraction arithmetic even after years of instruction. A recent theory of fraction arithmetic (Braithwaite, Pyke, & Siegler, in press) hypothesized that this poor learning of fraction arithmetic procedures reflects poor conceptual understanding of them. To test this hypothesis, we performed three experiments examining fourth to eighth graders' estimates of fraction sums. We found that roughly half of estimates of sums were smaller than the same child's estimate of one of the two addends in the problem. Moreover, children's estimates of fraction sums were no more accurate than if they had estimated each sum as the average of the smallest and largest possible response. This weak performance could not be attributed to poor mastery of arithmetic procedures, poor knowledge of individual fraction magnitudes, or general inability to estimate sums. These results suggest that a major source of difficulty in this domain is that many children's learning of fraction arithmetic procedures develops unconstrained by conceptual understanding of the procedures. Implications for education are discussed.

Keywords: Cognitive Development; Numerical Cognition; Conceptual Understanding; Fractions; Arithmetic

Do Children Understand Fraction Addition?

Understanding fractions is critically important to mathematical development. Attesting to this importance, individual differences in fractions knowledge in fifth grade predict individual differences in algebra and overall mathematics achievement in tenth grade, even after controlling for numerous other predictors, including children's whole number arithmetic knowledge, their IQ, and their family's SES (Siegler et al., 2012). This importance of fractions extends beyond school: In a recent study of a large, representative sample of American white and blue-collar workers, 68% said they used fractions, decimals, or percentages on their job (Handel, 2016).

Despite the importance of fractions and the several years children spend studying them, many children fail to master them. Fraction arithmetic presents a particular challenge (Byrnes & Wasik, 1991; Fuchs et al., 2014; Hecht & Vagi, 2010; Jordan et al., 2013; Lortie-Forgues, Tian, & Siegler, 2015; Newton, Willard, & Teufel, 2014). For example, percent correct on fraction arithmetic problems involving all four operations was only 46% among sixth graders and 57% among eighth graders in Siegler and Pyke (2013), and it was only 32% among sixth graders and 60% among eighth graders in Siegler, Thompson, and Schneider (2011).

Prior studies of fraction arithmetic have focused primarily on procedural knowledge – ability to solve problems using standard algorithms. However, conceptual understanding seems at least as important, as indicated by strong relations between it and overall math achievement, even when procedural knowledge is statistically controlled (Hecht, 1998; Hecht, Close, & Santisi, 2003; Siegler et al., 2011). Although conceptual and procedural knowledge of mathematics are related, the association is far from perfect; people may correctly execute mathematical procedures without understanding why they work, or they may understand the logic of mathematical procedures but execute them inaccurately.

After reviewing the literature on fraction arithmetic, Braithwaite, Pyke, and Siegler (in press) hypothesized that most children's procedural knowledge of fraction arithmetic develops unconstrained by conceptual understanding. This hypothesis was central to their computational model of fraction arithmetic learning, FARRA. FARRA solves fraction arithmetic problems by selecting from a range of procedures, both correct and incorrect. The incorrect procedures superficially resemble the correct ones, but the incorrect rules lack critical information about when they should be used, leading to frequent overgeneralization. FARRA has no knowledge of the conceptual basis of the procedures; instead, it relies on trial and error to learn which procedures are most likely to yield correct answers for problems having particular features, such as equal denominators. This process yields learning that is slow and that asymptotes well below perfect accuracy.

Despite, or perhaps because of, FARRA's lack of conceptual understanding, it accurately simulated numerous aspects of children's fraction arithmetic performance. For example, its relative accuracy on different types of problems was highly correlated with that of children, it generated the same types of errors as children, and it displayed similar relations between problem features and strategy choices as children. The fact that a model without conceptual understanding of fraction arithmetic generated performance highly similar to that of children lends plausibility to the hypothesis that children also lack such understanding. However, there is little empirical evidence that directly tests this hypothesis.

One exception is Siegler and Lortie-Forgues' (2015) study of sixth and eighth graders' knowledge of the direction of effects of fraction arithmetic operations. The children judged, without calculating, whether answers to fraction arithmetic problems involving each arithmetic operation were larger than the larger of the two operands – for example, whether $\frac{31}{56}+\frac{17}{42}>\frac{31}{56}$.

Children were very accurate (89% correct) on addition problems (for which the correct answer was always “true”), seemingly indicating good conceptual understanding of fraction addition.

However, this high accuracy might not indicate understanding, but rather a superficial generalization that fraction arithmetic works just like natural number arithmetic. Such a generalization would explain both children’s highly accurate direction of effects judgments with fraction addition and subtraction (because the directions of effects for addition and subtraction are the same for all positive numbers) and their below chance accuracy on multiplication and division problems with operands less than one (because unlike for natural numbers, multiplying fractions less than one yields answers smaller than either operand, and dividing by a number less than one yields answers greater than the number being divided).

In the present study, we directly tested the hypothesis that children lack conceptual understanding of fraction addition, using several methods that did not allow superficial generalizations from natural number arithmetic. For example, we asked children to provide numerical estimates of fraction sums – an approach previously used to assess understanding of whole number addition (Dowker, 1997; Gilmore, McCarthy, & Spelke, 2007). The rationale for this approach is that understanding fraction addition implies knowing that the magnitude of the sum of fractions equals the sum of the addends’ magnitudes. It might seem trivially obvious that the estimated sum should equal the sum of the estimates of the individual addends, but as will be seen, many children lack such understanding, even after years of fraction arithmetic instruction.

Accurate estimation of sums is an important skill in its own right. For whole numbers, manipulations that improve estimation accuracy for individual numbers also improve addition performance and learning (Booth & Siegler, 2008; Siegler & Ramani, 2009). A likely reason is that accurate estimation of a sum promotes rejection of implausible answers and the procedures that generate them, and stimulates search for procedures that yield plausible answers. Similarly, a child who accurately estimates fraction sums can reject common errors such as 1/2 + 1/3 = 2/5, on the grounds that the proposed sum, 2/5, is too small, smaller than one of the addends.

We also examined relations between individual differences in the accuracy of estimates of individual fractions and the accuracy of estimates of the sums of fractions. If children understand fraction addition, then more accurate estimation of magnitudes of fraction sums should accompany more accurate estimation of individual fractions.

Experiment 1

Experiment 1 examined fourth and fifth graders' judgments of the magnitudes of individual fractions and fraction sums. These age groups were chosen because children in the United States are typically taught how to add fractions with equal denominators in fourth grade and how to add fractions with unequal denominators in fifth grade (CCSSI, 2010).

Method

Participants. Participants were 148 children, 74 fourth graders and 74 fifth graders, attending four elementary schools near Pittsburgh, PA. Informed consent was obtained from the parents of all children who participated. The experimenters were two female research assistants.

Materials. The individual fractions that children were asked to estimate included two sets of 12 fractions apiece, with each set containing three fractions in each quartile from 0-1.

The fraction addition estimation task also included two sets of 12 problems. The addends in each problem were between 0 and 1 and had unequal denominators; correct answers ranged from 1/2 to 1 1/2. The task was to choose the alternative closest to the correct sum, given the choices 1/2, 1, and 1 1/2. On each problem, one alternative was at least .167 closer to the correct sum than the next closest alternative (mean = .263 closer.) To illustrate what this average difference means, 5/10+1/8 is equal to 0.625, which is 0.25 closer to the best estimate, 1/2, than to the next best estimate, 1. Each alternative was the best estimate of the sum equally often.

Children received one of the two sets of individual fractions and one of the two sets of fraction addition problems; the two sets of each type were presented equally often. The sequence of items within each set and the order of addends within each addition problem were randomized. All stimuli are included in the Supporting Information, Part A.

Procedure. Both tasks were presented on a laptop computer. The tasks were administered one-on-one, with children working at their own pace. Children estimated individual fraction magnitudes first and fraction sums second. On the individual fraction estimation task, children clicked the mouse to indicate where each fraction belonged on a 0-1 number line. On the addition estimation task, children were presented 1/2, 1, and 1 1/2, and asked to click the mouse on the number closest to the sum. They were instructed not to exactly calculate the answers, but to choose the best estimate by imagining adding the sizes of the addends together. Average response time was 9.3 seconds ($SD = 5.2$) for the individual fraction estimation task, and 13.4 seconds ($SD = 8.5$) for the addition estimation task.

Results

Estimation of individual fractions. Estimation accuracy was assessed using percent absolute error (PAE), defined as the absolute value of the difference between the estimate and the correct response, divided by the size of the range of possible answers. Lower PAEs indicate more accurate estimates. Average PAE was 17.9%.

Effects of grade, stimulus set, and their interaction on PAEs were assessed using linear regression. A Shapiro-Wilk test indicated that PAEs diverged from a normal distribution, $p < .001$. Therefore, significant terms in the regression were identified using bootstrapping, which does not require the assumption of a normal distribution (Erceg-Hurn & Mirosevich, 2008). We used bootstrapping to estimate a distribution of values for each coefficient in our regression (Efron & Tibshirani, 1986; Neal & Simons, 2007). Ten thousand simulated replications of the experiment were conducted by randomly sampling participants, with replacement, from the experimental data, using *boot* from the boot package in *R* (Canty & Ripley, 2016). Each simulation was analyzed using the above regression; the results of these analyses were used to generate a 95% confidence interval for each regression coefficient. Coefficients whose 95% confidence interval (*CI*) excluded zero were considered to indicate significant effects.

This analysis indicated that PAE improved from fourth grade (20.8%) to fifth grade (14.9%), $B = -0.059$ (95% *CI* of $B = [-0.096, -0.022]$). Stimulus set had no effect, and did not interact with grade in school.

Estimation of fraction sums. Judgments of the nearest sum were correct on 44% of trials. A Shapiro-Wilk test again indicated that accuracies diverged from a normal distribution, $p < .001$, so percent correct was compared to chance performance (i.e., 33% correct) by generating a 95% confidence interval on mean accuracy, using the bootstrapping procedure described above. Children's accuracies were higher than chance, as indicated by a 95% confidence interval excluding 33%, in both fourth grade (mean = 39%, 95% *CI* of mean = [36%, 43%]) and fifth grade (mean = 49%, 95% *CI* of mean = [43%, 54%]). However, almost half of children (47% of fourth graders and 43% of fifth graders) scored at chance (33% correct) or below.

A similar bootstrapping procedure indicated that fifth graders were more accurate than fourth graders, $B = 0.096$ (95% *CI* of $B = [0.026, 0.163]$). Stimulus set had no effect on the accuracy of estimates of sums, nor did it interact with grade.

Individual differences. To assess the relation between knowledge of the magnitudes of individual fractions and knowledge of the magnitudes of fraction sums, children's accuracies on the addition estimation task were regressed against their PAEs on the individual fraction estimation task. Significance was assessed using bootstrapping, as above. After controlling for grade, PAE for estimates of individual fraction magnitudes predicted percent correct choices of the closest sum, $B = -0.608$ (95% *CI* of $B = [-0.878, -0.312]$), uniquely explaining 10.5% of the variance. Even children who estimated magnitudes of individual fractions quite accurately often estimated sums inaccurately, though those who estimated sums accurately almost always estimated individual fractions accurately as well (see top left of Figure 1).

========================= Figure 1 about here =========================

Discussion

Nearly half of children performed no better than chance on the estimation of sums task, suggesting poor understanding of fraction addition. However, the inaccurate estimation of sums might have reflected the unfamiliarity of the task of choosing the closest alternative to a sum or the children's limited experience with fraction addition. Similarly, the relatively modest relation between accuracy on the two tasks might have reflected differences between the continuous number line estimation task and the discrete task of choosing the response alternative closest to the sum. These issues were addressed in Experiment 2.

Experiment 2

The sixth and seventh graders who participated in Experiment 2 estimated the location on number lines of both individual numbers and sums for both fractions and whole numbers. The main prediction was that estimates of individual numbers would be more accurate than estimates of sums for both fractions and whole numbers, but the difference would be greater for fractions.

Such an interaction would suggest that children have difficulty understanding fraction addition, above and beyond their difficulty with individual fractions or with estimation of sums in general.

A second prediction involved correlations between precision of children's estimates of individual numbers and sums. Understanding addition implies knowing that the sum of two addends has a magnitude equal to the sum of the individual addends' magnitudes. To the extent that children understand this, precision of estimates of sums should vary with precision of estimates of individual numbers, leading to positive and reasonably strong correlations between the two. However, if children understand how fractions sum less well than how whole numbers sum, this correlation should be weaker with fractions than with whole numbers.

A third prediction involved understanding of direction of effects for fraction and whole number addition. To assess this understanding, the numbers that appeared as addends for addition estimation were also used on the individual number estimation tasks. Beyond removing the possibility that differences in the fractions used in the two tasks could lead to differences in their difficulty, this approach allowed us to calculate how consistently each child's estimate of the sum of two addends was at least as large as their *own* estimates of both individual addends in the problem. Even children who do not accurately represent the magnitudes of addends could perform well on this measure, as long as they understood that sums of positive numbers are greater than the individual addends. Poorer performance on this measure with fractions than with whole numbers would be another indicator of a specific difficulty understanding fraction addition.

Finally, to test whether understanding of rational number addition increases with greater rational number experience, participants in Experiment 2 were sixth and seventh graders, and thus older and more experienced with fraction addition than participants in Experiment 1.

Method

Participants. Participants were 101 children, 41 sixth graders and 60 seventh graders, attending a middle school near Pittsburgh, PA. The study was conducted as part of children's regular math classes; parents were notified of the study in advance and given the option to opt out of their children's participation. A female research assistant administered the tasks.

Materials. The individual number estimation tasks involved a 0-1 number line for fractions and a 0-1000 line for whole numbers. Each task involved estimation of the values of 18 numbers; whole numbers were generated by multiplying each fraction by 1000, which resulted in fractions and whole numbers occupying equivalent or virtually equivalent locations on the number lines.

For each type of number, six items were in the lowest quartile of the distribution: the fractions in this quartile had values between 0 and 0.25, and the whole numbers had values between 0 and 250. Both fractions and whole numbers had four values in each subsequent quartile of the number line.

The estimation of sums task included 16 items. In the fractions version, the pairs of addends had unequal denominators; both were among the individual fractions whose magnitudes children estimated. Answers ranged from 0-1, and appeared equally in each quartile of that range. The whole number version was exactly parallel except for adjustments so that none of the individual whole numbers or sums had a unit digit of zero.

The stimuli within each set were randomly ordered, separately for each child, with the constraint that the correct answers could not fall in the same half of the numeric range (0-1 for fractions, 0-1000 for whole numbers) on more than three successive trials. The order of addends within each addition problem was also randomized for each child. All stimuli are included in the Supporting Information, Part B.

Procedure. The study was conducted in a whole class format in a computer lab at the children's school. The tasks were presented on desktop computers, with each child working on a different computer and proceeding at a self-paced rate.

The number or addition problem whose magnitude was to be estimated was presented above a number line. Its endpoints were marked 0 and 1 for the fraction tasks, and 0 and 1000 for the whole number tasks. Children clicked a mouse to indicate each number's/sum's location on the line. Whether children performed whole number or fraction tasks first was randomized. For each type of number, children always estimated locations of individual numbers before sums.

All tasks followed the same procedure as the individual fraction number line task in Experiment 1, except that on the tasks involving estimation of sums, children were instructed not to calculate exact answers but rather to imagine adding the addends together. Average response time was 4.8 seconds (*SD* = 4.3) for individual whole number estimates, 7.3 seconds (*SD* = 7.9) for estimates of whole number sums, 5.8 seconds (*SD* = 6.2) for individual fraction estimates, and 8.0 seconds (*SD* = 10.4) for estimates of fraction sums.

Results

Percent absolute error. PAEs were calculated in the same way as for the individual fraction estimation task in Experiment 1, using 0-1 as the answer range for the two fraction tasks and 0-1000 as the answer range for the two whole number tasks. Children's average PAEs were 6.4% (min = 4.1%, max = 31.5%) for individual whole number estimates, 9.4% (min = 2.6%, max = 29.6%) for whole number sum estimates, 13.9% (min = 2.7%, max = 51.6%) for individual fraction estimates, and 28.0% (min = 8.2%, max = 48.1%) for fraction sum estimates.

PAEs were analyzed using a linear mixed model, with participant as a random effect and number type (whole number or fraction), estimation task (individual number or sum), grade (6 or 7), and task sequence (fractions or whole numbers first) as fixed effects. The analysis (and all subsequent mixed model analyses) used *lmer* from the lme4 package in *R* (Bates, Maechler, Bolker, & Walker, 2015). Shapiro-Wilk tests indicated that PAEs diverged from a normal distribution for individual whole number estimation, individual fraction estimation, and whole number sum estimation, ps < .001 (though not for fraction sum estimation, $p = .543$). Therefore, significant effects on PAEs were identified using bootstrapping, as in Experiment 1.

As expected, PAE was higher (less accurate) for the two fraction tasks (21.0%) than for the two whole number tasks (7.9%), $B = 0.134$ (95% *CI* of B = [0.118, 0.148]). This effect was larger in sixth grade (fraction tasks: 23.2%, whole number tasks: 8.2%) than in seventh grade (fraction tasks: 19.4%, whole number tasks: 7.7%), $B = 0.032$ (95% *CI* of B = [0.002, 0.060]). Further, PAE was higher for estimates of sums (18.7%) than for estimates of individual numbers (10.2%), $B = 0.085$ (95% *CI* of B = [0.075, 0.095]).

Critically, number type interacted with estimation task, $B = 0.109$ (95% *CI* of B = [0.086, 0.134]). As predicted, the discrepancy between fractions and whole numbers was much greater for estimation of sums than for estimation of individual numbers (Figure 2). The particularly high PAE for fraction sum estimation was not a consequence of one or a few particularly difficult items: The lowest PAE (averaged across children) for any of the 16 fraction sums was higher than the highest PAE for any of the 18 individual whole numbers, any of the 16 whole number sums, and 17 of the 18 individual fractions. No other effects or interactions were found.

========================== Figure 2 about here ==========================

To make clear the degree of inaccuracy of children's estimates of fraction sums, the mean PAE on each estimation task was compared to the mean PAE that would have resulted from marking the midpoint of the answer range (i.e., 0.5 for the fraction tasks, 500 for the whole number tasks) regardless of what number was presented. This midpoint strategy would have yielded PAEs of 26% on the two individual number tasks and 25% on the two sum estimation tasks. Bootstrapping was used to compare children's PAEs to those of the midpoint strategy.

Children's PAEs were far lower (i.e., more accurate) than those of the midpoint strategy for individual whole numbers (6.4% versus 26%, 95% *CI* of children's PAE = [5.5%, 7.2%]), whole number sums (9.4% versus 25%, 95% *CI* of children's PAE = [8.3%, 10.4%]), and individual fractions (13.9% versus 26%, 95% *CI* of children's PAE = [11.7%, 16.0%]). However, mean PAE for fraction sums was actually *higher* than that yielded by the midpoint strategy (28.0% versus 25%, 95% *CI* of children's PAE = [26.4%, 29.5%]). Thus, children's estimates of fraction sums were less accurate than they if they had simply guessed the midpoint on every trial.

Another way of assessing children's estimates of fraction sums is to compare them to the results of a common incorrect strategy for adding fractions, in which the numerators and denominators of the addends are added separately to obtain the sum (e.g., 3/5+1/8=4/13; Ni & Zhou, 2005). This strategy would have yielded a PAE of 31% on the fraction addition estimation task. Children were more accurate than this, but only slightly (28.0% versus 31%, 95% *CI* of children's PAE = [26.4%, 29.5%]).

Individual differences. To test whether individual children's accuracy of estimates for specific numbers was related to their accuracy of estimates for sums involving the same numbers, and whether these relations differed between fractions and whole numbers, PAEs of sums were submitted to a linear mixed model with participant as a random effect and grade, individual number estimation PAE, and number type (fraction or whole number) as fixed effects. Significance was assessed using bootstrapping, as above.

The effect of individual number estimation PAE on sum estimation PAE differed between fractions and whole numbers, as indicated by an interaction between individual number estimation PAE and number type, $B = 0.392$ (95% CI of $B = [0.148, 0.719]$). Therefore, sum estimation PAEs for fractions and whole numbers were separately regressed against grade and individual number estimation PAE. In both regressions, individual number estimation PAE predicted sum estimation PAE after controlling for grade, indicating that children who estimated individual numbers more precisely also estimated sums more precisely. However, the effect of individual number estimation PAE on sum estimation PAE was far weaker for fractions, $B = 0.327$ (95% CI of $B = [0.185, 0.466]$), than for whole numbers, $B = 0.720$ (95% CI of $B = [0.495, 1.058]$). Individual number estimation PAE did not interact with grade for either fractions or whole numbers.

Direction of effects. Each sum estimate was classified as respecting the direction of effects principle if it was at least as large as the child's estimates of both addends in the corresponding individual number estimation task. For example, if a child's estimates for 3/5 and 1/8 in the individual fraction estimation task were equal to 0.49 and 0.14, then that child's estimate for 3/5+1/8 in the fraction addition estimation task would be classified as respecting the direction of effects principle if its value was greater than or equal to 0.49.

Children's estimates met this lenient criterion on fewer than half (48.4%) of sum estimation trials. That is, on slightly more than half of trials, children's estimates of fraction sums were smaller than their estimates of one of the addends. By contrast, the large majority (85.1%) of their whole number sum estimates were consistent with the principle. In a linear mixed model analysis with participant as a random effect, number type (whole number or fraction), grade (6 or 7) and task sequence (fractions or whole numbers first) as fixed effects, and significance assessed using bootstrapping, fewer estimated fraction sums than whole number sums were consistent with the principle, *B* = 0.365 (95% *CI* of *B* = [0.316, 0.414]). No other effects or interactions were found.

Discussion

The findings from Experiment 2, like those from Experiment 1, indicate that children have difficulty estimating fraction sums. Experiment 2 extended the finding to older children (sixth and seventh graders) and a different task (number line estimation). The results indicate that the difficulty estimating fraction sums does not merely reflect poor understanding of individual fraction magnitudes or of the choice task. Children were less accurate on both estimation tasks with fractions than with whole numbers, but the discrepancy was much greater for sums than individual numbers. Further, the relation between precision of individual number estimates and precision of addition estimates was much weaker for fractions than for whole numbers.

Most striking, children frequently violated the direction of effects principle when estimating fraction sums, though they rarely did so when estimating whole number sums. This finding suggests that many children do not understand, or cannot apply, the basic logic of addition in the context of fractions. The fact that each child’s estimate of each addend’s magnitude was compared to that child’s estimate of the sum of the same addends made these violations of the direction of effects principle especially striking.

Experiment 3

Experiment 3 was designed to address two issues raised by Experiments 1 and 2. First, children's weak understanding of fraction addition could reflect either general difficulty with rational numbers or specific difficulty with fractions. To find out, we compared the accuracy of children's estimates of decimal and fraction sums.

Second, children's frequent violations of the direction of effects principle in Experiment 2 could merely reflect estimates of sums being constrained to the same range (0-1) as the addends. This constraint may have induced children to generate smaller estimates of sums, and therefore to more often violate the direction of effects principle, than they would have without such a constraint. To test this possibility, and to demonstrate the generality of the findings regarding lack of understanding of fraction addition on a third task, participants in Experiment 3 generated estimates in a free-response format, with no constraint on the range of their estimates.

Method

Participants. Participants were 13 seventh and 34 eighth graders attending a middle school near Pittsburgh, PA. Although the seventh graders were younger, all were enrolled in an advanced mathematics class (Algebra I), whereas 29 of the 34 eighth graders were drawn from a less advanced class (Introduction to Algebra). Because of these differences, older and younger children's performance was not compared. Informed consent was obtained from children's parents. The experimenters were two female research assistants.

Materials. The fraction addition estimation task included eight problems. All addends were proper fractions (fractions with magnitudes between 0 and 1); the two addends within each problem had unequal denominators. Two problems had sums in each quartile of the range 0-2.

The decimal addition estimation task also included eight problems, which were created by replacing the fractions in the fraction addition set with equivalent decimals (rounded, when necessary, to two or three decimal places). For each problem in the decimal task, one addend had two decimal digits and the other had three decimal digits.

All stimuli are included in the Supporting Information, Part C.

Procedure. Tasks were administered one-on-one. The experimenter presented the child a printed packet of problems and asked for an oral estimate of the answer to each problem, in whatever numerical format the child preferred. Estimates of fraction sums were most often fractions (80%), but were occasionally decimals or whole numbers (14%), percentages (5%), or other formats (1%). Estimates of decimal sums were almost always decimals or whole numbers (96%), but were occasionally percentages (2%) or fractions (2%).

To discourage exact computation, children were not allowed to write anything. After stating an estimate, children were asked to report how they generated it. Analyses of these verbal reports are included in the Supporting Information, Part D.

Problems were presented in two blocks; the first block included half of the fraction addition problems and half of the decimal addition problems, and the second included the remaining problems of each type. Whether children received fractions or decimals first within each block, and which half of the problems they received in each block, were counter-balanced across students. Problems within each block were randomly ordered for each child.

Children were randomly assigned to the hint or no hint groups. Prior to beginning the fraction problems in the second block, children in the hint group were told, “When you see a problem, first think about how big the first fraction is. Then think about how big the second fraction is. Finally you can add the two and come up with an answer.” Prior to beginning the decimal problems in the second block, they were given identical instructions except with the word “decimal” substituted for “fraction.” These instructions were not given to children in the no hint group. Because this manipulation had no effect on any outcome measure, we do not discuss it further.

Results

Absolute error. The main measure was absolute error (AE), the absolute value of the difference between the child's estimate and the sum on each trial. Responses greater than 2 constituted implausible estimates, because addends were always less than 1; thus, such responses were excluded from the analysis of AE (4.7% of trials, including 5.6% of fraction trials and 3.7% of decimal trials). One child who did not advance a plausible response for any fraction sum was excluded from the analysis.

AEs were analyzed using a linear mixed model, with participant as a random effect and task sequence (fractions or decimals first), group (hint or no hint), block (1 or 2), and number type (decimal or fraction) as fixed effects. Shapiro-Wilk tests indicated that PAEs diverged from a normal distribution for fraction sum estimation, $p = .017$, and decimal sum estimation, $p < .001$, so bootstrapping was used to identify significant effects, as in Experiments 1-2.

Estimates of sums were more accurate for decimals than for fractions (AEs = 0.266 and 0.376), $B = 0.109$ (95% *CI* of B = [0.047, 0.172]). Thus, children's difficulty estimating fraction sums partly reflected a difficulty specific to fractions, though the absolute error was quite high for decimals as well. No other effects or interactions were found.

As with PAEs in Experiment 2, bootstrapping was used to compared children's AEs with fractions and decimals to the AEs that would have resulted from responding on every trial with the midpoint of the plausible answer range, in this case 1. This midpoint strategy yielded an AE of 0.372 on the fraction estimation task and 0.374 on the decimal estimation task. Thus, AEs for estimates of fraction sums were equivalent to that yielded by the midpoint strategy (0.376 versus 0.372, 95% *CI* of children's AEs = [0.314, 0.439]). In contrast, AEs for estimates of decimal sums were more accurate than those yielded by the midpoint strategy (0.266 versus 0.374, 95% *CI* of children's AEs = [0.205, 0.326]).

Although estimates of decimal sums were more accurate than those yielded by the midpoint strategy, they were much less accurate than could have been achieved using a slightly more sophisticated approach: adding the tenths digits of the addends and ignoring all other digits. For example, applying this strategy to the problem 0.875+0.34 would yield the estimate 0.8+0.3=1.1. This strategy would have yielded an AE of 0.126, much more accurate than children's AE for decimal sums (0.126 versus 0.266, 95% *CI* of children's AEs = [0.205, 0.326]).

Direction of effects. Estimates of fraction and decimal sums were classified as respecting the direction of effects principle if they were at least as large as both addends (no data were excluded in this analysis). This criterion was met on 53.8% of fraction addition trials, similar to the 48.4% obtained in Experiment 2. Thus, children's estimates of fraction sums frequently violated the direction of effects principle, even without any constraint on the answer range. By contrast, 77.9% of decimal sum estimates were larger than both addends, considerably more than the corresponding percentage with fractions and approaching the 85.1% for whole numbers in Experiment 2. A linear mixed model analysis with the same model structure as the PAE analysis above and with significance assessed using bootstrapping, found that the percent of sum estimates that respected the principle was greater for decimals than for fractions, B = 0.238 (95% *CI* of B = [0.142, 0.329]). No other effects or interactions were found.

Discussion

Children's estimates of fraction sums were quite inaccurate, as in Experiments 1 and 2, considerably less accurate than their estimates of decimal sums. Thus, difficulty estimating fraction sums is at least partially specific to fractions, rather than general to rational numbers.

However, estimates of decimal sums were also inaccurate; children could have estimated much more accurately by summing the tenth place digits in the addends and ignoring the rest of the decimals. Thus, the difficulty also seems partially general to rational number addition.

The seventh and eighth graders' estimates of fraction sums in Experiment 3 violated the direction of effects principle on roughly half of trials, despite the open response format. This argued against attributing children's frequent violations of the direction of effects principle in Experiment 2 to estimates of fraction sums being constrained to the same range (0-1) as the addends in that experiment.

General Discussion

Across three experiments, children's estimates of fraction sums were highly inaccurate. In Experiment 1, 45% of children were no better than chance in identifying the closest fraction to the sum. In Experiments 2 and 3, children's estimates of fraction sums were no more accurate than guessing the midpoint on every trial. These findings indicate a lack of conceptual understanding of even the most basic arithmetic operation, addition, in the context of fractions.

Several explanations of this poor performance can be excluded. The poor performance cannot be attributed to poor procedural knowledge, because none of our estimation tasks required use of fraction arithmetic procedures. The poor performance also cannot be attributed to general deficiency at estimating sums, because children performed substantially better when estimating sums of whole numbers (Experiment 2) and decimals (Experiment 3). The performance also cannot be attributed to task peculiarities, because performance was poor across three quite different addition estimation tasks (i.e., multiple choice, number line, and free response). Finally, the inaccurate estimates of fraction sums cannot be explained in terms of poor understanding of individual fraction magnitudes (Experiment 2).

Particularly striking, children frequently violated the direction of effects principle when estimating fraction sums. These results inform the interpretation of a previous study of conceptual understanding of fraction arithmetic (Siegler & Lortie-Forgues, 2015). In that study, eighth graders correctly judged that sums of fractions were larger than the addends that generated the sums. As described in the Introduction, this finding could reflect either conceptual understanding of fraction addition or reliance on the superficial strategy of answering in the same way as if whole numbers were involved. The latter, less optimistic, interpretation seems correct in light of the present findings.

The results support the hypothesis underlying Braithwaite et al.'s (in press) FARRA model of fraction arithmetic: For many children, development of procedural knowledge in this domain seems unconstrained by conceptual understanding. Further, the results disambiguate between two possibilities consistent with this hypothesis. Children might have conceptual understanding of fraction arithmetic operations, but fail to use this understanding while executing fraction arithmetic procedures, perhaps due to working memory demands, or they might simply lack such understanding. The present findings were obtained on tasks that did not require computation; thus, it seems likely that children do not have this conceptual understanding of fraction arithmetic, rather than having but not using it.

Results from the present study highlight differences in the development of whole number and fraction knowledge. With whole numbers, conceptual understanding of arithmetic develops in advance of procedural knowledge. Preschool and primary school children understand arithmetic principles in which they have received no formal instruction (Rasmussen, Ho, & Bisanz, 2003; Sherman & Bisanz, 2007) and accurately estimate whole number sums which they cannot calculate exactly (Dowker, 1997; Gilmore et al., 2007). First to third graders can use estimation to evaluate the plausibility of addition answers (Rousselle & Noël, 2008).

By contrast, the present findings suggest that conceptual understanding of symbolic fraction arithmetic develops after eighth grade, if ever. Many children showed poor understanding of the most basic fraction operation, addition, even after years of practicing it. These results dovetail with recent evidence for poor conceptual understanding of fraction multiplication and division among eighth graders and pre-service teachers, including many who correctly solved fraction multiplication and division problems (Siegler & Lortie-Forgues, 2015).

A second difference between fraction and whole number arithmetic development involves relations between knowledge of numerical magnitudes and conceptual understanding of arithmetic. In the present study, accuracy of fraction magnitude estimates correlated positively with accuracy of addition estimates (Experiments 1 and 2), but the relation was much weaker for fractions than whole numbers (Experiment 2). We propose that this difference reflects differences in conceptual understanding of fraction and whole number addition. Most children understand whole number addition reasonably well, so accurate estimates of individual whole numbers enable them to generate accurate estimates of whole number sums. In contrast, many children have poor conceptual understanding of fraction addition, and consequently cannot accurately estimate fraction sums, even if they accurately estimate the magnitude of each addend.

This proposal has clear implications for educational practice. Because children have reasonable understanding of whole number addition, improving their representations of individual whole number magnitudes facilitates their learning of whole number addition (Booth & Siegler, 2008; Siegler & Ramani, 2009), presumably by allowing children to generate better estimates of whole number sums. However, improving children's ability to estimate and calculate fraction sums likely requires improving not only knowledge of individual fraction magnitudes but also understanding of what fraction addition does. In short, children must re-learn the conceptual basis of addition in the context of fractions, despite knowing it already in the context of whole numbers. How to achieve this is an important question for educational practice and for theories of numerical development as well.

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Figure Captions

Figure 1. Relations between each child's percent correct judgments of best estimate of fraction sum and that child's mean PAE for estimates of individual fractions (Experiment 1). Lower PAE indicates more accurate estimates.

Figure 2. PAEs for estimates of individual numbers and sums for fractions and whole numbers (Experiment 2). Lower PAEs indicate more accurate estimates. Error bars indicate standard errors.

Figure 1:

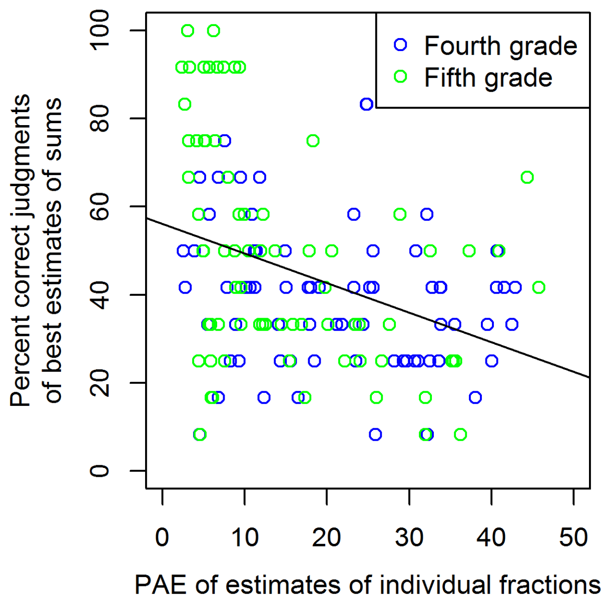
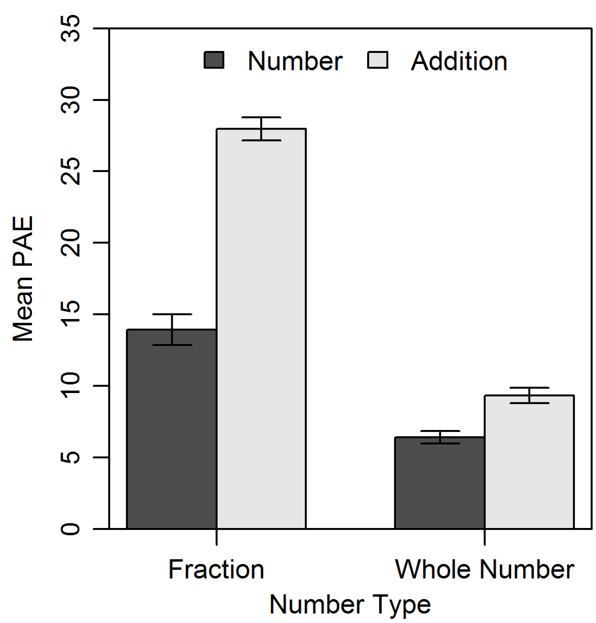


Figure 2:



Do Children Understand Fraction Addition?

Supporting Information

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Part A: Stimuli for Experiment 1

Stimuli for Individual Fraction Estimation Task

Stimulus Set	Fraction	Value
A	1/7	0.1429
A	1/5	0.2
A	2/9	0.2222
A	2/8	0.25
A	3/10	0.3
A	4/10	0.4
A	4/8	0.5
A	3/5	0.6
A	2/3	0.6667
A	6/8	0.75
A	7/9	0.7778
A	6/7	0.8571
B	1/9	0.1111
B	1/6	0.1667
B	2/10	0.2
B	2/7	0.2857
B	2/6	0.3333
B	2/5	0.4
B	3/6	0.5
B	5/9	0.5556
B	6/10	0.6
B	3/4	0.75
B	4/5	0.8
B	7/8	0.875

Stimuli for Fraction Addition Estimation Task

Stimulus Set	Problem	Answer Value	Best Estimate	| answer – estimate |
A	5/10+1/7	0.6429	1/2	0.14286
A	4/10+2/8	0.65	1/2	0.15
A	4/8+1/6	0.6667	1/2	0.16667
A	5/10+1/6	0.6667	1/2	0.16667
A	8/10+1/9	0.9111	1	0.08889
A	9/10+1/9	1.0111	1	0.01111
A	3/4+1/3	1.0833	1	0.08333
A	3/5+1/2	1.1	1	0.1
A	5/6+2/4	1.3333	1 1/2	0.16667
A	5/7+2/3	1.381	1 1/2	0.11905
A	3/4+2/3	1.4167	1 1/2	0.08333
A	3/4+4/6	1.4167	1 1/2	0.08333
B	5/10+1/8	0.625	1/2	0.125
B	4/9+1/5	0.6444	1/2	0.14444
B	4/10+1/4	0.65	1/2	0.15
B	5/9+1/10	0.6556	1/2	0.15556
B	7/10+2/9	0.9222	1	0.07778
B	8/10+2/9	1.0222	1	0.02222
B	2/3+2/4	1.1667	1	0.16667
B	2/3+1/2	1.1667	1	0.16667
B	5/6+1/2	1.3333	1 1/2	0.16667
B	3/4+3/5	1.35	1 1/2	0.15
B	4/5+4/7	1.3714	1 1/2	0.12857
B	4/5+2/3	1.4667	1 1/2	0.03333

Part B: Stimuli for Experiment 2

Stimuli for Individual Number Estimation Tasks

Fraction	Value	Matched Whole Number
1/10	0.1	98
1/9	0.11111	111
1/8	0.125	125
2/7	0.28571	286
1/3	0.33333	333
5/9	0.55556	556
3/5	0.6	597
3/4	0.75	752
5/6	0.83333	833
1/18	0.05556	56
1/14	0.07143	71
2/17	0.11765	118
4/13	0.30769	308
7/16	0.4375	438
6/11	0.54545	545
11/19	0.57895	579
11/12	0.91667	917
14/15	0.93333	933

Stimuli for Addition Estimation Tasks

Fraction Problem	Answer Value	Matched Whole Number Problem	Answer Value
1/8+1/10	0.225	125+98	223
1/8+1/9	0.2361	125+111	236
2/7+1/10	0.3857	286+98	384
1/3+1/10	0.4333	333+98	431
1/3+2/7	0.619	333+286	619
3/5+1/8	0.725	597+125	722
3/4+1/9	0.8611	752+111	863
3/5+2/7	0.8857	597+286	883
1/14+1/18	0.127	71+56	127
2/17+1/14	0.1891	118+71	189
4/13+2/17	0.4253	308+118	426
7/16+1/18	0.4931	438+56	494
11/19+2/17	0.6966	579+118	697
7/16+4/13	0.7452	438+308	746
11/12+1/18	0.9722	917+56	973
6/11+7/16	0.983	545+438	983

Part C: Stimuli for Experiment 3

Stimulus Set	Number Type	Problem	Answer Value
A	Decimal	.17+.286	0.456
A	Decimal	.875+.34	1.215
A	Decimal	.643+.95	1.593
A	Decimal	.72+.182	0.902
A	Fraction	1/6+2/7	0.4524
A	Fraction	7/8+1/3	1.2083
A	Fraction	9/14+18/19	1.5902
A	Fraction	13/18+2/11	0.904
B	Decimal	.176+.27	0.446
B	Decimal	.69+.462	1.152
B	Decimal	.14+.667	0.807
B	Decimal	.889+.75	1.639
B	Fraction	1/7+2/3	0.8095
B	Fraction	8/9+3/4	1.6389
B	Fraction	11/16+6/13	1.149
B	Fraction	3/17+4/15	0.4431

Part D: Verbal Strategy Reports for Experiment 3

In Experiment 3, after stating each estimate, children reported the strategy they used to generate it. Two experimenters independently categorized children's computational estimation strategies into three categories: *rounding then adding, adding without rounding,* and *unclear*. Trials in which participants used rounding then adding or adding without rounding strategies were further classified acording to whether the addition procedure used was correct or incorrect. The two experimenters initially agreed on 96% of trials and reached agreement on every trial after discussions. Examples of strategies categorized into each of the above types are shown in Table D1.

Table D1. Description and examples of each estimation strategy (Experiment 3).

Strategy	Addition Procedure	Examples
Rounding then adding	Correct	(Problem: 9/14 + 18/19.) "9/14 is close to 1/2, and 18/19 is almost 1. I added 1/2 and 1, and I got 1 1/2."
		(Problem: 0.14 + 0.667.) "I rounded 0.14 to 0.1 and 0.667 to 0.7. Then I added 0.1 and 0.7, and I got 0.8."
	Incorrect	(Problem: 1/6 + 2/7.) "I rounded both 1/6 to 1/7. Then I added 1/7 and 2/7, and got 3/14."
		(Problem: 0.69+0.462.) "I rounded 0.69 to 0.7, and 0.462 to 0.5. Then I added them and got 0.12."
Adding without rounding	Correct	(Problem: 1/7 + 2/3.) "I found a common denominator of 7 and 3. It's 21. Then I multiplied 7 by 2 and 3 by 1 and added them for the numerator, that's 17."
		(Problem: 0.72 + 0.182.) "I added 0.72 and 0.18, and got 0.9. Then I brought down the 2, and got 0.902."
	Incorrect	(Problem: 9/14 + 18/19.) "I added 9 and 18, got 27 for the numerator; then I added 14 and 19, got 33 for the denominator."
		(Problem: 0.17 + 0.286.) "I added the 7 and 6, 1 and 8."
Unclear	N/A	"I just guessed."

For each strategy, bootstrapping was used to compare the average percent of trials on which the strategy was used on fraction and decimal problems. Rounding then adding was used more often with decimal problems (58.5%) than with fraction problems (35.4%, mean difference = 23.1%, 95% *CI* of the difference = [14.6%, 31.4%]). Adding without rounding was used more often with fraction problems (55.8%) than with decimal problems (39.1%, mean difference = 16.8%, 95% *CI* of the difference = [6.6%, 26.6%]). Unclear strategies were used more often with fraction problems (8.8%) than with decimal problems (2.4%, mean difference = 6.4%, 95% *CI* of the difference = [1.3%, 10.6%]).

Next, separately for the rounding then adding and adding without rounding strategies, the percentage of trials on which children used correct addition procedures was calculated separately for fractions and decimals. For each strategy, only children who used that strategy at least once for each number type were included in the analysis ($N = 29$ for rounding then adding, and $N = 27$ for adding without rounding). Bootstrapping was used to compare the average rate of correct addition procedure use between fractions and decimals. For trials on which children used rounding then adding, the rate of correct addition procedure use did not differ between fractions 63.3%) and decimals (72.8%, mean difference = 9.4%, 95% *CI* of the difference = [-6.3%, 24.4%]). For trials on which children used adding without rounding, the rate of correct addition procedure use again did not differ between fractions (23.3%) and decimals (37.2%, mean difference = 13.8%, 95% *CI* of the difference = [-5.7%, 32.7%]).

In summary, children adopted rounding then adding strategies more often with decimals than with fractions, but for any given strategy, they were not significantly more likely to use correct addition procedures for decimals than for fractions. Together, these facts suggest that children's higher accuracy when estimating decimal rather than fraction sums may have been driven, in part, by their greater use of rounding for decimal sums than for fraction sums.