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What Works in Gifted Education Mathematics Study: Impact of Pre-differentiated and Enriched Curricula on General Education Teachers and Their Students

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The participating university includes the University of Virginia, as well as a research unit at the University of Connecticut.

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Note to Readers...

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ABSTRACT

Heterogeneous elementary school mathematics classrooms contain an astonishing array of student learning needs, including differences in abilities, cultures, and languages. Although many teachers strive to respond to student diversity, sensing that a "one-size-fits-all" curriculum fails to reach all learners, the time and knowledge demands of differentiation often preclude teachers from making meaningful adjustments that enhance learning. Consequently, effective differentiation of curriculum and instruction is absent in most classrooms in the United States.

Prior research found that providing teachers with professional development and pre-differentiated and enriched curricula—developed according to exemplary practices in gifted education—has proven effective for students identified as mathematically talented. However, applying differentiation and enrichment strategies to all students has previously found mixed results. The impact of this type of curriculum intervention on teacher learning has also not been well-documented. In addition, in conjunction with professional development, educative curricular materials designed to provide teachers with guidance while promoting teacher learning offers promise in increasing the probability of effectively implementing pre-differentiated curricula. Therefore, the present study seeks to determine how exposure to pre-differentiated and enriched curricula incorporating educative curriculum materials affects students' achievement as well as teacher and administrator responses to the intervention.

A 2-year multi-site cluster randomized control trial study (randomized by school for participants recruited during the first year and randomized by classroom for participants recruited during the second year) recruited a national sample of 4,530 grade 3 students in 216 classrooms from 62 schools across 17 states. All treatment teachers participated in professional development on differentiated instruction and enrichment practices, as well as specific training in using the components of the three differentiated mathematics units.

Treatment and control students completed standardized pretest measures of cognitive abilities and mathematics achievement, as well as a standardized posttest measure of mathematics achievement. Students in the treatment group also completed unit pretests and posttests for each of the three differentiated mathematics units and selected items from the out-of-level grade 4 National Assessment of Educational Progress (NAEP). Researchers observed treatment and control classrooms to evaluate the fidelity of implementation of the curricular units. Treatment teachers completed logs with their responses to each of the units and participated in focus groups. Administrators completed open-ended questionnaires to assess their reactions to participating in the study.

Quantitative results did not show significant differences overall between treatment and control students on the standardized mathematics achievement test after accounting for pretest scores, although several measurement issues clouded the results, including a significant ceiling effect on the standardized posttest and a lack of content alignment between the standardized assessment and the treatment units. However, the multilevel analyses revealed significant three-way interaction effects between treatment, pretest scores, and school-average pretest achievement. The treatment appeared to be most effective for high achieving students in low achieving schools. Additionally and perhaps most importantly, replacing 16 weeks of traditional mathematics curriculum with high-level investigative mathematics did not negatively impact student gains on traditional standardized mathematical assessments. Treatment students made substantial gains from pretest to posttest on the researcher-developed unit tests, and they scored above the national average on the out-of-level NAEP items.

Qualitative results demonstrated that the treatment teachers and administrators from participating schools responded positively to the curriculum intervention. Teachers and administrators appreciated how the mathematics units provided pre-differentiated lessons with guidance on forming instructional groups based upon preassessment data. This facilitated the implementation of differentiated and enriched instruction, as many teachers and administrators noted the benefits and ease of providing students with challenging lessons appropriately matched to students' instructional levels. While many participants shared positive reactions to the curriculum intervention, some did express concerns over "covering" the state standards while also participating in the study. In comparison to control teachers, it was evident that treatment teachers engaged students more often in challenging, hands-on, and real-world lessons as well as in mathematical discourse through the use of higher-level divergent questioning. The educative nature of the intervention curriculum materials in conjunction with the provision of professional development for all participating treatment teachers indicated teacher change and learning in the areas of instructional practices, understanding of how students learn mathematics, and expectations of students. Administrators hoped that their teachers would continue to differentiate instruction, challenge students at all instructional levels, and apply authentic student learning to their lessons.

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EXECUTIVE SUMMARY

The What Works in Gifted Education Mathematics Study employed a multisite cluster randomized control trial to assess the impact of pre-differentiated and enriched curricula on grade 3 students receiving mathematics instruction in regular classroom settings. Researchers gathered data on three cohorts of teachers and students across 2 years of implementation.

Differential and enriched curricula have long been advocated for students identified as gifted and talented (e.g., Bell, 1920; Hollingworth, 1926; Ward, 1961). Some consistent adaptations propounded have been a reduction of time spent on repetitious, skilled-based tasks, acceleration through regular curriculum, grouping with like-ability peers, and an emphasis on connections among disciplines (VanTassel-Baska & Brown, 2007). Several recent implementations of differentiated and enriched curricula with elementary students identified as gifted and talented have shown enhanced achievement both in mathematics (Gavin, Casa, Adelson, Carroll, & Jensen Sheffield, 2009) and in high-fidelity classrooms in language arts (Azano et al., 2011).

Expanded views on the nature of giftedness and talent development (e.g., Gardner, 1983; Renzulli, 1978; Sternberg, 1985), as well as the need to provide diverse students with access to higher-level (Bloom, 1956) learning opportunities necessary for career success (Bellanca & Brandt, 2010; Collins & Halverson, 2009; Tomlinson, 1996; Trilling & Fadel, 2009), have created interest in offering enriched and differentiated curricular and instructional experiences to students in heterogeneous classrooms. Previous research on intensive, sustained attempts to increase student achievement through enriched and differentiated curricular and instructional approaches has shown positive results (Beecher & Sweeny, 2008; Reis, McCoach, Little, Muller, & Kaniskan 2011; Tomlinson, Brimijoin, & Narvaez, 2008). Although larger studies have shown some success applying these principles to students in mixed-ability classrooms (Brighton,

Hertberg, Moon, Tomlinson, & Callahan, 2005; Reis et al., 2010; Tieso, 2002), the evidence has been somewhat equivocal, with treatment effects benefiting certain students in certain settings.

Components of prior models developed in gifted education guided the development of the current What Works in Gifted Education Mathematics Curricula. Based on the differentiation of instruction model (Tomlinson & Jarvis, 2009), unit lessons were tiered according to multiple student readiness levels as well as being differentiated by interest and learning preference. The tiered units enabled teachers to flexibly group students for instruction. The units drew upon Kaplan's depth and complexity model (2009) to emphasize the use of mathematical language, trends and patterns, and the "big ideas" of the content. From the Schoolwide Enrichment Model (Reis & Renzulli, 2009), unit developers embedded real-world investigative projects intended to impact an authentic audience. The mathematics units reflected many of the same types of scaffolding, extension activities, and sociocultural learning present in the Project M^3: Mentoring Mathematical Minds curriculum intervention (Gavin et al., 2009).

Heterogeneous elementary school mathematics classrooms contain an astonishing array of student learning needs, including differences in abilities, cultures, and languages. Although many teachers strive to respond to student diversity, sensing that a "one-size-fits-all" curriculum fails to reach all learners, the time and knowledge demands of differentiation often preclude teachers from making meaningful adjustments that enhance learning. Consequently, effective differentiation of curriculum and instruction is absent in most classrooms in the United States.

Prior research found that providing teachers with professional development and pre-differentiated and enriched curricula—developed according to exemplary practices in gifted education—has proven effective for students identified as mathematically talented (Gavin et al., 2009; Gavin, Casa, Adelson, & Firmender, 2013; Gavin, Casa, Firmender, & Carroll, 2013). However, applying differentiation and enrichment strategies to all students has previously found mixed results The impact of this type of curriculum intervention on teacher learning has also not been well-documented. In addition, in conjunction with professional development, educative curricular materials designed to provide teachers with guidance while promoting teacher learning offers promise in increasing the probability of effectively implementing pre-differentiated curricula. Therefore, the present study sought to determine how exposure to pre-differentiated and enriched curricula incorporating educative curriculum materials affects students' achievement as well as teacher and administrator responses to the intervention.

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What Works in Gifted Education Mathematics Study: Impact of Pre-differentiated and Enriched Curricula on General Education Teachers and Their Students

CHAPTER 1: Introduction

Jennifer L. Foreman

In the aftermath of the Sputnik launch, U.S. educational policies increased focus on student achievement in mathematics, science, and technology to maintain a competitive edge as a world economic and political power. The Soviet Union created a credible threat to the security of our nation, and national leaders recognized the need to cultivate the talents of American youth to withstand it. The ensuing National Defense Education Act (NDEA) of 1958 promoted the use of ability grouping, acceleration, and special programming for advanced students to pursue this competitive edge. Enthusiasm for these policies waned after the immediate threat had passed, yet critics of the education system continued to warn of the United States' persistently unfavorable performance in mathematics and science on international comparisons of student achievement. Although U. S. citizens have come to accept that the world of the 21st century and beyond is "flat" (Friedman, 2005), it is not completely certain what this means for future generations.

Just as global competition for jobs poses an external threat to the future of U. S. citizens, achievement gaps between students of different ethnic, racial, and social class groups threatens the nation from within. Some denouncers of student ability grouping (Oakes, 1985; Sapon-Shevin, 1994) argue that the types of arrangements fostered by policies like NDEA perpetuate these gaps. Even advocates of advanced learners concede that unacceptable achievement gaps exist among the highest performing students of different racial and social class groups (Plucker, Burroughs, & Song, 2010; Wyner, Bridgeland, & Dijulio, ND). Several decades of research consistently point to the under-representation of African Americans, Hispanic/Latino students, Native Americans, and students from low-SES backgrounds in special programs (such as gifted and talented programs) for high ability students (Gentry, Hu, & Thomas, 2008).

How can the United States retain a status as a great nation when such vast disparities exist in the lifetime opportunity structures for the diversity of the American people? The authorization of the federal No Child Left Behind legislation at the turn of the 21st century recognized that these disparities, if left unabated, could lead to outcomes that were not only inequitable for groups of individuals, but also unsustainable for the nation as a whole.

Quite recently, the Obama administration has again raised the call for renewed emphasis on excellence in what has commonly become known as STEM (science, technology, engineering, and mathematics) education. However, in conjunction with efforts to reduce achievement gaps among American students, the education system must seek solutions that create an optimal balance between external and internal risks to the stability and security of America's future. The access schools provide American students to challenging, engaging learning opportunities cannot be only for a select few. All the Nation's children must have exposure to the kinds of knowledge, understandings, and skills that will enable them to lead lives as contributors to America's continued economic, political, and social well-being.

Educational Needs of Tomorrow's Citizens

Over the past decade, authors and academics have given voluminous amounts of attention to "21st century skills" that students will need to function in an information-laden, technology-driven world. The Partnership for 21st Century Skills (2011) refers to these skills as a fusion of "the three Rs" with "the 4 Cs"—namely, critical thinking and problem solving, communication, collaboration, and creativity and innovation. Despite the current trendiness of touting these skills, it would be difficult to distinguish them from the age-old educational virtues of reasoning well, judging information wisely, solving problems in new ways, and communicating and working effectively with others. William James (1885) identified progressive levels of knowledge before the turn of the 20th century, and Bloom's popular Taxonomy of Educational Objectives (1956) in the cognitive domain has been with us for more than half a century, as well. Although a progression of escalating educational demands has been known to educators for some time, what has changed is the population of students for whom lower and higher levels of knowledge is judged to be appropriate educational programming. For many years and in many schools even today, there is often a fracture between the basic skills that comprise the general education curriculum and the higher order thinking skills that are the purview of gifted education programs.

In a review of the research on the efficacy of gifted education curriculum models, VanTassel-Baska and Brown (2007) identified four "best practices:"

- . . . [G]roup gifted students instructionally by subject area for advanced curriculum work that would be flexibly organized and implemented based on students' documented level of learning within the subject area.
- . . . [E]mbed multiple higher level thinking models and skills within core subject area teaching.
- . . . [U]se inquiry as a central strategy to promote gifted student learning in multiple modalities.
- . . . [U]se student-centered learning opportunities that are issue- or problem-based and relevant to the student's world. (pp. 351-352)

However, as even veteran advocates of gifted education have argued (Borland, 2003, 2005; Tomlinson, 2012) it has become difficult to sustain arguments that the latter three of these "best practices" for gifted education are fundamentally and qualitatively different from what all students need and deserve in curriculum and instruction.

During the last quarter of the twentieth century, a number of researchers of giftedness and talent development began advocating for a new way of approaching the conception of giftedness (Gardner, 1983; Renzulli, 1978; Sternberg, 1985). This reconceptualization held broad implications for change in the design and delivery of educational services, as well. Instead of envisioning giftedness as a permanent, enduring trait that exists in some people and not in others, these theorists recognized the many interacting components of the individual and the environment that must come together for giftedness to be manifested. Categorizing students in this way became perceived by some to be not only politically indefensible but also lacking correspondence with empirical evidence about the nature of human potential. The movement away from simplistic dichotomies of qualitatively different "gifted" and "non-gifted" students led to advocacy for the provision of services traditionally reserved for students identified as "gifted and talented" to all students. Models such as the Schoolwide Enrichment Model (Renzulli & Reis, 1997) and the Differentiation of Instruction Model (Tomlinson, 1999, 2001) were developed with the recognition that all students deserve and benefit from regular exposure to curricular and instructional experiences beyond basic skill development (Reis, Gentry, & Park, 1995; Renzulli, 2005).

Theoretical Framework

The theoretical framework for the What Works in Gifted Education Mathematics Study focuses on teachers, learners, and the curricula. Teachers are "expected to teach meaningful content that helps students to meet new learning goals in the context of authentic activities, while addressing the needs of diverse learners and ensuring that all students are successful" (Davis & Krajcik, 2005, p. 3). All students can learn to high levels, and they must have access to challenging curriculum (Haycock, 2001) to ensure they are actually learning rather than waiting to learn as they are exposed repeatedly to concepts they have already mastered. The "ideal act of learning" illustrates the dependence and interdependence of the teacher, learner, and curricula (Renzulli & Reis, 1997) (see Figure 1.1). Teachers' knowledge of the discipline and pedagogical skills are important to understand the content and methodology of the discipline and to capitalize on the learner's abilities, interests, and learning styles.

The theoretical framework is supported by the following review of literature that expands the conceptual basis for designing challenging, differentiated mathematics curricula for all grade 3 students, using Vygotsky's (1978) zone of proximal development as a necessary consideration in curriculum development. Complex concepts require presentations in multiple ways by varying the extent of scaffolding needed to guide students' knowledge, understanding, and application of concepts.

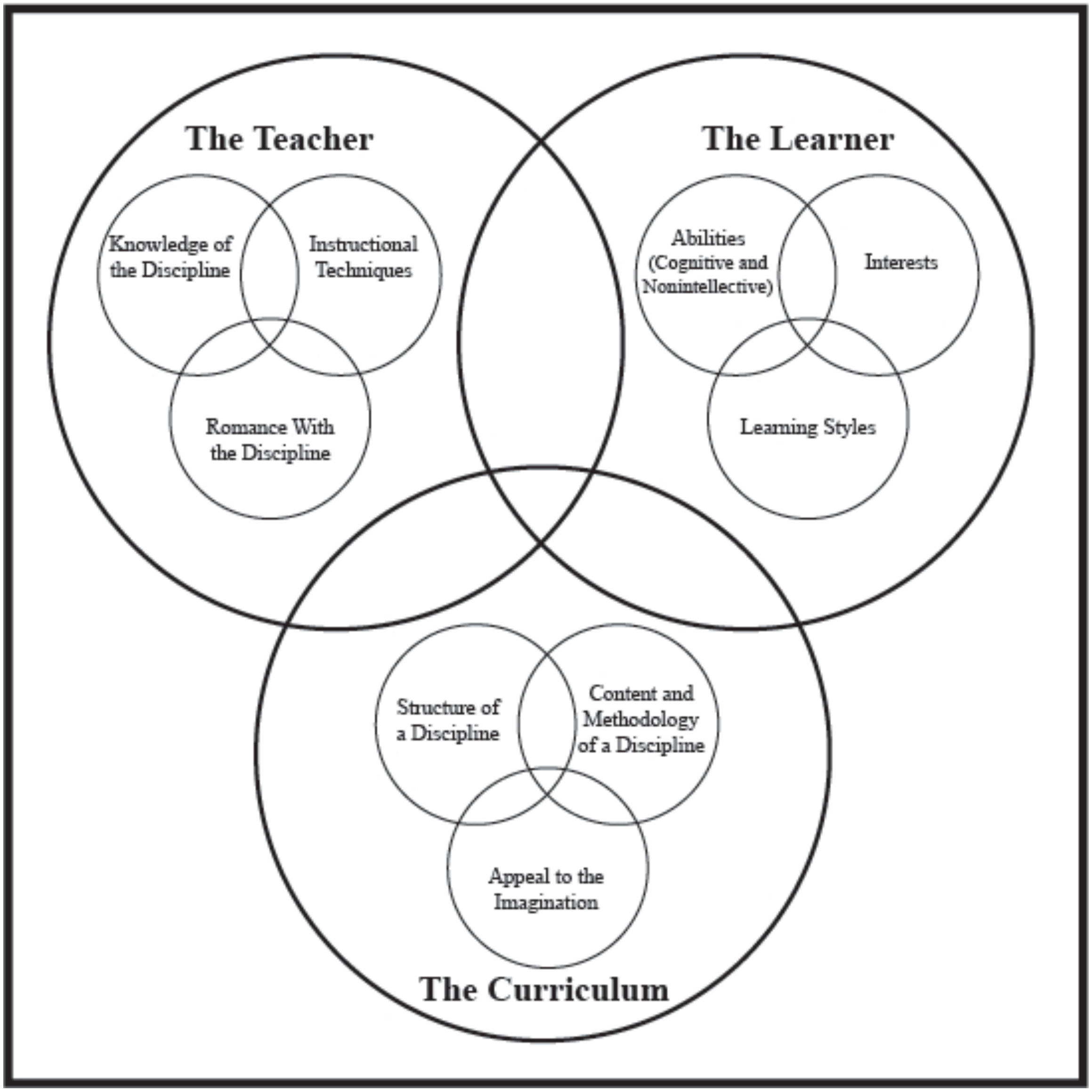

Figure 1.1. Figural representation of the act of learning (Renzulli & Reis, 1997).

CHAPTER 2: Review of the Literature

Jennifer L. Foreman
Jennifer Savino
Cindy M. Gilson

The review of literature provides an overview of the theoretical, curricular, and instructional models popularized in the field of gifted and talented education that influenced the present study. In addition to these overviews, identification procedures, teacher change, and learning through educative curricula are addressed. The focus of the chapter then turns to the importance of high-end thinking in elementary school mathematics classrooms with special emphases on nurturing mathematical talents, exploring teachers' mathematical understanding, and mathematical discourse.

Guiding Models for the What Works in Gifted Education Mathematics Curricula

Researchers at the University of Connecticut created model-based mathematics curriculum units with specific modifications and differentiation. These units were designed to be responsive to the academic diversity of mathematically talented students and all other students in general education classrooms. Elements of three curricular models in the field of gifted and talented education were combined and utilized to develop these units: the Differentiation of Instruction Model from Carol A. Tomlinson, the Depth and Complexity Model from Sandra N. Kaplan, and the Schoolwide Enrichment Model from Joseph S. Renzulli and Sally M. Reis. The study sought to examine the impact of this mathematics curriculum intervention on the achievement of general education classroom students and the professional development of their teachers.

Differentiation of Instruction Model

The Differentiation of Instruction Model (Tomlinson, 1999) is designed to provide rich and engaging curriculum matched to the diverse interests, readiness levels, and learning profiles of individual students (see Figure 2.1). The model assumes that there is no distinct, single curriculum that is appropriate for gifted learners, but rather that all students require educational experiences suited to their individual needs. The model proposes three main aspects of the learning experience that can be differentiated according to learner differences—content, process, and products. Content refers to the subject material that a student works with in the classroom. Content can be altered to a greater or lesser degree of complexity, depth, and level of abstraction based on learners' current readiness levels. Process refers to the dynamic teaching and learning interaction, which can be adjusted in terms of pace and responsive options. Finally, the products of student learning can be differentiated to expect greater or lesser levels of expertise, more close-ended or open-ended assignments, and greater or lesser levels of student independence.

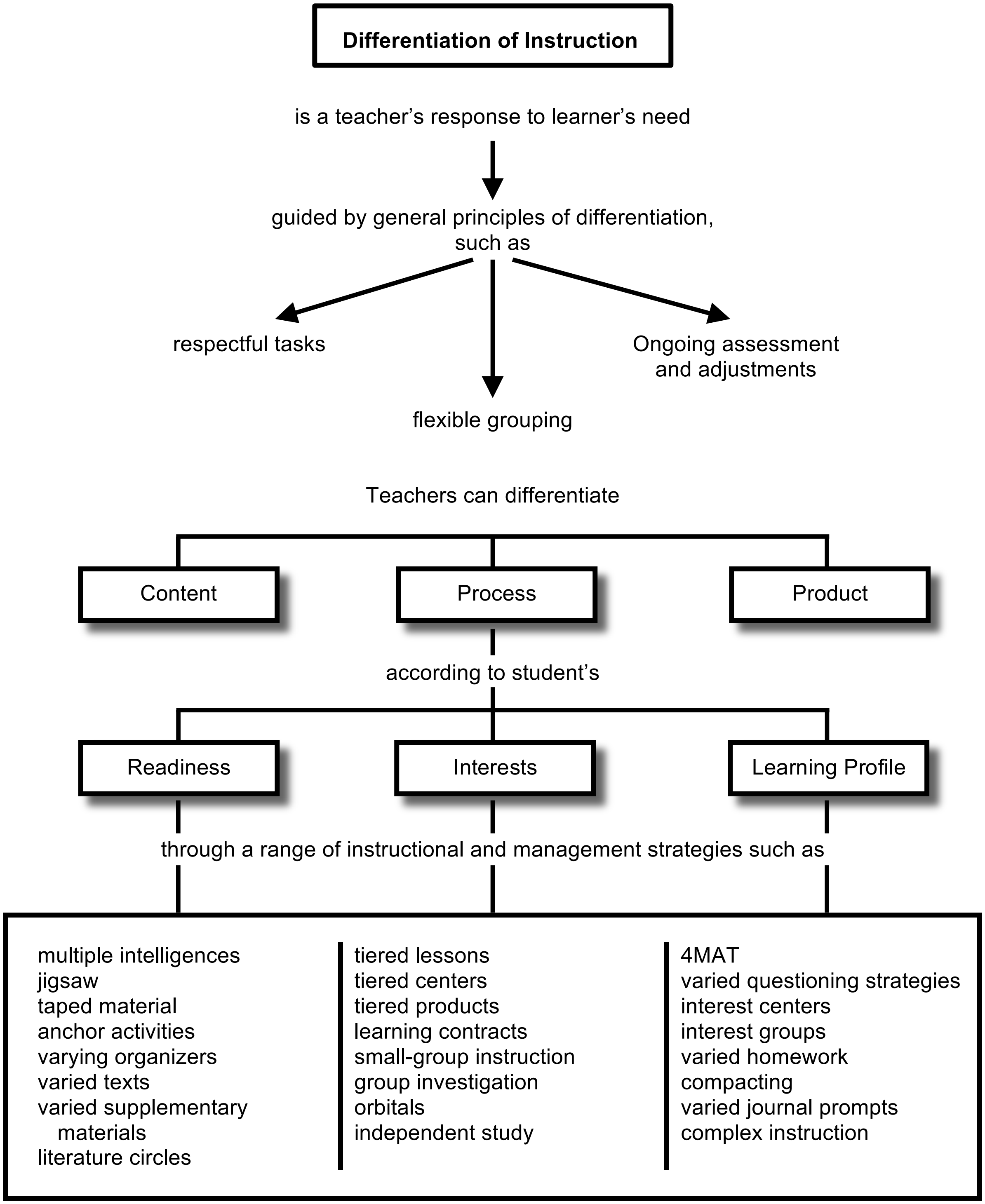

Figure 2.1. Tomlinson's Differentiation of Instruction Model.

Teachers who differentiate instruction effectively can provide opportunities for students to process information and demonstrate understanding in ways appropriate for their individual needs. They expose students to rich, engaging curriculum around concepts and principles to allow for in-depth and enduring understanding. Differentiated learning experiences provide opportunities for students to work in areas of strength as well as areas of need. They also foster learners who are articulate about their strengths and needs and who take active responsibility for their own learning. Although research on the effectiveness of differentiated instruction on improving students' achievement is still emerging, some studies (Brimijoin, 2001; Tieso, 2002) suggest that both students identified as gifted and those not identified as gifted, as well as students from different socioeconomic backgrounds, who receive differentiated learning experiences have increased academic achievement. Another study through The National Research Center on the Gifted and Talented (Brighton, Hertberg, Moon, Tomlinson, & Callahan, 2005) did find modest improvements in student achievement when teachers used an assessment-based treatment to differentiate for their students. While more research is clearly warranted to assess the effectiveness of differentiated instruction, the gains to student learning will need to be examined longitudinally as teachers and schools learn how to implement the model fully. Done correctly, differentiated instruction is a deep, systemic change to enhance student learning over the long-run, not necessarily a quick fix that brings immediate results. Figure 2.1 summarizes how the Differentiation of Instruction Model functions in the classroom context.

Depth and Complexity Model

The second model that was used to develop the curricular units for the present study was Kaplan's Depth and Complexity Model (Kaplan & Gould, 1998; see Figure 2.2). The Depth and Complexity Model presents strategies that promote questioning, utilize thinking and problem-solving, and organize information and planning for teachers and students alike. The dimensions of depth and complexity allow all teachers the opportunity to define, implement, and evaluate their differentiation of instruction and to plan learning experiences that provide activities suited to the content and learners' needs. The elements of the model are interrelated and mutually reinforcing.

Depth includes a set of eight elements that help facilitate learning within a discipline at differing levels of sophistication. The first element, *Identify the Rules*, focuses on defining organization elements and identifying and describing factors in the content being learned. The second element, *Statement of Trends*, involves the identification of changes over time and attention to causal factors and events that occur in the topic under consideration. Third, *Ethical Considerations* enable students to identify and analyze the ethics of an idea or event and categorize the ethical elements of the idea or event. Fourth, *Note the Patterns* investigates the order of events and helps students identify patterns and predict future occurrences. Fifth, *Recognize the Details* supports students' elaborations and descriptions of nuances of the topic. Sixth, the *Language of the Discipline* element promotes the use of appropriate terminology as students aim to think like practicing professionals. The seventh element, *Define Unanswered Questions*, calls for clarification, discovery and exploration, and evidence to support what remains unknown about a topic. Lastly, *Big Ideas* refer to generalizations, principles, and theories that can be induced from the specifics of a range of phenomena.

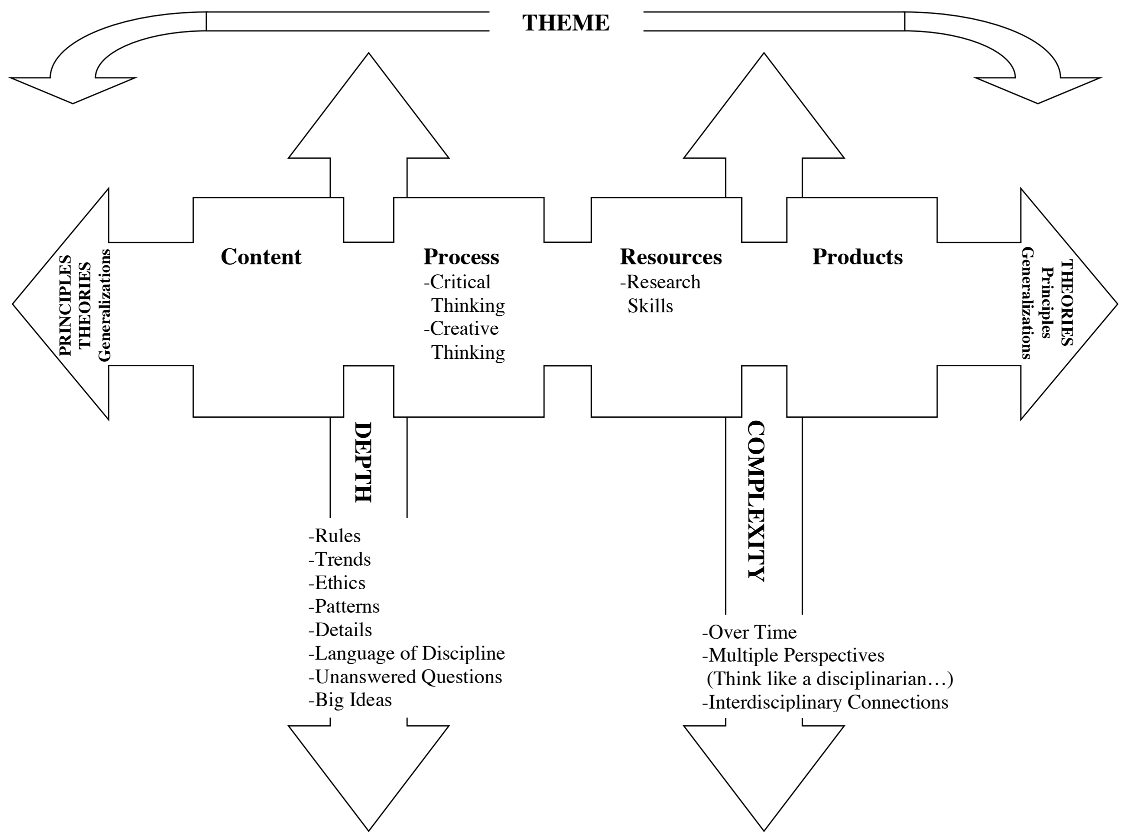

Figure 2.2. Kaplan's Depth and Complexity Model.

Complexity is the set of three elements that helps facilitate learning content or subject matter by focusing on the relationship between various disciplines, analyzing how disciplines have changed over time, and examining various issues from a variety of perspectives. The over time aspects emphasize the relationships among ideas and knowledge between past, present, future, or within a time period. Multiple perspectives present opposing viewpoints, as well as differing roles and knowledge. Third, interdisciplinary relationships allow students to explore the function of knowledge within the discipline, and between and across disciplines. Figure 2.2 illustrates how the elements of Kaplan's Depth and Complexity Model tie together to form a holistic structure for the development of differentiated curriculum and instructional strategies.

Schoolwide Enrichment Model

The Schoolwide Enrichment Model (SEM; Renzulli & Reis, 1997) is widely implemented as an enrichment program used with academically gifted and talented students and a magnet theme/enrichment approach for all schools interested in high-end learning and developing the strengths and talents of all students (see Figure 2.3). The SEM focuses on enrichment for all students through high levels of engagement and the use of enjoyable and challenging learning experiences that are constructed around students' interests, learning styles, and preferred modes of expression. Separate studies on the SEM have demonstrated its effectiveness in schools with differing socioeconomic levels and patterns of program organization. The SEM has been implemented in over 2,500 schools across the country (Burns, 1998) and programs using this approach have been widely implemented internationally. The effectiveness of the model has been studied in over 20 years of research and field-testing (Gubbins, 1995; Reis & Renzulli, 2010; Renzulli & Reis, 1994).

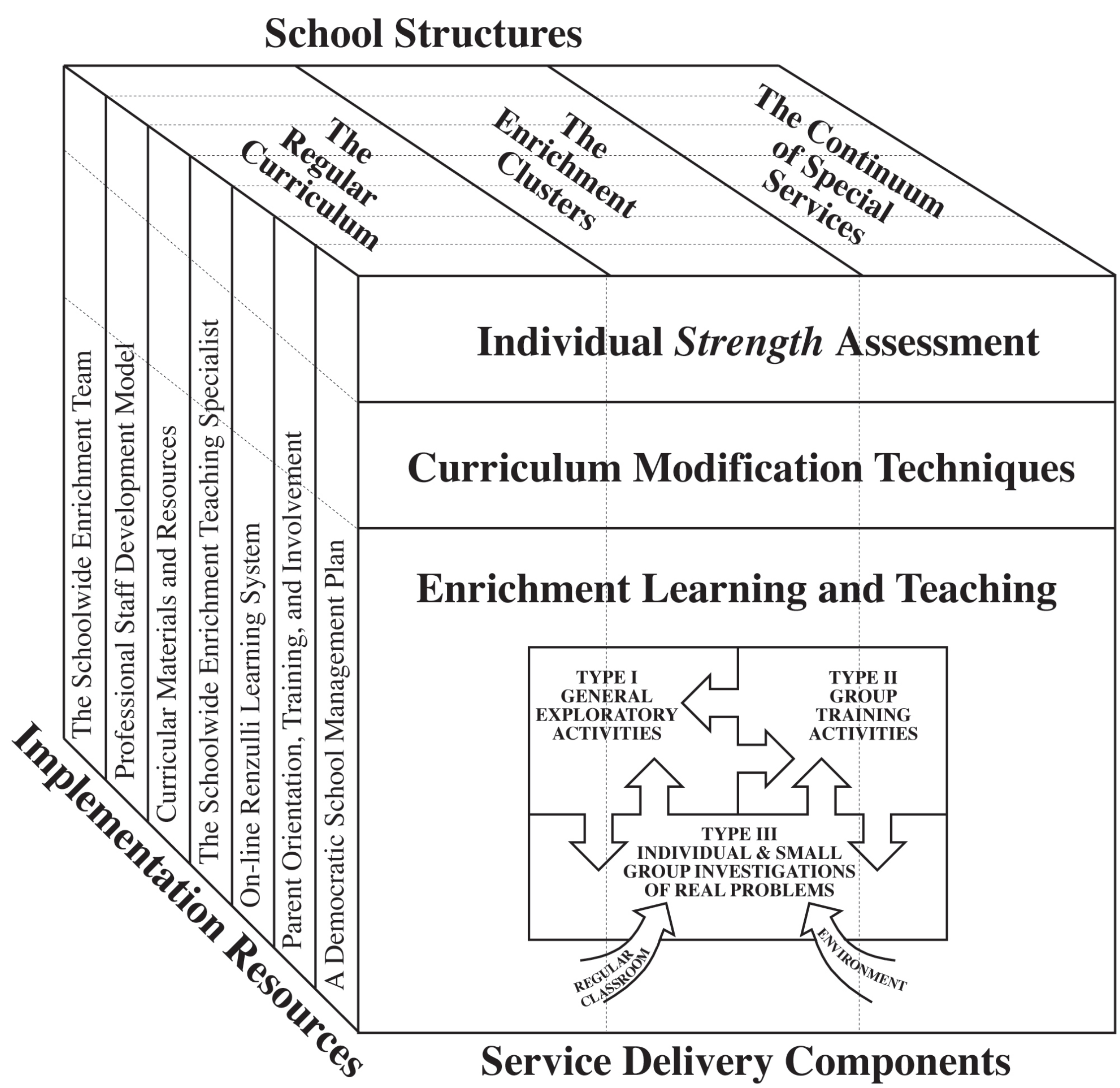

Figure 2.3. The Schoolwide Enrichment Model.

The Schoolwide Enrichment Model offers opportunities for modifying and differentiating curriculum in a variety of ways. Schools implementing the model can adjust levels of learning to challenge the academic abilities of all students. The model also calls for an increase in the number and frequency of in-depth experiences with specific content-related issues and problems. One key feature of the Schoolwide Enrichment Model is called curriculum compacting, in which teachers use formal and informal preassessment techniques to eliminate mastered curriculum (see Reis, Burns, & Renzulli, 1992). Once students have been preassessed, teachers and students can create a contract to replace mastered curriculum with more challenging, in-depth opportunities for content acceleration and research projects. The model also endeavors to provide students with options to modify or differentiate content, process, products, or the learning environment, similar to Tomlinson's model.

The Schoolwide Enrichment Model focuses on goals that are an outgrowth of a specific conception of giftedness and how gifted behaviors might be enhanced in all students over time through a process of talent development. All students should have opportunities to be exposed to a variety of disciplines, topics, and issues. Students should also develop critical and creative thinking skills, learning-how-to-learn skills, and advanced reference and communication skills. As these skills are developed, students should be provided opportunities for applying interests, knowledge, creative ideas, and task commitment to a problem or area of study. Students who show the ability and motivation to extend their learning receive opportunities to acquire advanced-level understanding of the knowledge (content) and methodology (process) used in various disciplines to develop products that are directed toward bringing about a desired impact on specific audiences.

Throughout the talent development process, students will develop self-directed learning skills in the areas of planning, organization, resource utilization, time management, decision making, and self-evaluation. Additionally, students develop task commitment, self-confidence, and feelings of creative accomplishment. To give students the chance to begin thinking as professional practitioners in a field, the model promotes investigative activities in which students assume the role of the first-hand inquirer; they think, feel, and act like a practicing professional at a "junior level." The real-world oriented projects that are the culmination of this sequence emulate products that have been accomplished by history's great thinkers and doers. Figure 2.3 summarizes the layers of opportunities provided to students participating in the Schoolwide Enrichment Model's offerings.

The specific conception of giftedness undergirding the Schoolwide Enrichment Model is the Three-Ring Conception of Giftedness (Renzulli, 1978). The Three-Ring Conception of Giftedness focuses on students whose giftedness is best described as an interaction of three traits (well-above-average abilities, creativity, and task commitment) brought to bear on a specific topic, problem, or issue leading to a product or a service. Rather than focusing on giftedness as a stable trait of some individuals but not others, the three clusters of traits converge "in certain people, at certain times, under certain circumstances." The Three-Ring Conception of Giftedness contends that individuals capable of developing high levels of gifted behavior are those possessing or capable of developing this composite set of traits and applying them to any potentially valuable line of human performance. Persons who manifest or are capable of developing an interaction among the three clusters require a wide variety of educational opportunities and services that are not ordinarily provided through regular instructional programs. Therefore, the focus of gifted education is to increase the likelihood of this convergence by strengthening each of the clusters of traits while providing opportunities for them to come together. The houndstooth pattern in which the Three-Ring Model is set represents those non-cognitive influences, such as optimism and a sense of destiny, that are also found in persons who have accomplished great things. Figure 2.4 illustrates the Three-Ring Conception of Giftedness.

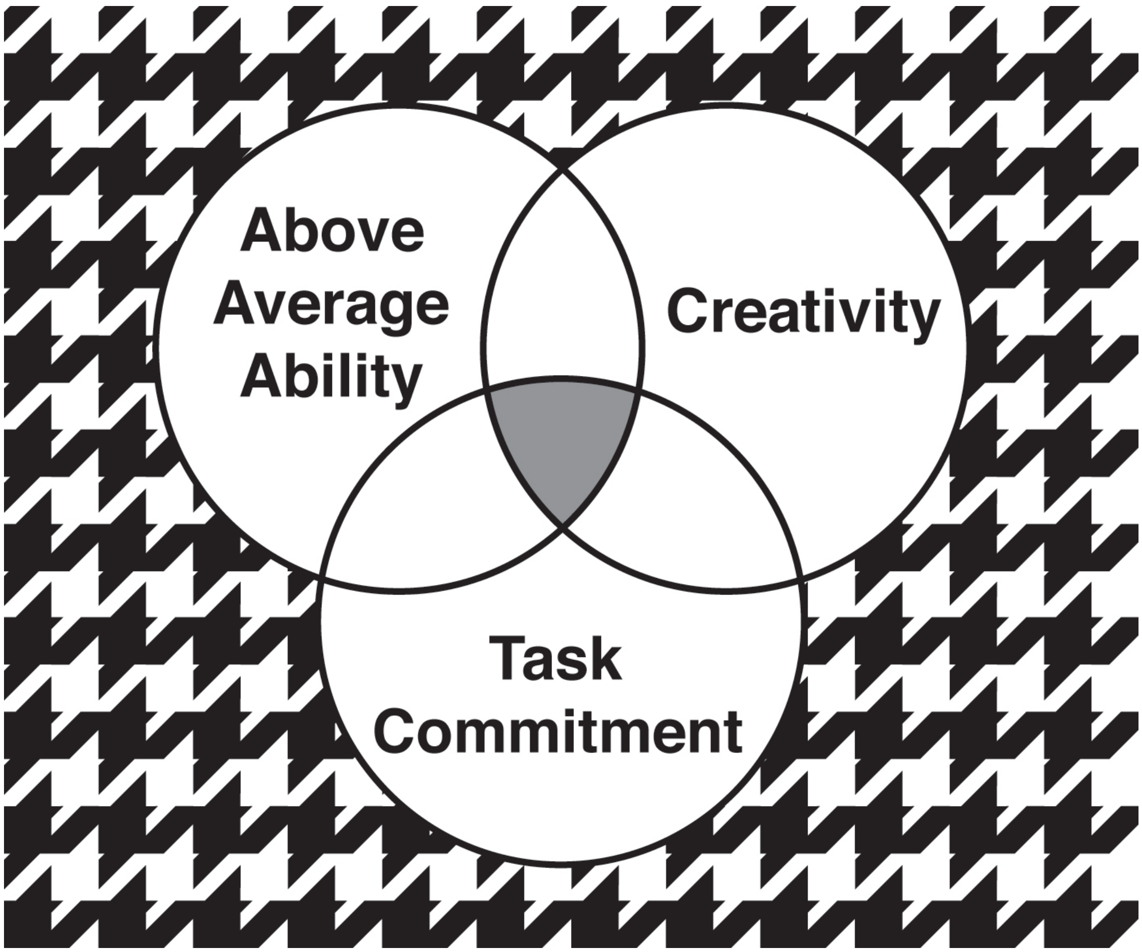

Figure 2.4. Three-Ring Conception of Giftedness.

The Three-Ring Conception of Giftedness promotes reflecting on and assessing students' obvious and latent talents and abilities rather than using one measure and a specific cut score to determine if a child is gifted.

Identification Procedures

A traditional psychometric conception of giftedness (Robinson, 2005) certainly simplifies the question of how students should become identified for special gifted education services. When only intelligence, as measured by standardized IQ tests such as the Stanford-Binet V and Weschler's Intelligence Scale for Children, is considered as indicative of giftedness, identification can be tidily done by determining which test will be used and what the cut score will be for admission into special programming. Students are tested, their scores are compared to the cut score, and they are either accepted for or rejected from programming on this basis. Issues of measurement error can be entertained in the process, but it is relatively straightforward and transparent.

However, from a broadened conception of giftedness such as those presented in the models discussed above, identification for special programming will need to include a more comprehensive review of students' strengths and weaknesses in a variety of contexts and include assessments from a broad variety of sources, including traditional IQ and achievements tests as well as tests of other traits such as creativity and motivation. Teacher, parent, peer, and self reports also add to the review so that a more complete and nuanced view of a student's educational needs can be determined. While the Schoolwide Enrichment Model includes very specific guidelines for a broadened, multiple criteria identification system, models such as Tomlinson's Differentiation of Instruction Model function without a need for categorical identification of students. If teachers can assess and meet individual students' needs in the general education classroom without a need to label and separate discrete categories of students, then individualized solutions might be tailored to enhance each student's talent development without needing formal identification procedures. However, teachers will need considerable support and continuing education to create the systemic change necessary to embrace this type of educational model.

These models and the expanded conception of giftedness applied in gifted education programs and services herald a change in teaching and learning in which the importance of challenging all learners predominates the core philosophies. To illustrate how the changes can and should be integrated into curricula was accomplished by creating educative curricula for the mathematics units in algebra, geometry and measurement, and graphing and data analysis.

Promoting Teacher Change and Learning Through Educative Curricula

Educative curricula are gaining increasing attention as a promising intervention for supporting and promoting teachers' continuing professional growth (Davis & Krajcik, 2005). What makes educative curriculum materials different from traditional curriculum materials is that they go beyond just providing teachers with a repertoire of instructional strategies. Davis and Krajcik point out the term *educative* indicates teachers are learners, therefore educative curriculum materials are specially designed to develop teachers' subject knowledge and pedagogical content knowledge for topics, and pedagogical content knowledge for disciplinary practices (e.g., mathematical inquiry). When designing this type of curriculum, it is important to keep in mind challenges, such as differentiating for students based upon preassessment data, that teachers encounter to ensure the practicality and applicability of the educative components. The mathematics curricular units in the current study were designed with the principles of educative curricular materials in mind by incorporating explanations of how the unit lessons' specific objectives connect to real-world big ideas, justifications for the use of particular instructional practices (e.g., Talk Moves, hands-on learning), explanations of how to provide differentiated instruction using preassessment data, and potential reactions to the lessons by the students.

The present study sought to combine successful elements of these three models of gifted education to create educative mathematics curricular units that would promote learning and enjoyment of students in regular heterogeneous classrooms and their teachers. Because gifted behaviors are often seen as domain specific, the ways in which they can be promoted within the subject area of mathematics points to areas of emphasis and intervention. What might the development of gifted behaviors look like in the context of elementary school mathematics curriculum?

High-end Thinking in Elementary School Mathematics Classrooms

Although higher-order thinking skills cut across curricular fields of knowledge and student grade levels, their application to elementary mathematics curriculum and instruction is particularly vital. The National Council of Teachers of Mathematics' inclusion of both content and process standards in its 2000 Standards "encourages students to problem solve, communicate, reason, make connections, and use different representations as they engage with mathematics" (Gavin et al., 2007, p. 569). The recent results from the mathematics portion of the 2009 Programme for International Student Assessment (PISA) indicate that, in addition to being below the international average scale score overall, the United States also produced a significantly smaller percentage of students than the international average who achieve at the highest level of PISA's performance standards—a level that denotes the ability to "conceptualise, generalise, and utilize information based on their investigations and modeling of complex problem situations" (Organization for Economic Co-operation and Development [OECD], 2009, p. 122). Yet how many problems that will need to be solved in these young people's futures will not be of this complex nature? Although teaching correct mathematical procedural skills to elementary students is also imperative, waiting until middle or high school to emphasize generalizations and abstractions may cause misconceptions about mathematics that are difficult to remedy in later years (Carpenter, Franke, & Levi, 2003). Instead, teachers should begin at the youngest grade levels to support students' conceptual understandings that allow them to reach levels of abstraction that are utilized by students traditionally referred to as "mathematically talented."

Nurturing Mathematical Talent

To nurture mathematical talent in students, it is critical to understand how talented young mathematical minds work. Two elements of the study's mathematics curriculum, problem-solving tasks and collaborative endeavors—though not mutually exclusive—were appropriately integrated as learning activities within the investigative sections of many of the lessons. Although these elements were not included solely to satisfy the needs of mathematically gifted students, they are frequently discussed in the literature examining this population and will be discussed here. Ultimately, these elements were extracted from the literature, modified appropriately, and included in the curriculum with the potential to advance the mathematical development of all students.

Much mathematical research has been dedicated to illuminating the transition from concrete thinking to abstract reasoning, particularly as it relates to the ability to identify and explicate generalizations. One particular study used a qualitative case study approach to examine this transition through an analysis of student journal entries and interviews. The data were gathered over a 3-month period and documented the problem-solving and reflective processes of nine students as they worked through a series of increasingly complex mathematical problems designed to facilitate abstraction and lead to generalization (Sriraman, 2003). The four previously identified mathematically gifted students' accurate and consistent identification of similarities across problems, in-depth analyses, and metacognitive reflection, all sustained by their inquisitive persistence, led them to the overall generalization. Inconsistencies in comprehension, applied procedures, and articulation, as well as a lack of verification that resulted in unjustified satisfaction kept the non-identified students from reaching this generalization. Although the students in this study were in a ninth-grade accelerated algebra class, the resulting tenet applies to all levels of mathematical abstraction: "There exists a relationship between mathematical giftedness, problem-solving ability, and the ability to generalize" (Sriraman, 2003, p. 151). Many of the problems that shape the investigative element of the present study's curricula were designed with this relationship in mind. Like the problems in the Sriraman study, the present study's learning activities required students to metacognitively explicate their thoughts and justify their solutions. Similarly, many problems in both studies were intentionally sequenced and demanded a return to and modification of previous strategies, which would elucidate key similarities on the way to an overall generalization.

In solving similarly complex and adequately yet appropriately frustrating problems, mathematically gifted students relied upon another metacognitive trait—self-knowledge of their collaborative needs in efficiently and successfully arriving at a solution. Another study used an exploratory case study design to examine the relationship between six mathematically gifted pre-teen students' preferences to collaborate and the level of challenge of the assigned problem or task (Diezmann & Watters, 2001). One researcher served as a nonparticipant critical observer who witnessed the dialogue and behavior of these students during a one-hour problem-solving session facilitated by another researcher. Analyzing these observations and the archival data of students' completed worksheets led the researchers to determine that mathematically gifted students preferred collaboration when presented with particularly challenging problems. These students sought cognitive or affective support in interactions with both their teacher and their peers. The four problems were presented in order of increasing difficulty; students completed the first two problems efficiently and accurately independently, but they went to the teacher for either clarification or scaffolding for the third problem. By the time they reached the fourth problem, the students conducted a group discussion to reach an accurate consensus regarding the solution.

Researchers discovered that students in this collaborative endeavor provided scaffolding for each other, demanded explanation and support for proposed answers, and demonstrated the metacognitive ability to evaluate and revise their thinking and their conclusions. In this collaborative setting, students worked at a more advanced cognitive level than they would have been able to independently on a similarly challenging task; by working with and observing other mathematically talented students who also needed a support system to overcome challenges, these students grew in their confidence and self-efficacy. The researchers, however, temper their presentation of these findings with a sobering emphasis on the delicate nature of accurately determining readiness, successfully matching task difficulty to capability, and maintaining student motivation throughout these difficult tasks. The implications presented by the researchers mirror the efforts of the present study not only to determine student capabilities but also, in respecting these capabilities, to provide a diverse array of problems or tasks that address the variegated needs of a typical heterogeneous classroom.

While respecting the fact that students in heterogeneous regular classrooms may have differing optimal levels of task challenge, the What Works Mathematics Curricula offered students opportunities to collaborate with peers to receive support for solving appropriately challenging problems. The curriculum included many tiered assignments, a form of differentiation entailing multiple activities that all promote the same conceptual understanding. Students were flexibly grouped to collaborate on these activities with other students demonstrating similar incoming readiness levels, interests, or learning styles.

Elementary Teachers' Mathematical Understanding

In addition to what students bring to the classroom, student achievement can also be influenced by what their teachers bring to the classroom. Nye, Konstantopoulos, and Hedges (2004) indicated that as much as 21% of variance in students' achievement scores could be attributable to teacher effectiveness. This influence was higher for mathematics than for reading. Unfortunately, the picture research paints of elementary school pre-service and in-service teachers' predispositions and preparation regarding elementary mathematical content is far from reassuring.

One study from the National Center for Research on Teacher Education at Michigan State University challenged prevalent misconceptions of mathematics teacher education that strongly and wrongly influence how pre-service teachers are—or are not—prepared for the mathematics classroom (Ball, 1990). Misconceptions that were addressed revolved around the tendency to assume that "traditional school mathematics content is simple" (p. 462) and, thus, that the subject, conceptual, and practical mathematical knowledge development of pre-service teachers was adequate. In fact, undergraduate education program training on mathematical knowledge was so deficient that, in responding to mathematical questions posed by the researcher, both elementary education majors and secondary mathematics education majors had to access knowledge acquired in their own primary and secondary mathematics classes. Additionally, citing this knowledge in their responses did not reveal long-established mathematical understanding but rather exposed their understandings as based on little else but haphazard, unconnected, and meaningless rules and procedures.

To challenge the misconceptions that lead to these inadequacies in mathematics teacher education, the researcher conducted a longitudinal study of 217 pre-service elementary education majors and 35 secondary mathematics education majors that examined participants' perspectives on mathematics and the teaching of mathematics as well as their mathematics subject matter knowledge and mathematics self-efficacy. Data for Ball's study came from questionnaires, interviews, and observations that spanned the beginning of the teacher education program to the first year of teaching. Results indicated that the mathematical "understandings" of both the elementary education major participants and the secondary mathematics education major participants were limited to the ability to apply their knowledge of mathematical procedures to perform calculations. Participants were neither able to explicate the mathematical principles underlying the procedures they used nor capable of discussing the meaning of the mathematics beyond the rules. They viewed mathematics as "a collection of arbitrary rules to be memorized" (Ball, 1990, p. 460) and mathematical knowledge as comprised solely of "discrete bits of procedural knowledge" (p. 459) rather than a unified field of study that requires thoughtful selection and purposeful application of meaningful procedures to solve complex problems, respects differences in perspective and process, and facilitates discussion. However, if these pre-service teachers had never experienced mathematics this way at any level of schooling and their teacher education programs had never presented mathematics this way, it is not difficult to determine how such narrowly compartmentalized views of mathematics were propagated.

Although the teachers did recognize these deficiencies and relatively accurately projected how they could inevitably hamper instruction, the passivity and simplicity with which these teachers pursued mathematical understanding (if they pursued it at all) paralleled the manner in which the non-generalizing students in the Sriraman study (2003) approached problem-solving tasks. Cognitively, both populations viewed mathematics solely as an accumulation (rather than a more-unified "collection") of arbitrary rules to be memorized and random procedures to perform on numbers. Behaviorally, each population demonstrated premature satisfaction with their mathematical endeavors, whether solving assigned problems or seeking meaning (or not seeking meaning). The student participants often guessed haphazardly without testing their reasoning and did not return to the problem once they had reached an initial conclusion. Both elementary and secondary teacher participants' responses to researchers' questions about explaining particular mathematical ideas centered on rules and procedures; they "lacked explicit and connected conceptual understanding of mathematical ideas and procedures" (Ball, 1990, p. 461) and thus were not able to reason beyond the simple application of the rule or procedure. These pre-service teachers exhibited similar premature satisfaction with their depth of knowledge as the students in the Sriraman study and never returned to their mathematical understandings to revise and refine them even though they admitted that they were worried about responding to conceptual questions from students (Ball, 1990). They were satisfied with their inculcation of general rules to explicate mathematical ideas and demonstrated minimal curiosity or perseverance in their conceptual development despite their fears of the classroom; would these pre-service teachers ever be able to successfully lead students to mathematical generalization?

A positive answer to this question might lie in an analysis of the pre-service teachers' feelings towards mathematics. Although these pre-service teachers shared commonly narrow mathematical perspectives, they did vary substantially in their mathematical affect; these differences reflected the participants' majors. All secondary mathematics education participants expressed enjoyment of and efficacy in mathematics. Half of the elementary education major participants demonstrated little to no enjoyment of and low efficacy in mathematics; of these participants, over one-third admitted to avoiding mathematics whenever possible. Avoiding mathematics will not lead to increased conceptual understanding; positive affect towards mathematics is a precursor to learning the generalization skills necessary to inspire students to do the same.

Although the Sririman study's non-generalizing students were also non-identified students and parallels can be drawn between these students and the pre-service teachers in the Ball study, the similarities discussed above have not been presented to imply that the prospective teachers studied were not mathematically gifted. Rather than focusing on the causes of these deficiencies in metacognition and persistence, future studies should examine the prevalence of these mathematical behavioral characteristics in mathematics students of all ages and encourage appropriate modifications to mathematics programs and curricula.

Another study of focused entirely on the mathematics anxiety and efficacy of undergraduate elementary education students (Gresham, 2008). Data were gathered from 156 pre-service teachers in the form of an anxiety scale and an efficacy belief instrument and interviews about both topics that were analyzed using grounded theory. Quantitative results indicated that there existed a significant, negative relationship between the teachers' mathematics anxiety and their mathematics efficacy. Anxiety was found to be the basis for participants' mathematics teaching beliefs as it related to their efficacy concerning mathematics teaching practices. The efficacy demonstrated by some participants was overcome by their anxiety, and they ultimately doubted their ability to teach mathematics effectively.

Participant data indicated that negative mathematics experiences led to negative attitudes toward mathematics that led to higher levels of mathematics anxiety; conversely, positive experiences led to positive attitudes that led to lower anxiety levels. Interestingly, the degree of negativity of the high-anxiety participants can be determined by the prevalence of one word throughout all of their responses to questions about how they felt about mathematics: "hate" (Gresham, 2008, p. 177). Conversely, low-anxiety participant responses contained positive terms such as "love," "like," "easy," "fun," and even "hugged" (p. 178). Similar stratifications in responses were observed when participants were questioned about their mathematics understandings: "frustrated," "embarrassed," "worry," "struggle," and the negative connotation of "challenge" (pp. 180-181) characterized the high-anxiety participants' responses while "love," "supportive," "proud," and the positive connotation of "challenge" (p. 181) represented the low-anxiety responses. High-anxiety participants did express concern over possible student perception of their negative attitudes and how these perceptions would affect their ability to teach.

The researcher concluded a discussion of the results by focusing on a positive future for the mathematics classroom. He first cited previous research (Hoy, 2000) in which pre-service teachers' fears were built upon unrealistic expectations and attitudes regarding challenges they projected they would encounter in the mathematics classroom. Supporting this, the researcher re-cites his own findings that indicate the pre-service teachers studied, regardless of anxiety levels, all professed a belief in the importance of proven, effective mathematics teaching strategies: using manipulatives, connecting mathematics to students' "real-world" realities (p. 182), and providing motivation and support. When teachers utilize these practices, student mathematics anxiety decreases, and, although it is not explicitly stated in the research examined here, teacher mathematics anxiety should be expected to decrease in kind.

Teachers' mathematical knowledge influences efficacy and practice and, consequently, student achievement. A study of approximately 700 elementary teachers and their first and third grade students examined the effects of teacher knowledge—particularly, teacher knowledge of teaching mathematics—upon student achievement (Hill, Rowan, & Ball, 2005). Student assessments, parent interviews, and teachers' logs and questionnaires were used to show that student mathematics achievement gains were positively predicted by teacher knowledge of how to teach mathematics and that teacher content knowledge influenced the teaching of even the most minimal levels of mathematics. In their discussion of the results, the researchers focused on the inequitable distribution of intellectual resources related to teaching knowledge across both ethnic and socioeconomic groups and how this can negatively affect student achievement in disadvantaged schools.

A smaller study produced case studies of two fourth-grade teachers implementing a new mathematics textbook (Remillard, 2000). This study examined the nature of mathematics teaching and how mathematical knowledge can change and consequently influence practice. A year-long collection of data through interviews and classroom observations and analyses of the resulting field notes, artifacts, and interview transcripts afforded insight into how curricular materials and their respective professional development sessions can positively influence teacher learning. The researcher found that, similar to the necessary engagement of students in their own learning processes, teacher engagement in the curriculum learning process facilitated mathematics understanding and encouraged appropriate adjustments to classroom practice. Thus, although it is not explicitly stated in the study's discussion, teacher engagement in the learning process that leads to an improvement in practice has the potential to lead to increased student engagement and, possibly, understanding and achievement.

In an overall analysis of teacher practices that evolved from engagement with the new curricular material, the study (Remillard, 2000) delineated three types of "reading" that influenced teacher learning: active reading of the text through the lens of how it could facilitate development of student understanding; reading, or attending to and interpreting, student readiness and progress; and reading tasks, or critically analyzing and altering existing or inventing new learning activities. These “readings” were most effective in context—when teachers applied the understandings and ideas acquired from them to their own classroom pedagogical decisions and practice. The teachers in the study were able to do so successfully because their learning of the new curricular material involved engagement in active reading, analyzing and altering their mathematical knowledge, developing an understanding of the curriculum development industry itself, and their subsequent decision making based on all of this information. Ironically, this approach to acquainting teachers with a new textbook does not encourage a helpless reliance upon the text as the sole curriculum but rather provides teachers with the knowledge and insight to critically analyze materials, determine how they can be altered to best address students’ needs, and understand how they can be combined with other elements to augment the mathematics curriculum—curriculum that transcends the textbook.

The findings of this study can not only influence the way new curricular materials are introduced but also how professional development sessions are conceptualized and carried out. The key influential factor is engagement through presentation in context. Teachers need to see immediately how new curricular materials or particular concepts or methodologies directly relate to them, their students, and their classrooms and need to practice applying them accordingly.

Effective professional development practices were also examined from the perspective of teachers, as with the previous study, but on a much larger scale in a national study of approximately 1,000 mathematics and science teachers (Garet, Porter, Desimone, Birman, & Yoon, 2001). Survey data gathered were teacher self-reports of behavior and the professional development experiences that influenced or did not influence that behavior. Data were analyzed using multiple regression to determine how various professional development characteristics influenced teacher learning.

Similarly to the Remillard (2000) study, results demonstrated that teachers considered professional development opportunities to be most effective when they featured active learning of content knowledge and allowed for integration within the context of existing practices, both within the classroom and the school as a whole. Providing teachers prolonged, rigorous exposure to and active practice with content knowledge integrated within the contexts of their own students, classrooms, and schools makes professional development more effective. Learning through application has the potential to make the information more meaningful for teachers who will, in turn, more deeply internalize it, integrate it, and utilize it. The study is unique as it is one of the few studies to offer striking empirical support for prolonged, integrated professional development that allows for collaboration among similar groups of teachers and thus encourages professional discourse—all elements that can lead to improved understanding, awareness, and practice.

The researchers concluded their discussion with a call for professional development that follows these research-validated practices. They lament that many districts' professional development programs focus on quantity, or breadth, over quality, or depth, and suggest the re-allocation of priorities as well as funding to significantly improve professional development opportunities and their influence upon teachers.

As the previous study's results were derived from teacher self-reporting of their own professional development experiences and resulting behavior, it is important that similar results be confirmed with studies using different methodologies. A follow-up study used multilevel modeling to estimate the effects of particular elements of professional development on practice (Desimone, Porter, Garet, Yoon, & Birman, 2002). The researchers conducted three waves of a longitudinal survey over the course of 3 years in studying how professional development practices influenced teaching practices, particularly in mathematics and science. Many of the study's findings, in addition to providing a clear-cut path for the revision of professional development practices, also serve to reinforce or further define particular results of the Garet et al. (2001) study.

The Desimone et al. (2002), study reinforced the importance of teachers' active participation in their own learning during and after professional development, contrary to antiquated theories conceptualizing learners as "passive recipients of information" (p. 101). The study also extended the concept of collaboration and collective participation within professional development to the specific subject of technological skill development, finding that when teachers rely on colleagues to help refine their skills, it expedites the process. The study also emphasized the importance of professional discourse in the development of teachers' practice of holding intellectual, informed discussions concerning (and validating) their own roles as educators as well as how their own and their students' roles as thinkers and learners interact. Reminiscent of the call for a focus on depth of topic (quality) rather than breadth of topics (quantity) in professional development found in the Garet et al. (2001) study, the Desimone et al. study produced additional support for professional development that solely focuses on one teaching practice at a time. The study's findings demonstrated increased use of a deeply-learned practice in the classroom regardless of prior practice and transcendent of subject or level taught.

Mathematical Discourse

Just as it is important for teachers to actively expand their own mathematical understandings throughout their careers, they can adopt classroom techniques that promote active inquiry and conceptual development among their students. A reform that has garnered support from National Council of Teachers of Mathematics (NCTM; 2000) is the use of increased mathematical discourse in elementary mathematics classrooms. Discourse can involve communications between teachers and students or among students. In contrast to more traditional modes of mathematics instruction in which teachers lecture and students complete seatwork, these communications provide opportunities for children to develop higher-level understandings of mathematical concepts. Hiebert and Wearne (1993) summarized the theoretical argument for the ability of classroom discourse to influence learning: “As students express their beliefs and opinions with their classmates, defend them in the face of questions, and question others’ ideas, they are likely to recognize incongruities and elaborate, clarify, and reorganize their own thinking” (p. 396). The What Works curricular units sought to promote mathematical discourse through teachers’ use of “Talk Moves,” as well as requiring students to explain their mathematical reasoning in verbal and written form. Several prior research studies indicate the effectiveness of mathematical discourse in enhancing students’ conceptual understandings of mathematics.

Even before TIMSS (National Center for Education Statistics [NCES], 2005, 2009) and PISA (OECD, 2009) international assessments caused alarm at the inability of America’s classrooms to provide students with globally competitive mathematics proficiencies, researchers had begun examining how different forms of instructional strategies impacted students’ mathematical achievement. Hiebert and Wearne (1993) investigated the impact of student tasks and classroom discourse on second-grade students’ mathematical understandings. In a school with six second-grade classes (two comprised of average to high achieving students and four comprised of low to average achieving students), four classrooms (labeled classrooms A, B, C, and E) were assigned to the control condition in which teachers proceeded with their regular instructional practices. Two classrooms—one classroom of higher achieving students (labeled classrooms F) and one classroom of lower achieving students (labeled classroom D)—were assigned to the treatment condition. The treatment involved being provided with mathematics instruction by teachers who were hired by the researchers and trained to provide more conceptual tasks and more conceptual-level discourse in their classrooms. The topics of instruction for both treatment and control students were place value and the addition and subtraction of whole numbers.

The researchers conducted weekly classroom observations of all six classrooms for the 12-week duration of the intervention. They calculated the proportion of class time each class spent on mathematical discourse. In addition, the observation audiotapes were transcribed to enable the researchers to establish codes for different types of classroom questioning and discourse. The four broad categories of questions that emerged from the classroom observation were: “recall” questions, “describe strategy” questions, “generate problem” questions, and “examine underlying feature” questions (Hiebert & Wearner, 1993, p. 402.) The frequency of the usage of these types of questions in the six classrooms was calculated. Additionally, the researchers generated an assessment to measure students’ learning of place value and addition and subtraction of whole numbers. This assessment was administered to students in the six classrooms in September, December, and May to determine whether patterns of student learning were related to instructional features of the classrooms in which they were taught.

Students in the intervention classrooms (D and F) spent more time on average than their control classroom counterparts on group discussion and less time on individual seatwork. These classrooms were also found to spend a greater length of time discussing each problem during whole group discussion and to have students spending a greater length of time per problem when engaged in seatwork. Although the number of words spoken overall varied considerably by classroom, students in the intervention classrooms were observed as contributing more words to the classroom discussion per lesson than students in the control classrooms. In terms of the four broad categories of questions established, treatment classrooms pursued a greater proportion of “describe strategy” and “examine underlying features” questions than the control classrooms. Students in classroom D (the intervention classroom of low to average performing students) showed higher performance on all item types on the spring assessment than the students of similar initial achievement who attended the control classrooms (A, B, and C). The differences in final achievement between the two higher-initial achievement classrooms were less pronounced.

Even in mathematics classrooms in which teachers establish discourse-oriented social norms (such as explaining thinking, sharing strategies, and collaborating with peers), there remain differences in the application of these norms to broadened conceptual thinking in mathematics. Kazemi (1998) and colleagues distinguished between “high press” and “low press” (p. 410) classrooms in which teachers use questioning and discourse strategies to solidify and expand their students’ mathematical understandings. The researchers utilized the framework of “sociomathematical norms” (Cobb & Yackel, 1996, p. 459) as a starting point for their investigation of “press” for building deeper mathematical learning. The researchers identified two teachers (each of whom taught 2 sections of upper elementary mathematics), both of whom used mathematical discourse in their instruction, but who did so in different ways that were classified as “low press” in the one case and “high press” in the other case. They conducted in-depth observations of the four classes to elucidate how the discourse functioned in both sets of classrooms.

Kazemi (1998) described four main sociomathematical norms that were present in the “high press” classrooms:

> Explanation consisted of mathematical arguments, not simply procedural summaries of the steps taken to solve the problem; errors offered opportunities to reconceptualize a problem and explore contradictions and alternative strategies; mathematical thinking involved understanding relations among multiple strategies; and, collaborative work involved individual accountability and reaching consensus through mathematical argumentation. (p. 411)

The students in the classrooms exhibiting these norms demonstrated higher performance on assessments of conceptual thinking and problem solving in mathematics. These findings indicate that classroom discourse is one necessary component to enhance students’ mathematical understanding, and teachers’ skillful use of sociomathematical norms determine whether discourse will be translated into genuine depth of learning by students.

As was previously mentioned, the educational system in general and gifted education programs in particular have wrestled with providing equitable resources for all students while simultaneously promoting excellence. Access to the types of mathematical discourse that nurture conceptual mathematical learning must be provided to the increasingly diverse student population attending today's classrooms. Due to historical and current mathematics achievement gaps between White or Asian students and African American or Hispanic students, research should endeavor to understand what instructional features are most effective for teaching mathematics to these students. Some critics (Oakes, Ormseth, Bell, & Camp, 1990) have argued that many classrooms with African American and Hispanic students rely too heavily on teacher-centered pedagogy and the acquisition of basic skills, while failing to engage students in the types of problem solving and conceptual understanding necessary for success in the current knowledge economy. An exploratory qualitative study of two ethnically diverse mathematics classrooms at an urban magnet school (White, 2003) followed two teachers who were trained to offer constructivist methods of instruction with particular focus on high-level questioning and classroom discourse.

Classroom observations revealed a number of successful techniques employed in the two classrooms to reach students of many different ethnic backgrounds. The teachers demonstrated their valuing of all students' ideas by listening to their reasoning without passing immediate judgment about the mathematical correctness of their thinking. They also established classroom climates in which students felt comfortable discussing and even disagreeing with one another. Questioning focused on students' process strategies for arriving at a solution rather than on the solution itself. To use students' reasoning to push for greater understanding, the teachers offered students the opportunity to correct their own mistakes by viewing other students' strategies. In addition, they helped facilitate the abstraction of concrete mathematical situations by asking students to translate problems solved with manipulatives into symbolic representations.

The two teachers incorporated students' prior informal mathematical knowledge into novel problem solutions. In one classroom, a student used prior knowledge drawn from playing cards to solve a problem that involved quantities of 26 (half a deck of cards) and 55 (three more than a deck of cards). The teachers encouraged multiple and unusual ways of arriving at an answer rather than insisting on one particular algorithm, as might be typically found in more traditional behaviorist-focused instruction. Finally, the teachers' function as facilitators rather than knowledge dispensers created classroom environments in which students were expected to generate knowledge, evaluate the ideas of others, use mathematical argumentation to support their ideas, and ultimately arrive at a classroom consensus on the relative worth of different ideas. This type of environment enabled students of all ethnicities to take ownership in the process of knowledge creation, rather than assuming the role of passive recipient. Although this qualitative study was unable to link classroom practices with student performance data, it provided rich examples of what high-quality mathematical instruction might look in elementary classrooms with ethnically diverse students.

In their recent review of the literature on the pedagogical use of discourse in elementary and secondary mathematics classrooms, Walshaw and Anthony (2008) induced four broad themes that influenced the effectiveness of mathematical discourse on student outcomes. The authors used Engeström's conceptual framework to explain how discourse functions in the context of mathematics classrooms. First, they determined that classrooms must establish clear participating rights and obligations for students to benefit from mathematical discourse. In other words, more talk in mathematics classrooms is not necessarily beneficial to students without pre-existing structures and expectations. Second, the authors induced that successful discourse involves teachers supporting and extending students' reasoning, not merely evoking students' ideas in a non-evaluative way. This requires teachers to listen carefully to students' incoming knowledge, conceptions, and arguments to advance reasoning toward mathematically sound solutions. Third, the review of literature suggested that "fine-tuning" (p. 532) students' mathematical thinking through language was an effective pedagogical strategy. Teachers who use mathematical language with which students can accurately and efficiently describe their reasoning processes enhance students' ability to communicate with precision about mathematical ideas. Finally, teachers can affect positive outcomes by helping shape the mathematical argumentation in which their students engage.

A rich literature is amassing regarding the impact of classroom discourse on students' conceptual mathematical understandings. As discourse continues to reform elementary mathematics classrooms, researchers must recruit larger samples of students and propose generalizable research designs to determine if what has been documented in the microcosm of individual classrooms will also be effective at a larger scale.

CHAPTER 3: Methods and Procedures

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Purpose of the Study

The overarching purpose of the study was to determine the impact of pre-differentiated and enriched curricula on heterogeneously grouped grade 3 students and their teachers. To understand how such curricula unfold in practice, three cohorts of teachers over 2 years received professional development and implemented three mathematics units with their grade 3 students. Impacts on students' mathematical learning on proximal (researcher-developed) tests and distal (national standardized) tests were analyzed. Additionally, to explore and describe participating teachers' and administrators' responses to the curriculum intervention, inductive qualitative methods were used to reveal recurrent themes as well as atypical but salient responses.

Recruitment and Selection

The National Research Center on the Gifted and Talented (NRC/GT) research team used multiple recruitment efforts to develop a list of potential schools interested in participating in the mathematics research study. The research team shared information about the What Works in Gifted Education Mathematics Study through contacts with State Department of Education consultants; lists of gifted and talented coordinators from Market Data Retrieval (professional company that maintains K-12 databases); presentations at conferences; direct mailings of recruitment flyers; announcements on the NRC/GT website; creation of a DVD highlighting the study's protocol and videos of teachers demonstrating lessons; and email and telephone contacts with participants involved in Neag School of Education's professional development conferences and institutes.

Grade 2 Responsibilities

- Administrators must submit a School Profile form and a letter of intent to be considered for participation in the study. The School Profile form includes questions regarding instruments used to identify gifted students, smallest and largest number of grade 2 teachers per school; and existence of district Institutional Review Board.
- Administrators and teachers complete the approved university Institutional Review Board forms to participate in the research study.
- Parents review the approved Information Sheet outlining the study's requirements and determine if their children will participate. Non-

participating students will not be involved in norm-referenced achievement and ability assessments.

- Students from interested schools will be tested in the spring with the Iowa Tests of Basic Skills (ITBS) and the Cognitive Abilities Test (CogAT). Extant data from norm-referenced achievement and ability tests may be considered for substitution of these specific achievement and ability measures to avoid additional testing.
- Teachers will be asked to complete learning, motivation, creativity, and mathematics rating scales on 5-7 classroom students with potential academic talent.

Grade 3 Responsibilities

- Treatment teachers will be asked to implement units in algebra, geometry and measurement, and graphing and data analysis based on the National Council of Teachers of Mathematics Standards and Focal Points. The units require 16 weeks to implement in general education classrooms. The content of each unit is differentiated based on gifted and talented education models developed by Carol A. Tomlinson, Sandra N. Kaplan, and Joseph S. Renzulli and Sally M. Reis. All supplies and manipulatives are provided.
- Treatment teachers will participate in 2 days of professional development as the research team shares the study's protocol; demonstrates the implementation of the units; explains the scoring of the pretest and posttest unit tests; highlights the DVDs accompanying each unit; previews the professional development digital libraries that include PowerPoint presentations from the onsite training, teachers' logs, extension lessons, and relevant websites annotated for teachers and students; and shares parent resources related to the mathematics units.
- Treatment group teachers will be asked to complete teachers' logs at the completion of each unit to provide information to the research team about the implementation of the curriculum.
- Treatment and control group teachers must agree to classroom observations.
- Administrators and treatment group teachers must agree to participate in interviews or focus groups.
- Treatment and control group teachers must administer the ITBS—Math Problem Solving and Data Interpretation subtest in the spring of the implementation year.
- Treatment group teachers must administer criterion-referenced unit tests before and after implementing each curricular unit.
- Treatment group teachers must administer a subset of algebra, geometry and measurement, and graphing and data analysis items from the grade 4 released items from the National Assessment of Educational Progress.
- Control group teachers will receive DVDs of the mathematics curriculum, professional development digital libraries, videos, professional development presentations, and parent resources.

A total of 63 schools submitted the required documentation (School Profiles, Letters of Intent, Administrator and Teacher Consent Forms), and they were deemed eligible for study participation. Prior to assignment of teachers to the treatment or control condition, several schools decided not to participate. The central office administrator in one district decided not to include 13 schools in the study due to their local focus on improving state test scores; administrators from three other schools in different districts were new to the school and chose not to participate; an additional group of three administrators from different districts were also concerned about state or local test scores. One school decided not to continue with the implementation of the study after the first professional development and partial completion of one unit as local monthly mathematics' assessments indicated treatment group students' scores were not competitive with control group students' scores.

Teachers from 43 schools comprised the candidates for assignment to treatment or control conditions. Treatment teachers were required to implement three curricular units in algebra, geometry and measurement, and graphing and data analysis, which would supplant the district's adopted mathematics curricula. Control group teachers continued with the district's adopted mathematics curricula or "business as usual." All teachers' names from the 43 schools were written on slips of paper and designation for the treatment or control condition was determined by flipping a coin for each teacher's name within each school. Of the 141 teachers, 84 were assigned to the treatment condition; 57 assigned to the control condition. In two instances of co-teaching, the co-teachers were assigned to condition as a single unit.

Research Questions

A series of quantitative and qualitative research questions guided the implementation of the What Works in Gifted Education Mathematics Study. The questions required the collection of a substantial amount of data to document the intervention from the perspectives of student achievement and reasoning; administrators' and teachers' reactions to the impact and effectiveness of the intervention; teachers' assessments of students' results on criterion-referenced unit tests and items from the National Assessment of Educational Progress; and teachers' evaluation of a subset of students' learning, motivation, creativity, and mathematics characteristics using the Scales for Rating the Behavioral Characteristics of Superior Students (Renzulli et al., 2004). The quantitative and qualitative questions follow.

Cohort I and Cohort II Research Questions

The following questions guided the research study with Cohorts I and II.

1. What is the impact of implementing differentiated mathematics curricula in algebra, geometry and measurement, and graphing and data analysis on the achievement of grade 3 students?

a. Are there differences in posttest mathematics achievement, as measured by the Iowa Tests of Basic Skills (ITBS) Math Problem Solving and Data Interpretation subtest or the Stanford Achievement Test mathematics subtest, of students receiving the curriculum intervention and comparison students, after accounting for pretest levels of achievement and status as gifted relative to their peers?
b. Do student level variables such as quantitative cognitive ability, gender, and status as under-represented predict achievement on the unit pretests and posttests?
c. How much do treatment students improve their content-specific achievement from pretest to posttest on the researcher-developed unit tests?
d. How do grade 3 treatment students compare with a nationally representative grade 4 sample on a selection of mathematics items from the National Assessment of Educational Progress?

2. How do treatment teachers and administrators respond to their access to pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis?

For Cohort III, the following research questions and hypothesis were addressed:

Research Question 1
What is the impact of creating pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis on the achievement of grade 3 students, after controlling for pretest achievement scores?

Hypothesis 1
Treatment group students involved in pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis will outperform control group students on the Iowa Tests of Basic Skills—Math Problem Solving and Data Interpretation subtest, after controlling for pretest achievement scores.

Research Question 2
Are there practically meaningful gains between the pretest and posttest criterion-referenced unit test scores in algebra, geometry and measurement, and graphing and data analysis for the treatment group students?

Hypothesis 2
There will be large gains in the treatment group students' performance criterion-referenced unit tests between pretest and posttest.

Research Question 3
Will at least 50% of the grade 3 treatment group students involved in the mathematics curriculum master the content, concepts, and skills typically addressed by grade 4 students?

Hypothesis 3
At least 50% of the treatment group students will master each item in algebra, geometry and measurement, and graphing and data analysis from the grade 4 National Assessment of Educational Progress.

Description of the What Works Mathematics Curricula

Description of Units

Mathematics units developed for Project M^3: Mentoring Mathematical Minds (Gavin et al., 2009) served as guides for developing curricula, as they emphasized best practices in mathematics as well as gifted and talented education. The content of the units was accelerated for mathematically promising elementary students, which allowed students to investigate mathematics concepts in depth. Given that the current mathematics research study focused on all grade 3 students, a working edition of the algebra unit from Project M^3 that included exemplars of differentiated lessons was further developed for the What Works in Gifted Education Mathematics Study. The overall philosophy of Project M^3 and many of the elements of lesson design reflected the principles of teaching and learning espoused by Tomlinson, Kaplan, and Reis and Renzulli.

The research curricula for this study included three University of Connecticut What Works Mathematics Curricula carefully designed as a 16-week intervention to make differentiation accessible for grade 3 teachers. The in-depth curricula included three separate teacher manuals and Student Mathematician Journals for the following units: (a) *Awesome Algebra* (Gavin et al., 2008a, 2008b; Gavin et al., 2009a, 2009b); (b) *Geometry & Measurement for All Shapes & Sizes* (Cole, Heilbronner, et al., 2009a, 2009b; Gubbins, Cole, Heilbronner, Corbishley, & Savino, 2008a, 2008b); and (c) *Greening Up With Graphing: Recycle, Reduce, & Reuse* (Cole, Rubenstein, et al., 2009a, 2009b; Corbishley et al., 2008a, 2008b). Conceptual understanding, real-world investigation, students as practicing mathematicians, and development of mathematical language and communication through discourse and writing were emphasized in the design of the units. A brief overview of each unit is provided below.

The *Awesome Algebra* (Gavin et al., 2008a, 2009a) Unit encourages students to study patterns and determine how they change, extend or repeat, and grow. Students then move beyond this to organize and analyze information systematically to develop generalizations about the patterns. Students are also encouraged to use variables to construct rules for predicting patterns. In this unit, arithmetic is viewed from an algebraic perspective and students are challenged to look at basic number concepts in new ways.

The *Geometry & Measurement for All Shapes & Sizes* (Cole, Heilbronner, et al., 2009a; Gubbins et al., 2008a) Unit integrates measurement concepts within the study of geometry. Students discover the relationships between geometric figures through hands-on explorations and discussions. They think like practicing mathematics professionals by inventing methods to render perfect circles or grapple with the differences between terms such as “estimated” and “exact” as related to measurement of geometric shapes. This unit also promotes an interdisciplinary approach to learning. For example, students might take the stage as actors and read and write stories and poems about geometry.

In the *Greening Up With Graphing: Recycle, Reduce, & Reuse* (Cole, Rubenstein, et al., 2009a; Corbishley et al., 2008a) Unit, students formulate questions and collect, organize, and display relevant data. Students also learn to utilize appropriate analytical methods to derive meaning from their investigations, making inferences and predictions based on their results. In addition to oral and written communication of ideas, students learn to represent ideas through pictures, tables, graphs, and other displays.

The beginning of each What Works Mathematics Unit includes an introduction, alignment of the unit lessons to the NCTM (2000) and Focal Points (2006) curricular standards, a weekly pacing chart, a description of the unit and lesson format, and biographies for each of the famous mathematicians for the students’ groups. As the units were intended to serve as educative professional development, the rationale for each unit, information about how the units support differentiated instruction, and how to use each component of the units was clearly explicated. For example, in the *Awesome Algebra* (Gavin et al., 2009a) unit, a rationale for mathematical discussion as an important instructional practice is provided along with a detailed description of particular strategies to support classroom discourse such as “talk moves” (Chapin, O’Connor, & Anderson, 2003).

Additionally, as the University of Connecticut What Works Mathematics Curricula were based on models of differentiated instruction (Kaplan, 1998; Renzulli & Reis, 1985, 1997; Tomlinson, 1996), grouping students by instructional readiness as determined by performance on preassessments was a key component that was integrated within each of the units. All three of the What Works Mathematics Curricula included preassessments in the form of unit tests accompanied by answer keys, and rubrics.

The mathematics units were designed in such a way that teachers could use information from the pretests to assign students to work on tiered assignments that are provided in the mathematics units. The students’ groups were given names of famous mathematicians, along with their brief biographies to share their accomplishments at various stages of their lives and to promote mathematics as created knowledge emerging from theoretical perspectives:

- *Awesome Algebra:* Leonardo Fibonacci, Diophantus of Alexandria, and Sonya Kovalevsky
- *Geometry & Measurement for All Shapes & Sizes:* Pythagoras, Hypatia, and Euclid
- *Greening Up With Graphing: Recycle, Reduce, & Reuse:* Charles Babbage, Etta Falconer, and Galileo Galilei.

Of the 16 lessons in the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit (Cole, Rubenstein, et al., 2009a; Corbishley et al., 2008a), seven lessons incorporate tiered assignments or student pages. In the *Awesome Algebra* Unit (Gavin et al., 2008a, 2009a), of the eight lessons, seven had tiered assignments or student pages. In the *Geometry & Measurement for All Shapes & Sizes* Unit (Cole, Heilbronner, et al., 2009a; Gubbins et al., 2008a), 11 lessons of the 17 provide tiered assignments or student pages. To support teachers in utilizing the pretest data for these tiered assignments, tables are provided in the lessons that specify certain items from the unit tests, how many points students must score on the items to be assigned to a particular tiered assignment, and the corresponding pages in the Student Mathematician Journal.

In addition to using the unit tests as a preassessment for matching students to the tiered assignments, individual lessons provide detailed suggestions on how to use initiating activities to inform instructional decision-making. This may include instructions on how to monitor and assess student performance, criteria for matching students to tiered assignments, and specific student pages to assign based on the criteria.

To allow for easy implementation, each lesson across all three What Works Mathematics Curricula is formatted similarly. In the planning phase, the lesson opens with a description or list of the following:

- Big Mathematical Ideas
- Lesson Objectives
- Materials
- Mathematical Language
- Lesson Preview.

Big Mathematical Ideas provide a brief overview connecting lesson objectives to key concepts or big ideas to be woven into the lesson, along with a rationale for the lesson. In the second part of the lesson, the teaching phase, the following format supports the organization and flow of the lesson:

- **Initiate**—This is an introductory learning experience or activity designed to engage students.
- **Investigate**—These are activities for students to explore the lesson concepts through hands-on, real-world investigations.
- **Conclude**—Students communicate what they have learned in a variety of ways, which allows them to solidify new knowledge from the lesson.
- **Look Ahead**—This is a preview for the purpose of the next lesson.
- **Assess**—Teachers are provided with suggestions on how to formally and informally assess student progress. Assessment options include student pages from the Student Mathematician Journal, checkpoints, group discussions, or direct inquiry.

Following the planning phase and teaching phase for each lesson in the teacher manual are the student pages from the Student Mathematician Journals along with the answer keys for all closed-ended questions and sample student responses for any open-ended questions. Recognizing that some students may demonstrate a need for further challenge, several lessons include Super Challenges. These are challenging activities that present the lesson objectives in a more in-depth or complex manner for students.

The What Works Mathematics Curricula were piloted with teachers and their students who provided feedback on the content and organization of the lessons. For 2 years, treatment teachers from 17 states experimented with units and suggested further modifications and additions. The third edition of the units was pressed on DVDs and made available to all treatment and control teachers and their administrators. The knowledge and experiences of multiple research teams, grade 3 teachers, and administrators were invaluable for creating high quality, differentiated, and enriched curricula.

Length of the Intervention

The algebra, geometry and measurement, and graphing and data analysis units were designed using a lesson plan format: Big Mathematical Ideas, Lesson Objectives, Materials, Mathematical Language, Initiate, Investigate, Conclude, and Assess. The estimated time for each lesson (60 minutes) was based on piloting the units with local teachers who were not study participants. Each unit included a pacing chart for a Monday to Friday schedule of activities. Teachers needed to commit to the intervention for 16 weeks. Algebra lessons were provided for 5 weeks, geometry and measurement for 4 weeks, and graphing and data analysis for 5 weeks. One week was for flex time and one week was available for assessments. An example of the pacing chart for geometry and measurement is illustrated in Figure 3.1.

Treatment Group: Pre-differentiated and Enriched Curricula

Treatment classroom teachers received 2 days of professional development that included a summary of the goals of the research study and the development of the curricular units. This part of the session presented three models originally developed for gifted and talented programming that had informed the development of the units. The majority of the first session was spent facilitating treatment teachers' understanding of how to implement the curricular units with high fidelity. Teachers completed and practiced scoring unit pretests to develop familiarity with the scoring rubric. They participated in sample activities from each of the three units, including using the many manipulatives provided. Members of the research team provided instruction for teachers to complete and submit student unit scores as well as teachers' logs with their own reactions to the intervention. Finally, the research team helped teachers determine how to fit the research units into their own district mathematics curriculum to alleviate concern that standards required for statewide tests would be neglected during the implementation.

The second day of professional development occurred after the completion of one mathematics unit. Treatment teachers had multiple opportunities to become familiar with the study's research protocol, view model lessons, engage in application of concepts through use of provided manipulatives, become familiar with the "big mathematical ideas," experience the scoring of unit tests using rubrics, and learn about the resources available from the professional development libraries. Copies of all professional development presentations with scripted notes and resources were available on DVDs and CDs to ensure that the research team replicated the professional development consistently across research sites.

	Monday	Tuesday	Wednesday	Thursday	Friday
Week 1 Time to Shape Up! (Lessons 1-5)	**Lesson 1:** Classifying Two Dimensional Shapes **Tiered by Activity** -Pythagoras' Shapes -Hypatia's Shapes	**Lesson 2:** What's the Right Angle? **Tiered Student Pages** -Pythagoras -Euclid	**Lesson 3:** *The Greedy Triangle (Marilyn Burns' book The Greedy Triangle provided.)* **Interdisciplinary** Students use creative abilities to write quadrilateral stories.	**Lesson 4:** The Rectangles Only Club! **Tiered Student Pages** -Euclid -Hypatia	**Lesson 5:** Flipping, Turning, & Sliding **Extensions** Additional prompts and Student Pages to increase the level of challenge
Week 2 Time to Shape Up! (Lessons 6-10)	**Lesson 6:** Your Height Can Change Your Life **Tiered Student Pages** -Pythagoras -Hypatia	**Lesson 7:** Speaking Fractional Language **Students Grouped by Readiness**	**Lesson 8:** A World Without Congruence **Non-Differentiated Lesson**	**Lesson 9:** Same Size, Same Shape…What Does it Mean? **Tiered by Activity** -ACUTE (Pythagoras) -OBTUSE (Hypatia)	**Lesson 10:** Going in Circles (optional lesson) **Scaffolded Version** Extension questions to challenge students who finish early and a scaffolded task page
Week 3 Time to Shape Up! (Lesson 11) Living on the Edge (Lessons 12-15)	**Lesson 11:** Infinite Lines of Symmetry **Tiered by Activity** Pythagoras' Shapes Hypatia & Euclid Shapes	**Lesson 12:** The Ants Go Marching **Tiered Student Pages** -Hypatia -Euclid	**Lesson 13:** Ruler of the Ruler **Non-Differentiated Lesson**	**Lesson 14:** ?-Inch Ruler **Tiered by Activity** Measurement lengths determined by mathematician name	**Lesson 15:** Same Perimeter, Different Shape **Non-Differentiated Lesson**
Week 4 Living on the Edge (Lessons 16-17)	**Lesson 16:** A Fair Way to Shade **Tiered by Activity** Pythagoras, Hypatia & Euclid activities	**Lesson 17:** Square Units in an Un-square World **Non-Differentiated Lesson**	**Unit Project (Option 1):** A Geometry Scavenger Hunt **Unit Project (Option 2):** A Shapely Living Room	**Unit Project (Option 1):** A Geometry Scavenger Hunt **Unit Project (Option 2):** A Shapely Living Room	**Unit Project (Option 1):** A Geometry Scavenger Hunt **Unit Project (Option 2):** A Shapely Living Room

Figure 3.1. Geometry and measurement pacing chart.

Control Group: "Business as Usual"

Control group classrooms provided student pretest and posttest scores, but followed their districts' traditional mathematics curriculum over the course of the school year. At the completion of the curriculum intervention, control classroom teachers received DVDs containing the information presented in the treatment teachers' professional development session.

Qualitative Methods

To gain an in-depth understanding of the effect the research curricula had on treatment teachers' perceptions it was critical to employ multiple qualitative methods to document the intervention onsite or through forms with rating scales and guided questions. Obtaining such data was best served through a basic interpretive qualitative design (Merriam, 2002) using multiple data sources to ensure reliable findings. These data sources included classroom observations (Control Teacher Observation [CTO], Treatment Teacher Observation [TTO]), teachers' focus groups (FG, treatment), teachers' logs (TL, treatment), and administrators' interviews (AI, control and treatment). Observations were conducted using a researcher-designed observation form; in addition, the lessons were scripted to allow for an inductive analysis (Merriam, 2009).

The rigor of a qualitative data collection and analysis can be evaluated through three criteria: credibility, transferability (extrapolation), and dependability. Patton (2002) stated that credibility depends on rigorous methods, which consists of ensuring the collection of "high-quality data that are systemically analyzed" (p. 552). The data for the What Works Mathematics Study fit with this definition as they were collected from multiple sources, during multiple time periods, using multiple methods. These types of data could easily be triangulated across sources, methods, and time. Additionally, in the analysis stage, credibility was enhanced by having multiple coders and theme developers. All qualitative data were analyzed using Merriam's (2009) general analysis procedure, beginning with an inductive analysis, which allowed for all the team members' voices to be heard as to how they interpreted the data, as they were instructed to inductively create open codes. These open codes were generated to be all-inclusive, and then, the open codes were analyzed and condensed into analytical codes. Then the team moved towards a combination of inductive and deductive methods, which continued to allow freedom for members to adjust the pre-developed code when they saw fit or include any additional codes. The codes were created and agreed upon by five team members to ensure accurate data interpretation. Eventually, the data were analyzed using primarily a deductive approach, and new data sources did not require adaptations in the coding scheme, indicating that the codes accurately encompassed the data, and the team used a primarily deductive approach. Linkages between codes across data sources were analyzed to determine the major themes of the study. Team members conferred to check interpretations of the data after initial themes were identified. Overall, this careful attention to the data collection and analysis demonstrates the credibility of this study.

In addition to credibility, qualitative researchers are concerned with transferability, or as Patton (2002) prefers to call it, extrapolation, which encourages qualitative researchers to think beyond the current context and how the data may be applied in other settings. These data were collected from educators living in various states with various degrees of funding and rigor of their state standards. The fact that the sample of schools in the study were from different geographic locations and represented different levels of socio-economic status, yet a single coding scheme was able to capture the meaning of the data speaks to the potential of extrapolating the findings to other situations. However, these schools may have several characteristics in common that may be necessary for these results. For example, the teachers were allowed to participate in the study, indicating administrator buy-in to the program.

Even with credibility and transferability, a qualitative study needs to demonstrate dependability. Some of the same methods used to demonstrate credibility also apply to the demonstration of dependability such as multiple sources, multiple coders, and multiple methods of data collection across multiple points in time. In addition, however, dependability also can be demonstrated through an audit trail (Merriam, 1998), which is a recording of how the data were collected and analyzed, and how decisions were made throughout the process. The team created an audit trail through the use of research notes taken by the research team members and through the collective use of QSR International NVivo-9 software (2010), a qualitative data management program. The team utilized NVivo to revisit codes and save different versions of the coding process.

Quantitative Methods

The key purpose of the study was to determine if there was a difference between the mathematical achievement of students involved in 16 weeks of differentiated mathematics curricula in algebra, geometry and measurement, and graphing and data analysis and the achievement of students involved in the district's general education mathematics curriculum or "business as usual."

Cohort I participants were cluster randomized by school to examine the impact of the pre-differentiated and enriched curricula on student achievement. Twenty-one schools were randomly assigned to treatment or control, of which twenty submitted posttest achievement scores. A total of 970 Cohort I students had sufficient data to be included in the analytic sample. The same research design—cluster random assignment by school—enabled investigation of the research questions for Cohort II participants, which included 17 schools with a total of 45 teachers and 846 students. Cohort III's research design was a multisite cluster randomized control trial that assigned 141 grade 3 general education classroom teachers from 12 states and 43 schools across the country to treatment or control conditions.

Treatment group students completed selected items from the National Assessment of Educational Progress that reflected the same content as each of the units in algebra, geometry and measurement, and graphing and data analysis. Descriptive statistics were used to analyze the data. For the criterion-referenced unit tests, paired *t* tests were used to analyze the pretest and posttest data from the treatment group students. Cohen's *d* estimates of effect sizes utilized the pooled standard deviation as the standardizer, rather than using the standard deviation of the difference score to provide a more conservative and more comparable estimate of the effect.

Description of Instruments

Several instruments were adopted or created to assess the efficacy of using pre-differentiated and enriched curricula with grade 3 students in general education classrooms. Adopted instruments included norm-referenced achievement and reasoning tests, while the unit tests, administrators' interview questions, teacher focus group questions, and classroom observation forms were created by the NRC/GT research team. An overview of each instrument follows.

Reasoning Ability

Reasoning ability was measured for most student participants with the Cognitive Abilities Test (CogAT; Lohman & Hagen, 2001); however, three Cohort I schools and one Cohort III school administered the Otis-Lennon School Ability Test (OLSAT) for this purpose. The CogAT assesses students' developed abilities in reasoning and problem solving using verbal, quantitative, and nonverbal symbols. The 6th edition of CogAT was co-normed with the ITBS. Verbal, quantitative, and nonverbal reasoning scores are available for two subtests for grades K-2, and three subtests for grades 3-12. Primary grade (K-2) items do not require any reading and are paced by the teacher. Tests for grades 3-12 require some reading and the total testing time is 90 minutes for the 3 subtests, plus 15 minutes for distributing and collecting forms. The test was standardized on approximately 150,000 students in grades K-8 and 31,000 students in grades 9-12 from all 50 states. The sample of students represented various types of communities, ethnicity, race, and socioeconomic status. Internal consistency estimates ranged between .85 and .98 on the composite score. The test was subjected to content validity and construct validity and it correlates with achievement and other measures of ability. Factor analytic and bias studies were implemented. Criterion validity is .54-.87 (Lohman & Hagen, 2002).

The Otis-Lennon School Ability Test (OLSAT) is another widely used instrument to assess reasoning abilities correlated with academic achievement. The test is designed to measure verbal, quantitative, and figural reasoning skills. The developers (Otis & Lennon, 2003) state that "to learn new things, students must be able to perceive accurately, to recognize and recall what has been perceived, and to apply generalizations to new and different contexts" (p. 5). The OLSAT includes seven levels for students in grades K-12. The test was standardized using stratified random sampling of 463,000 students. Participants involved in this process also completed the Stanford Achievement Test, Ninth Edition. Reliability coefficients for the verbal, nonverbal, and total scores for each grade level ranged between .80 and .90.

Mathematics Achievement

Mathematics achievement was assessed by one of three well-known, research-based tests. Each of the tests is described briefly.

The Stanford Achievement Test is a norm-referenced and standards-based test, as it reflects the most recent content area standards proposed by various professional organizations as well as state academic standards. The test addresses multiple content areas in reading, mathematics, language, spelling, listening, science, and social science. Thirteen test levels are available for students in grades K-12. Fall norms are based on 110,000 students and spring norms are based on 250,000 in the standardization sample. The stratified cluster sample design for participating school districts was based on demographic and ethnic characteristics. The internal consistency reliability coefficients ranged from .80 to .90. Convergent reliability was determined by correlations between the Stanford 9 and Stanford 10, resulting correlations were in the .70-.80 range. There were also reasonable correlations with the Otis-Lennon School Ability Test.

The TerraNova is a norm-referenced test that reflects the concepts and skills taught in schools across the country. The test is available for grades K-12 in reading, language, mathematics, science, and social studies. The standardization sample was based on a nationally representative stratified random sample of over 3,000,000 students from different geographic regions of the county. Reliability coefficients ranged between .80 and .90. Criterion related validity was conducted with Stanford Achievement test, among others.

The ITBS provide a comprehensive assessment of student progress in the basic skills. They consist of a Complete Battery (reading, language arts, mathematics, social studies, and science), a Core Battery (reading, language, and mathematics), and a Survey Battery (shortened version of Core Battery).

Test content is aligned with the most current content standards, curriculum frameworks, and instructional materials. The test was standardized on a national sample of students K-9, with approximately 3,000 students per level per form completing the tests. Internal consistency estimates using Kuder-Richardson 20 (KR 20) varied between .79 and .98 (Hoover et al, 2003). Students in the standardization sample represented various types of communities, ethnicity, race, and socioeconomic status. The standardization sample included public, parochial, and non-parochial schools. Schools in the standardization were further stratified by socioeconomic status. Data from these sources were used to develop special norms for a variety of groups (e.g., race/ethnicity, public school).

Mathematics Pretest

The ITBS Level 8 Math Problems subtest was administered prior to the curricular intervention to obtain information on students' achievement in mathematics. The Level 8 ITBS subtest had 30 items. A small proportion of students completed other mathematics achievement pretests (the TerraNova, the Measure of Academic Progress [MAP], or the Stanford Achievement Test [SAT]). Because the achievement tests were on different scales, *z* scores for the scores on each of the four achievement tests were calculated so that students' pretest achievement could be compared across tests.

Mathematics Posttest

During the first year of the study, students in 16 schools took the Level 9 Math Problem Solving and Data Interpretation subtest of the ITBS as a measure of post-intervention achievement; four additional schools measured post-intervention achievement using the mathematics subtest of the Stanford Achievement tests. These scores were converted to z-scores for the relevant analyses. During the second year of the study, all Cohort II and Cohort III students in the sample analyzed in the current study took the 25-item Level 9 Math Problem Solving and Data Interpretation subtest of the ITBS. For Cohort II, the mean posttest scale score across schools was 200.8 (*SD*=21.5) for treatment students and 202.8 (*SD*=22.7) for control students. For Cohort III, treatment students' mean ITBS posttest score was 206.5 (*SD*=23.0) and control students' mean ITBS posttest score was 205.8 (*SD*=22.0). A substantial test ceiling was present for both treatment and control students in all cohorts on this measure, indicating that higher achieving students' actual achievement was not adequately measured. Given the ceiling effect, the analyses included both traditional regression and Tobit regression techniques, which could be compared. The Tobit model should provide less biased parameter estimates for regression models with censored data.

Criterion-referenced Unit Tests

The NRC/GT developed pretest and posttest criterion-referenced unit tests in algebra, geometry and measurement, and graphing and data analysis. The content emphasized on each unit test was chosen carefully to determine potential changes over time as the treatment group students practiced and applied the content, concepts, and skills. Closed-end and expanded response items and sub-items were scored using rubrics created by the research teams. The pretest items also foreshadowed the types of questions presented in the units.

National Assessment of Educational Progress Released Items

The National Assessment of Education Progress (NAEP) releases items used to assess specific academic progress of our Nation's students in several subject areas in grades 4, 8, and 12. Selected items from the grade 4 tests in algebra, geometry and measurement, and graphing and data analysis required advanced application of content, concepts, and skills typically designed for grade 4 students. The NAEP released test items were administered to treatment students only as a post-only assessment.

Teacher Focus Group Questions

Treatment teachers were presented with nine questions during the teachers' focus groups. Questions were posed prior to the completion of the intervention and when teachers completed at least two of the three curricular units. The focus groups were opportunities for study participants to reflect on their own teaching and learning practices and how the curricular units may have influenced their pedagogical content knowledge. Questions focused on topics such as the extent to which the math curriculum helped teachers accommodate diverse learners; the comparison between the approach to mathematics espoused in the What Works Mathematics Curricula and the school's textbook; and the extent to which the differentiated lessons assisted teachers with teaching the content (Appendix A).

The research team conducted the focus groups onsite by or across schools. Responses to questions were part of the field notes; there was no attempt to link individual's responses to a specific person within or across schools. Therefore, the qualitative analyses document the school when one school was present or list the focus group by a number when multiple schools were in attendance.

Administrators' Interviews

Administrators involved with the What Works in Gifted Education Study held roles as principals, assistant principals, gifted and talented coordinators, and curriculum coordinators. They responded to five questions focusing on teachers' reactions to the mathematics curricula, benefits to students, transfer of strategies or skills to other subject areas, support of other school initiatives, and involvement in research studies (Appendix A). Responses were recorded through onsite interviews, phone interviews, or email transcripts.

Teachers' Logs

Algebra, geometry and measurement, and graphing and data analysis teachers' logs allowed the researchers to gain information on the teachers' perspectives on the quality of the units, the impact on their teaching/learning, and the extent to which students were mastering the mathematical concepts. For the 9 to 10 Likert-type items on each teachers' log, the response scale varied from strongly disagree (1) to strongly agree (5). Items included, "This unit addressed my students' varied learning styles" and "I enjoyed teaching this unit" (Appendix A).

Internal consistencies for the closed-ended teachers' log responses were .85 for the algebra unit, .88 for the geometry unit, and .86 for the graphing unit. Open-ended questions included "Describe a specific student's reaction to the unit" and "Describe your experience with the math curriculum."

Treatment Fidelity

Determining the fidelity of implementation (Lastica & O'Donnell, 2007) was important to ensure that the curriculum as designed by the research team was implemented as intended and as outlined in the teachers' manuals. Lynch (2007) enumerates reasons for the focus on fidelity of intervention. Assessing fidelity of implementation is essential to determine:

- If the intervention was actually implemented (a fidelity check);
- The extent to which the intervention was implemented (using instruments that can provide a range on fidelity indicators);
- Whether there is a relationship between fidelity of implementation and student outcomes (an indication of the intervention "worth," requiring fine-tuned measures of fidelity capable of discriminating among classroom enactment of interventions); and
- Whether the intervention was much different than typical practices in "comparison" classrooms (necessary to understand the "value added" for an intervention). (p. 4)

With this focus as a guide, fidelity of implementation was assessed in multiple ways through onsite professional development, teachers' logs, weekly contacts via email or telephone with treatment group teachers; unit test completion; onsite observations; and the curriculum units as "educative curricula." It was critical to include multiple methods to check fidelity of implementation, as onsite classroom observations were limited to one visit due to the scope of the study in multiple schools and states.

Onsite Professional Development

The NRC/GT research team conducted 2 days of professional development: one day prior to the intervention and one day after the completion of one unit. Treatment teachers had multiple opportunities to become familiar with the study's research protocol, view model lessons, engage in application of concepts through use of provided manipulatives, become familiar with the "big mathematical ideas," experience the scoring of unit tests using rubrics, and learn about the resources available from the professional development libraries. Copies of all professional development presentations with scripted notes and resources were available on DVDs and CDs to ensure that the research team replicated the professional development consistently across research sites.

Teachers' Logs

The teachers' logs described under the section entitled Description of Instruments provided the documentation of teachers' perspectives on the implementation of the units and the students' responses to dealing with pre-differentiated and enriched curricula. This approach to data collection allowed us to gather additional insights into level of fidelity of the implementation of the curricula.

Weekly Contacts

Research team members served as liaisons to the treatment group teachers in the participating schools. They contacted the teachers weekly via email or phone to inquire about the status of the intervention and to respond to any questions. Information gleaned from this informal approach to assessing the fidelity of implementation ensured that the team was knowledgeable about the progress with the intervention.

Unit Test Completion

It was important to establish the extent to which teachers completed each of the pre-differentiated and enriched curricula units. We documented the unit test completion for each of the Cohorts.

Cohort I Unit Test Completion

Table 3.1 presents the number and percentage of Cohort I classrooms completing the curricular units in each treatment school. There were 31 treatment classrooms in Cohort I, of which 25 provided complete ITBS posttest scores for the primary quantitative analysis and an additional 5 provided Stanford posttest scores. The fidelity of implementation of completing the curricular units for Cohort I was high except for the graphing and data analysis unit. Eighty-four percent of the Cohort I treatment classrooms completed the algebra and geometry and measurement units, but only approximately two-thirds (68%) of the Cohort I classrooms completed the graphing and data analysis unit.

Cohort II Unit Test Completion

Table 3.2 presents the number and percentage of Cohort II classrooms completing the curricular units in each treatment school. There were 21 treatment classrooms in Cohort II, from which 20 provided complete ITBS posttest scores for the primary quantitative analysis. The fidelity of implementation of completing the curricular units for Cohort II was extremely high. Ninety-five percent of the Cohort II treatment classrooms completed all three of the units, with one classroom from Rosewood Park school that did not complete any of the units.

Table 3.1
Cohort I Classrooms Completing Curricular Units by Treatment School (*n*=31)

School Pseudonym (Number of Treatment Classrooms)	Algebra *n* (%)	Geometry and Measurement *n* (%)	Graphing and Data Analysis *n* (%)
Crowder Point (4)	*3 (75)*	4 (100)	*3 (75)*
Heritage (1)	1 (100)	1 (100)	1 (100)
Historic Cove (3)	3 (100)	3 (100)	3 (100)
Lakeshore (5)	5 (100)	*4 (80)*	*4 (80)*
New Horizon (3)	*0*	*0*	*0*
Pleasant View (2)	2 (100)	2 (100)	2 (100)
Rosewood Park (4)	4 (100)	*3 (75)*	*3 (75)*
Stone Mill (3)	3 (100)	3 (100)	3 (100)
Vista (4)	*3 (75)*	*3 (75)*	*1 (25)*
West Valley (2)	2 (100)	2 (100)	*1 (50)*
Total number (Percent) of classrooms completing units across schools	26 (84)	26 (84)	21 (68)

Note: Italicized entries indicate schools in which all teachers did not complete the mathematics units.

Table 3.2
Cohort II Classrooms Completing Curricular Units by Treatment School (*n*=21)

School Pseudonym (Number of Treatment Classrooms)	Algebra *n* (%)	Geometry and Measurement *n* (%)	Graphing and Data Analysis *n* (%)
Crowder Point (3)	3 (100)	3 (100)	3 (100)
Heritage (1)	1 (100)	1 (100)	1 (100)
Historic Cove (3)	3 (100)	3 (100)	3 (100)
Lakeshore (4)	4 (100)	4 (100)	4 (100)
Pleasant View (2)	2 (100)	2 (100)	2 (100)
Rosewood Park (4)	*3 (75)*	*3 (75)*	*3 (75)*
Stone Mill (3)	3 (100)	3 (100)	3 (100)
West Valley (1)	1 (100)	1 (100)	1 (100)
Total number (Percent) of classrooms completing units across schools	20 (95)	20 (95)	20 (95)

Note: Italicized entries indicate schools in which all teachers did not complete the mathematics units.

Cohort III Unit Test Completion

Table 3.3 presents the number and percentage of classrooms completing the curricular units in each treatment school. There were 81 classrooms with 83 teachers—as 2 classrooms were organized in co-teaching situations. The fidelity of implementation of the curricular units was 91% or more for the three mathematics units, with 91% of the treatment group teachers completing the algebra unit, 95% and 96% completing the geometry and measurement and graphing and data analysis units, respectively.

Table 3.3
Cohort III Classrooms Completing Curricular Units by Treatment School (*n*=81)†

School Pseudonym (Number of Treatment Classrooms)	Algebra *n* (%)	Geometry and Measurement *n* (%)	Graphing and Data Analysis *n* (%)
Apple Tree (1)	1 (100)	1 (100)	1 (100)
Bald Eagle (2)	2 (100)	2 (100)	2 (100)
Calder (2)	2 (100)	2 (100)	2 (100)
Casini (1)	*0*	*0*	1 (100)
Cedar Brook (1)	1 (100)	1 (100)	1 (100)
Centurion (1)	1 (100)	1 (100)	1 (100)
Cortana (2)	2 (100)	2 (100)	2 (100)
Deer Creek (2)	2 (100)	2 (100)	2 (100)
East Halsey (2)	2 (100)	2 (100)	2 (100)
East Point (1)	1 (100)	*0*	*0*
Evergreen Street (2)	2 (100)	2 (100)	2 (100)
Farnsworth (1)	1 (100)	1 (100)	1 (100)
First Sun (2)	2 (100)	2 (100)	2 (100)
Forge Hill (3)	3 (100)	3 (100)	3 (100)
Franklin Bridge (2)	2 (100)	2 (100)	2 (100)
George Washington (2)	2 (100)	2 (100)	2 (100)
Governor's Park (2)	2 (100)	2 (100)	2 (100)
Grand Arch (2)	2 (100)	2 (100)	2 (100)
Halcyon (2)	*0*	2 (100)	2 (100)
Haverbrook (1)	1 (100)	1 (100)	1 (100)
Lucasville (2)	2 (100)	2 (100)	2 (100)
Morrowind (3)	*2 (67)*	3 (100)	3 (100)
Mustang Ranch (1)	1 (100)	1 (100)	1 (100)
Newton (2)	*1 (50)*	2 (100)	2 (100)

Note: Italicized entries indicate which teachers did not complete the mathematics units.
† There are 81 classrooms and 83 teachers due to two co-teaching situations.

Table 3.3 (continued)
Cohort III Classrooms Completing Curricular Units by Treatment School (*n*=81)

School Pseudonym (Number of Treatment Classrooms)	Algebra *n* (%)	Geometry and Measurement *n* (%)	Graphing and Data Analysis *n* (%)
Northwest (2)	2 (100)	2 (100)	2 (100)
Old Toll Road (1)	1 (100)	1 (100)	1 (100)
Oyster Harbor (2)	*0*	2 (100)	2 (100)
Pegasus (2)	2 (100)	2 (100)	2 (100)
Savannah (1)	1 (100)	*0*	*0*
Seabreeze (2)	2 (100)	2 (100)	2 (100)
Shade Rock (2)	2 (100)	2 (100)	2 (100)
Shady River (1)	1 (100)	1 (100)	1 (100)
Shelbyfield (4)	4 (100)	4 (100)	4 (100)
Skinner (4)	4 (100)	4 (100)	4 (100)
Smithton (4)	4 (100)	4 (100)	4 (100)
Solsbury Valley (2)	2 (100)	2 (100)	2 (100)
Southeastern (3)	3 (100)	*2 (67)*	*2 (67)*
Springville (1)	1 (100)	1 (100)	1 (100)
Staten Ridge (2)	2 (100)	2 (100)	2 (100)
Sunnyside (2)	2 (100)	2 (100)	2 (100)
Sycamore (1)	1 (100)	1 (100)	1 (100)
Terracotta (2)	2 (100)	2 (100)	2 (100)
Vermillion (1)	1 (100)	1 (100)	1 (100)
Total number (Percent) of classrooms completing units across schools	74 (91)	77 (95)	78 (96)

Classroom Observation Scales

Classroom Observation Scales guided the onsite classroom observations (see Appendix A). The 12-item scales included a 5-point scale: N/A-not applicable; 1-Occurs to a lesser extent to 4-Occurs to a greater extent for the treatment classroom observations and a dichotomous scale of N/O-not observed; O-Observed for the control classroom observations. Items from the What Works in Gifted Education Observation Scales follow:

1. Students understand the “Big Idea” (i.e., understand concept in different context) of the lesson.
2. The materials provided for lesson implementation are used appropriately.
3. Appropriate mathematical language is used.
4. Students have a clear understanding of directions for lesson activities.

5. The students and/or teacher make(s) connections to prior concepts.
6. Student learning is assessed through observation, listening, and/or information gathering.
7. Students are grouped according to the suggested levels of differentiation.
8. Students are presented with challenging content.
9. Discourse (whole group, small group, peer) about mathematical problems occurs.
10. Students are invited to find multiple strategies or solutions to the mathematical problem.
11. Students are encouraged to explain or justify their responses.
12. Students are engaged in the lesson.

Observers took field notes of the mathematics lessons to determine if the items from the Classroom Observation Scales were evident as treatment teachers implemented a lesson from the algebra, geometry and measurement, and graphing and data analysis units. Observations of the control teachers' classrooms focused on whether these same elements of lesson design and implementation were evident in the classroom environment, curricular and instructional strategies and practices, and student engagement and responses to the content.

Mathematics Units as "Educative Curricula"

As the amount of time devoted to an in-depth focus on algebra, geometry and measurement, and graphing and data analysis contained in the What Works Mathematics Curricula are not typical of grade 3 curricula, it was important to ensure that teachers' pedagogical content knowledge would be developed or enhanced as they prepared for the implementation of the curricula by reviewing each unit's accompanying manual, detailed lesson plans, and assessments. Teachers need strong skills in mathematics, especially in academically diverse classrooms with students who may be well below or well above grade level. They also need pedagogical knowledge to allow them to use multiple instructional styles and pedagogical content knowledge to teach the content (Shulman, 1986). Teachers have agency of their own professional learning and they, too, must "make connections between ideas, in addition to adding new ideas about subject-area concepts, instructional approaches, students' likely ideas, or teaching principles" (Davis & Krajcik, 2005, p. 4). Three of Davis and Krajcik's Guidelines for developing educative curriculum provided a framework for the mathematics units for the What Works in Gifted Education Study:

- First, educative curriculum materials could help teachers learn and how to anticipate what learners may think about or do in response to instructional activities. (p. 4)
    - What Works Mathematics Curricula provided initiation questions to engage students in the content; offered guided questions as students dealt with "messy mathematical problems;" and included illustrative examples of students and teachers engaging in discourse about complex concepts.

- Second, curriculum materials could support teachers' learning of subject matter. (p. 4)
    - What Works Mathematics Curricula offered detailed explanations of the content and requisite skills as well as illustrations of how to connect the concepts to real-world situations.
- Third, curriculum materials could help teachers consider ways to relate units during the year. (p. 4)
    - What Works Mathematics Curricula included opportunities for students to design and implement projects. As teachers reflected on what was learned from one unit, they could transfer the application of knowledge, content, and skills to another unit as all units promoted problem solving, reasoning, communication, making connections, and designing and analyzing representations. (NCTM, 2006)

By designing educative curricula, the study guided the implementation of the lessons to achieve fidelity within and across treatment classrooms. The multiple methods used for assessing the fidelity of implementation of the What Works Mathematics Curricula served as proxies for actually witnessing lesson implementation multiple times.

CHAPTER 4: Cohort I Quantitative Results

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School-level Data

During the 2008-2009 school year, 21 schools representing seven states participated in a pilot study to determine the impact of implementing three pre-differentiated and enriched curricular units with grade 3 mathematics students of all ability levels. The pilot study research design specified the unit of randomization for study condition as the school. Ten of the 21 participating schools were assigned to receive the curriculum intervention for 16 weeks of the school year, while the other 11 were assigned to be in the control condition, implementing their regular "business as usual" grade 3 mathematics curricula for the entire school year.

Table 4.1 provides information on the schools' enrollments, proportions of underrepresented minority students, proportions of students eligible for free/reduced-priced meals through the National School Lunch Program (NSLP), and student to teacher ratios during the 2008-2009 school year. The term underrepresented minority refers to the research literature (see Gentry et al., 2008 for a summary), which has documented the underrepresentation of African Americans, Hispanic/Latino students, and Native American students in gifted education programs in the United States. The National Research Center on the Gifted and Talented has focused its research efforts on providing enriched educational opportunities for students from these groups to promote both excellence and equity for students of all races and ethnicities.

The schools participating in the pilot year of the What Works in Gifted Education Mathematics Study had a mean enrollment of 412.7 with a standard deviation of 152.5, and a median enrollment of 371. These schools' mean percentage of underrepresented minority students was 19.6 with a standard deviation of 18.5, and a median percentage of 13.8. This is somewhat lower than the percentage of students identified as being in racial groups other than White and Asian in the nationally representative American Community Survey of 2006-2010 (NCES, 2012), which estimates this number as approximately 30% of American public school children.

In terms of socioeconomic status, the mean percentage of students from Cohort I schools eligible for free/reduced-priced meals was 26.0 with a standard deviation of 19.65, and a median percentage of 20.5. Nationally, approximately 62% of students meet eligibility criteria for free/reduced lunch status during the 2008-2009 school year (National School Lunch Program, 2012). Therefore the Cohort I sample of schools contained more students of higher socioeconomic backgrounds than in the nation overall.

Additionally, the mean student to teacher ratio for the Cohort I schools was 16.3 to one with a standard deviation of 2.6 and a median ratio of 15.8 to one.

Table 4.1
Demographics[†] *for Cohort I Schools Participating in the What Works in Gifted Education Mathematics Pilot Study* (*N*=21)

School Pseudonym	Research Condition	Total Enrollment	White/ Asian (%)	Other Ethnicities (%)	Free/ Reduced Lunch (%)	Student Teacher Ratio
Acadia	Control	343	96.8	3.2	27.7	14.8
Beaumont	Control	364	89.7	10.3	4.7	14.5
Canyon Ridge	Control	371	94.9	5.1	7.5	20.2
Cove	Control	282	70.6	29.4	22.0	15.5
Crowder Point	Treatment	444	86.7	13.3	11.0	15.6
Heritage	Treatment	163	93.9	6.1	51.5	12.9
Historic Cove	Treatment	426	83.1	16.9	8.2	16.3
Lakeshore	Treatment	646	86.0	14.0	13.3	15.8
Lighthouse	Control	369	84.0	16.0	13.3	14.8
Mountain View	Control	704	57.7	42.3	45.1	16.4
New Horizon	Treatment	415	21.9	78.1	73.3	12.8
North Point	Control	411	93.9	6.1	27.3	21.4
Pleasant View	Treatment	294	88.4	11.6	39.1	20.9
Riverside	Control	318	72.7	27.3	20.1	14.7
Riverton	Control	301	86.4	13.6	10.8	19.3
Rosewood Park	Treatment	695	85.8	14.2	20.5	16.2
Sidewind	Control	702	94.3	5.7	2.0	16.6
Stone Mill	Treatment	395	71.7	28.3	22.3	15.7
Vista	Treatment	442	98.6	1.4	17.2	16.0
West Valley	Treatment	242	91.3	8.7	56.6	19.7
Woodbridge	Control	339	61.9	38.1	52.8	12.8

† Source: School Digger website: http://www.schooldigger.com (2008-2009 data).

Teacher-level Data

During the 2008-2009 pilot study, 31 teachers participated in the study from the schools assigned to treatment condition. Data were not collected on the teachers from schools assigned to the control condition during this year, and six of the treatment teachers did not return demographic information. However, demographic information describing the 25 remaining treatment teachers was collected and is summarized in Table 4.2. The treatment teachers in Cohort I were fairly similar to national averages in terms of gender distribution, which estimated 81.7% of elementary and middle school teachers to be female in 2011 (Bureau of Labor Statistics, 2012). Only one Cohort I treatment teacher identified as belonging to a race other than White, and one teacher failed to provide this information. The treatment teachers for Cohort I had substantial teaching experience, with the majority (68%) having more than 10 years total experience. In terms of grade 3 teaching experience, the Cohort I teachers were evenly split between those with greater and those with fewer than 10 years teaching experience. Most of the Cohort I treatment teachers held a Master's degree or Sixth Year certificate, with a substantial minority holding Bachelor's degrees.

Table 4.2
Cohort I Treatment Teachers Demographics (*n*=25)

Characteristic	Frequency (%)
Gender	
Male	3 (12)
Female	22 (88)
Race/Ethnicity	
Black/African American	1 (4)
White	24 (96)
Total Years Teaching Experience	
0-4	4 (16)
5-9	4 (16)
10-14	6 (24)
15+	11 (44)
Years Teaching Grade Three	
0-4	6 (24)
5-9	7 (28)
10-14	8 (32)
15+	4 (16)
Highest Degree Earned	
BA/BS	7 (28)
MA/MS	15 (60)
Sixth Year/Ed. Specialist	3 (12)
Ph.D./Ed.D.	0
Professional Diploma	0
Other	0

Student-level Data

A total of 1,366 students participated in the 2008-2009 school year implementation of the What Works in Gifted Education Mathematics Study. Of this initial sample, ability, pretest, and posttest data necessary for the multilevel analyses were available from 970 students. These students comprised the Cohort I analytic sample and represented 20 of the 21 schools originally recruited for the study's pilot year implementation. The Cohort I analytic sample included 465 students from schools assigned to the control condition and 505 students from schools assigned to the treatment condition. Table 4.3 contains a summary of these students' gender and ethnicity, as well as their state of residence during the 2008-2009 school year. Gender was almost exactly evenly split between males and females, and a large majority of participants (approximately 84%) identified as either White or Asian.

Table 4.3
Cohort I Treatment and Control Group Student Demographics (*N*=970)

Characteristic	Treatment Group (*n*=505) Frequency (%)	Control Group (*n*=465) Frequency (%)	Total (*N*=970)
Gender			
Male	249 (49.3)	244 (52.5)	493
Female	255 (50.5)	221 (47.5)	476
Gender not indicated	1 (0.2)	0	1
Total	505	465	970
Ethnicity			
American Indian/Alaskan Native	5 (1.0)	6 (1.3)	11
Asian/Hawaiian/Pacific Islander	29 (5.7)	11 (2.4)	40
Black/African American	33 (6.5)	32 (6.9)	65
Hispanic/Latino(a)	23 (4.6)	12 (2.6)	35
White	392 (77.6)	378 (81.3)	770
Multiple Ethnicities/Other	8 (1.6)	18 (3.9)	26
Ethnicity not indicated	15 (3.0)	8 (1.7)	23
Total	505	465	970
State of Residence			
Northeast	180 (35.6)	136 (29.2)	316
Southeast	66 (13.1)	40 (8.6)	106
Midwest	153 (30.3)	193 (41.5)	346
West	0	23 (4.9)	23
Southeast	106 (21.0)	73 (15.7)	179

When conducting randomized experiments, it is important to verify that treatment and control participants have comparable characteristics prior to the intervention. Establishing the equivalence of the treatment and control groups prior to the onset of the intervention reduces concerns about selection bias and strengthens the ability to make causal inferences. Although random assignment should minimize pretest differences between the groups, empirical pretest data can also support the assertion of equivalence. Table 4.4 contains information on differences between treatment and control students for a number of pretest ability and achievement assessments, as well as teacher rating scales and demographic information.

Cohort I treatment and control students differed significantly on the verbal, quantitative, and nonverbal portions of the CogAT, with the control students having a higher mean for each ability subtest. These differences also caused the composite CogAT score, which is the mean of the three subtests, to be significantly greater in favor of the control students. Control students also had significantly higher pretest scores on the ITBS Math Problem Solving subtest than their treatment counterparts.

Teachers rated the treatment students significantly higher on the Motivation subscale of the Scales for Rating the Behavioral Characteristics of Superior Students (SRBCSS) than control students. Students from the treatment condition also attended schools with significantly higher percentages of underrepresented minority students and students eligible for free/reduced-priced meals than the schools control students attended. The two school-aggregate variables (percent underrepresented minorities and percent free/reduced meal eligibility) were based on school-level data rather than student-level data.

Table 4.5 contains the bivariate correlations between seven pairs of student-level and contextual variables for students in Cohort I. Pretest achievement, pretest ability, and posttest achievement scores were highly related. For the aggregate contextual variables, school average ability and school average achievement were quite highly correlated. The schools' percentages of underrepresented minorities and percentages of students eligible for free/reduced-priced lunches were also substantially correlated. Higher school-level percentages of underrepresented minority students and lower-SES students were substantially negatively correlated with school aggregate measures of ability and achievement, but less so with student-level ability and achievement measures.

In addition to the nationally normed standardized achievement tests that were taken by both treatment and control students, treatment students completed researcher-developed unit tests before and after instruction of each of the three curricular units. Table 4.6 presents pretest and posttest scores that have been disaggregated based on three categories of school-level socioeconomic status to enable comparisons of different groups' scores on these measures of algebra, geometry and measurement, and graphing and data analysis achievement. Student gains from pretest to posttest are also presented for the three school-SES categories; Cohen's *d* effect size measures for these gain scores are included.

Table 4.4
Cohort I Students' Group Equivalence on Pretest Measures for Treatment and Control Groups

Variable	Condition	*M*	*SD*	*t*(*df*)	95% confidence interval for difference: Lower bound	95% confidence interval for difference: Upper bound
Student age at pretest	Control	98.73	4.76	1.41(967)	-0.17	1.04
(in months)	Treatment	98.29	4.79			
CogAT age score	Control	107.52	14.03	3.73(792)	1.75	5.65
(verbal)**	Treatment	103.82	13.93			
CogAT age score	Control	106.85	13.93	3.61(782)	1.63	5.53
(quant)**	Treatment	103.26	13.87			
CogAT age score	Control	105.94	12.82	2.11(782)	0.13	3.79
(nonverbal)*	Treatment	103.98	13.24			
CogAT age score	Control	108.43	13.28	4.28(743)	2.26	6.08
(composite)**	Treatment	104.26	13.29			
OLSAT age score	Control	103.80	16.21	-0.05(182)	-4.54	4.31
(verbal)	Treatment	103.90	13.75			
OLSAT age score	Control	102.65	12.90	0.14(182)	-3.68	4.24
(nonverbal)	Treatment	102.37	14.31			
OLSAT age score	Control	103.35	14.06	0.13(182)	-3.75	4.26
(total)	Treatment	103.10	13.30			
ITBS Level 8 Reading	Control	181.97	19.26	1.09(939)	-1.09	3.85
	Treatment	180.60	19.31			
ITBS Level 8 Math**	Control	179.28	21.35	2.94(967)	1.33	6.65
	Treatment	175.29	20.82			
SRBCSS Learning	Control	54.77	7.26	-1.71(244)	-3.20	0.22
scale	Treatment	56.26	6.33			
SRBCSS Motivation	Control	53.43	10.48	-2.79(244)	-5.49	-0.95
scale**	Treatment	56.65	7.34			
SRBCSS Creativity	Control	42.61	7.59	-1.81 (244)	-3.38	0.14
scale	Treatment	44.23	6.39			
Gender (dummy code)	Control	0.52	0.50	0.99(966)	-0.03	0.09
	Treatment	0.49	0.50			
Percentage non-White/	Control	12.27	10.13	-4.52(967)	-5.72	-2.26
non-Asian students**	Treatment	16.26	16.37			
Percentage FRL†	Control	16.57	11.62	-6.67(967)	-8.76	-4.78
eligibility**	Treatment	23.34	18.82			
Proportion of students	Control	0.26	0.44	0.73(967)	-0.03	0.08
receiving teacher nominations	Treatment	0.24	0.43			

† FRL indicates free/reduced priced school meals
* $p<.05$.
** $p<.01$.

Table 4.5
Cohort I Students' Summary of Bivariate Correlations Between Variables in the What Works in Gifted Education Mathematics Study† *Scores*

Variable	Pretest Achievement Score	Pretest Ability Score	School % Non-White and Non-Asian	School % FRL Eligibility	School Mean Pretest Score	School Mean Ability Score	Posttest Achievement Score
Pretest Achievement Score	1	0.73	-0.10	-0.21	0.29	0.33	0.72
Pretest Ability Score		1	-0.12	-0.21	0.31	0.29	0.70
School % Non-White and Non-Asian			1	0.57	-0.48	-0.29	-0.08
School % FRL Eligibility				1	-0.71	-0.63	-0.21
School Mean Pretest Score					1	0.89	0.25
School Mean Ability Score						1	0.27
Posttest Achievement Score							1

† All correlations are statistically significant at the *p*<.01 level.

Table 4.6
Pretest Achievement, Unit Test Scores, and Difference Scores of Students in Cohort I Schools Under Different Categories of Student Eligibility for Free/Reduced Lunch (FRL)

Measure	Schools With <15% Student FRL Eligibility (*J=7)*	Schools With 15%-40% Student FRL Eligibility (*J*=9)	Schools With > 40% Student FRL Eligibility (*J*=4)	All Schools (*J*=20)
Achievement Pretest Scale Score				
N	446	408	115	969
M	180.95	176.09	166.62	177.2
SD	20.56	21.12	19.66	21.16
Algebra Unit Pretest				
N	190	203	79	472
M	7.44	6.38	5.04	6.58
SD	5.44	5.16	3.86	5.15
Algebra Unit Posttest				
N	205	188	50	443
M	14.67	12.10	12.97	13.27
SD	5.43	5.91	5.02	5.73
Geometry and Measurement Pretest				
N	208	203	68	479
M	12.21	13.47	12.97	12.85
SD	4.64	4.42	4.06	4.50
Geometry and Measurement Posttest				
N	191	168	49	408
M	23.90	21.83	19.82	22.56
SD	4.11	5.37	5.34	5.00
Graphing and Data Analysis Pretest				
N	191	186	50	427
M	5.14	5.00	4.56	5.00
SD	2.52	2.42	2.64	2.49
Graphing and Data Analysis Posttest				
N	172	143	32	347
M	8.85	9.20	9.28	9.04
SD	2.97	3.30	2.68	3.09
Algebra Difference Score				
N	186	187	49	422
M	7.24	5.59	7.61	6.55
SD	4.67	5.37	4.15	5.00
Cohen's *d*	1.33	1.01	1.71	1.20

Table 4.6 (continued)
Pretest Achievement, Unit Test Scores, and Difference Scores of Students in Cohort I Schools Under Different Categories of Student Eligibility for Free/Reduced Lunch (FRL)

Measure	Schools With <15% Student FRL Eligibility (*n*=7)	Schools With 15%-40% Student FRL Eligibility (*n*=9)	Schools With > 40% Student FRL Eligibility (*n*=4)	All Schools (*N*=20)
Geometry/Measurement Difference Score				
N	190	165	48	403
M	11.77	7.87	7.17	9.63
SD	4.60	4.97	3.29	5.04
Cohen's *d*	2.69	1.61	1.52	2.02
Graphing/Data Analysis Difference Score				
N	171	143	32	346
M	4.00	4.06	3.53	3.98
SD	2.31	2.84	2.36	2.54
Cohen's *d*	1.46	1.42	1.50	1.43

Cohort I: Quantitative Results

In the first cohort of the What Works in Gifted Education Mathematics Study, the research team examined the impact of the pre-differentiated and enriched curricula on student achievement using a cluster randomized design. Twenty-one schools were randomly assigned to treatment or control, of which twenty provided students' posttest achievement scores. A total of 970 students had complete data to be included in the analytic sample for the multilevel analysis.

The Model

Two-level hierarchical models provided an appropriate method to analyze the data as students were nested within schools. Students' post achievement in mathematics was predicted by both student- and school-level variables.

At the student level, the models controlled for student initial achievement in mathematics and examined the impact of a student being gifted relative to his or her peers on mathematics achievement. Students' initial achievement in mathematics was measured by the ITBS Level 8 Math Problems subtest. Students' pretest achievement scores were converted to *z* scores based on the national percentile rank of the student. In addition to measuring students' prior mathematics achievement, all Cohort I students took assessments intended to measure reasoning abilities. These ability scores were used to create a dichotomous "gifted" variable. Most of the schools (*n*=17) used the CogAT, and the remaining three schools used the Otis-Lennon School Ability Test (OLSAT). A student was identified as gifted if he or she scored in the top 10% of students at his or her school on either the CogAT composite score or the OLSAT total score.

At the school level, the two-level models controlled for initial school achievement and examined whether treatment impacted student posttest achievement in mathematics. Most Cohort I schools (n=16) provided ITBS Level 9 Math Problem Solving and Data Interpretation subtest scores for the posttest measure of achievement, but four schools measured posttest mathematics achievement using the total mathematics subscale of the Stanford Achievement Tests. Because these two measures were on different scales, the posttest achievement scores were also converted to a z-score based on students' national percentile ranks. The initial school achievement variable was the mean z score of student achievement at each school. Treatment was dummy coded (0-control, 1-treatment). Given that the student and school achievement variables were already in z-score units, they were entered into the models uncentered.

The Results

The intercept represents the expected or model-predicted score for students who were of average initial achievement in mathematics, who were not gifted, and who attended a control school. For average pre-achievement students who attended schools assigned to control and that had average initial school achievement, the predicted posttest z-score was 0.17 (γ_{00}=0.17, SE=0.06). Similar students from schools assigned to treatment did not have statistically different outcomes from students in control schools (γ_{01}=-0.03, SE=0.06, p>.05). School aggregate achievement significantly predicted posttest ITBS scores, after controlling for student achievement, gifted status, and treatment (γ_{01}=0.21, SE=0.09, p=.03).

The student math achievement slope represents the impact that initial mathematics achievement had on the post ITBS mathematics assessment. The intercept of this slope was strong and positive (γ_{10}=0.57, SE=0.05, p<.001), indicating that prior student achievement was quite predictive of posttest math achievement. Students who were one standard deviation above the mean in initial mathematics achievement were expected to score more than half a standard deviation higher on the posttest measure of mathematics achievement. Treatment did not moderate the effect of pretest math achievement on posttest math achievement: The treatment effect on the pretest slope was not statistically significant (γ_{11}=0.05, SE=0.05, p>.05). School mean pretest math achievement also did not moderate the relationship between pretest math achievement and posttest math achievement (γ_{12}=0.10, SE=0.08, p>.05).

Finally, examined the effects of being in the top 10% in relative ability (by our definition gifted) in a school. Those students who were identified as gifted were expected to score almost one-third a standard deviation higher (γ_{20}=0.29, SE=0.08, p<.001) on the posttest measure of achievement, after controlling for all other variables in the model.

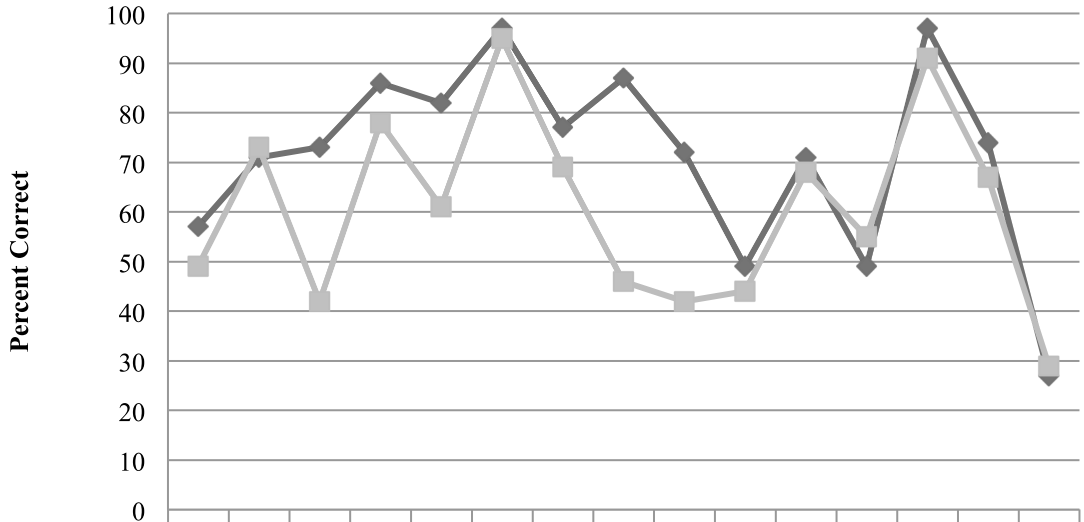
The results indicate that the treatment had no effect on students' posttest achievement nor on the relationship between pretest and posttest mathematics achievement. Instead, a student's starting score, being in the top 10% of ability relative to schoolmates, and his or her school's mean achievement were far more predictive of post mathematics achievement than treatment. However, there were two problems with the outcome measure. First, 8.4% of the students received the highest score possible on the posttest measures of achievement. Thus, the true growth of these students was likely not measured accurately and introduced measurement error into the model. The true post achievement of these students was not determined. Second, the skills that were taught in the treatment were not reflected on the ITBS and Stanford math subscales. Many of these skills were above grade level and not captured by the grade 3-level assessment of achievement. Additionally, the sample of schools in Cohort I was very small, which decreased the power of the statistical tests. Even so, the treatment effect in the final model was very small and negative, suggesting no appreciable treatment effect.

National Assessment of Educational Progress Results

The National Assessment of Educational Progress (NAEP) has provided a national benchmark of students' academic achievement in the United States for over 40 years. As part of participation in the NRC/GT mathematics study, grade 3 students who received the pre-differentiated and enriched curricula were administered 15 mathematics items from the NAEP. The items were chosen for their similarity to the conceptual knowledge contained in the research curriculum.

Although students involved in the study were in grade 3 during the curriculum implementation, the items on which they were tested were drawn from the grade 4 NAEP item bank. "Above-grade level" testing targets student achievement above the levels that could be measured by "at-grade level" items. For each item tested, the percentage of students who responded correctly to the item is shown compared to the national grade 4 percentages. Over 50% of the grade 3 students completed the grade 4 items correctly, with the exception of items 2 and 15 (see Figure 4.1). These results represent the 572 treatment students who participated in the first year of the study.

These differences were most pronounced when the content of the items most closely matched the units. For example, questions 3, 5, & 8 covered bar graphs, line graphs and perimeter, respectively. These concepts were addressed in multiple ways throughout the units. This provides further evidence of how students were able to learn when presented with challenging and differentiated curricula.



	Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
Study	57	71	73	86	82	97	77	87	72	49	71	49	97	74	27
Nation	49	73	42	78	61	95	69	46	42	44	68	55	91	67	29

Graphing & Data Analysis Geometry & Measurement Algebra

Figure 4.1. Comparison of percentage of correct responses on NAEP items by Cohort I treatment students and national sample.

Mathematics Curriculum Unit Test Results (2008-2009)

The What Works in Gifted Education Mathematics Study provided challenging pre-differentiated and enriched curricula based on the well-known models of gifted and talented education by Sandra N. Kaplan, Carol A. Tomlinson, Joseph S. Renzulli and Sally M. Reis. Regular education third grade classrooms participated in three units entitled *Awesome Algebra* (Gavin et al., 2008a, 2009a)*, Geometry & Measurement for All Shapes & Sizes* (Cole, Heilbronner, et al, 2009a; Corbishley et al., 2008a), and *Greening Up With Graphing: Recycle, Reduce, & Reuse* (Cole, Rubenstein, et al., 2009a; Gubbins et al., 2008a) during the 2008-2009 school year. Each unit was developed in line with the NCTM's (2000) Principles and Standards for School Mathematics, which provides guidance for educational decision makers in grades Pre-K through 12. The development of these units assumed that students in grade 3 possess the prior knowledge indicated by the standards for grades Pre-K-2 and extended this knowledge by focusing on the standards for grades 3-5. In addition, the authors of these units relied on the NCTM focal points, which provide additional specificity of content for grade 3.

Each unit had challenging content and a unit test with a high ceiling so all students could demonstrate growth. Despite the challenging content, across the sites all students showed significant gains from pretest to posttest in all three units (see Figure 4.2). These results represent the following number of treatment students who participated in the first year of the study, Algebra (n=480); Geometry & Measurement (n=462); and

Graphing & Data Analysis (*n*=399). These results demonstrate that all students can learn when presented with challenging, pre-differentiated and enriched curricula.

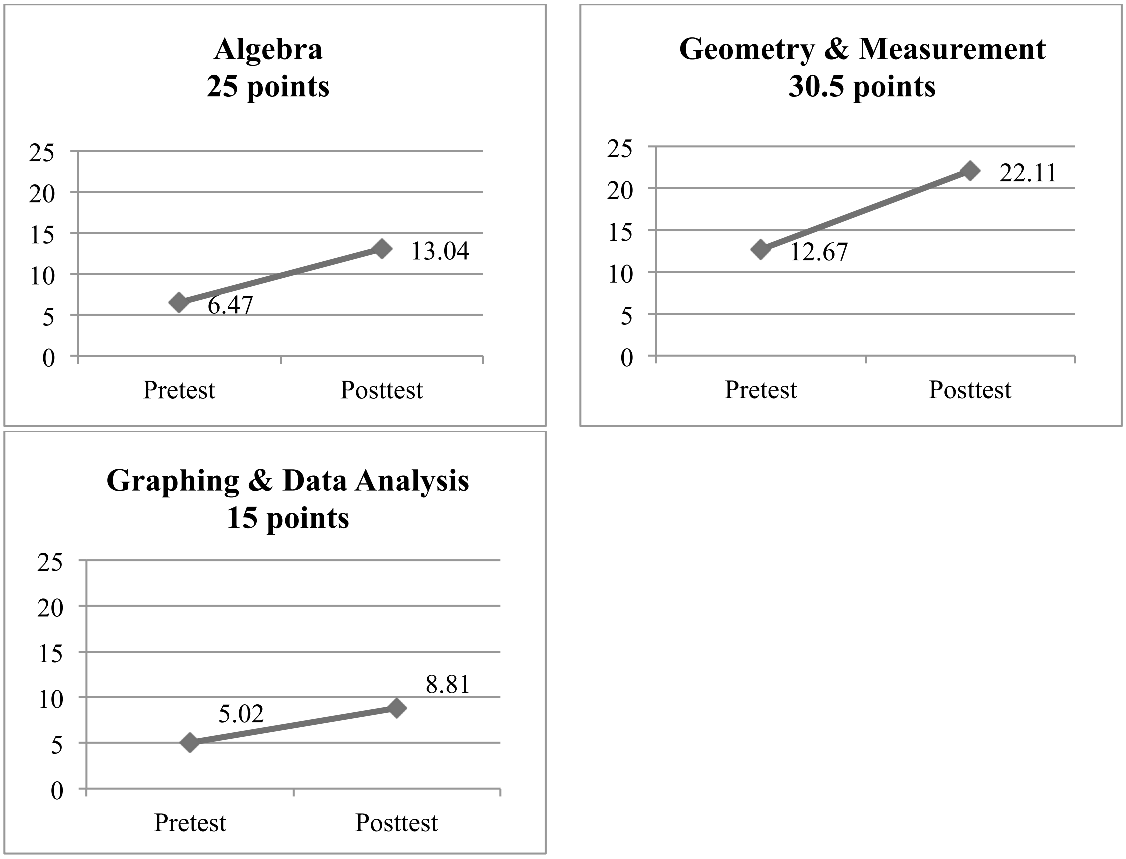


Figure 4.2. Average student gains on unit tests across all sites.

Since each test had a different total score, effect sizes provided a method for comparing change across different measures. Researchers hold different opinions about the appropriateness of specific formulas to calculate effect sizes for paired *t*-tests. The standardized Cohen's *d* using the pooled standard deviation as the standardizer was chosen as the most conservative approach. Cohen (1988) defined a Cohen's *d* of 0.2 as small, 0.5 as medium, and 0.8 or above as large. The growth of students on each test was considerably greater than Cohen's definition of large (see Figure 4.3). This further demonstrated that all students can make practically significant gains when presented with challenging pre-differentiated and enriched curricula based on gifted and talented pedagogy.

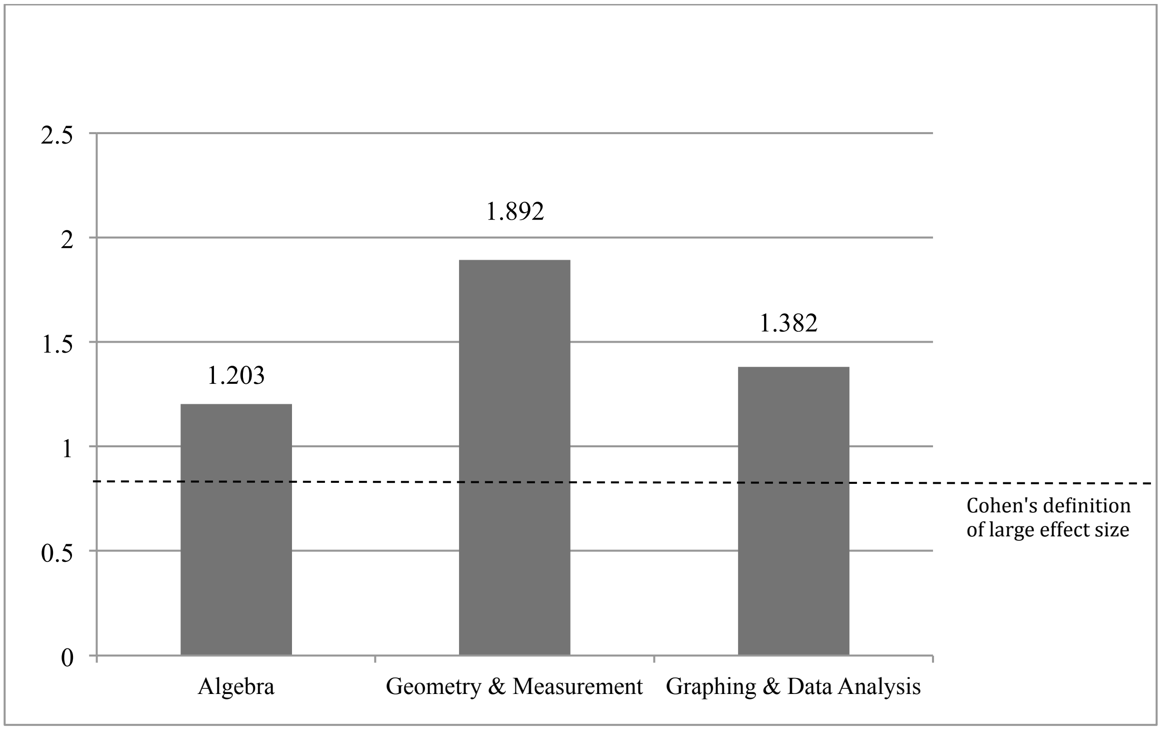


Figure 4.3. Effect sizes of Cohort I student gains.

Pretest and posttest unit data indicated that students successfully learned and applied the challenging content and concepts presented in the algebra, geometry and measurement, and graphing and data analysis units. Students' academic needs were met by the teachers' implementation of the pre-differentiated and enriched curricula based on the pedagogical models of gifted and talented education. The designers of each curricular unit used several strategies proposed by these models' authors as they designed lessons to engage both students and their teachers in teaching mathematics.

CHAPTER 5: Cohort I Qualitative Results

Cindy M. Gilson

Participating schools in Cohort I were randomly assigned to control or treatment conditions. Only treatment schools were observed. The data sources for the qualitative analysis of the Cohort I treatment schools included administrators' interviews, classroom observations, teachers' focus groups, and teachers' logs. As this was the first year of the study, the data set was relatively small comprising 14 treatment teacher observations. Of the 14 teacher observations, 12 included lessons from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit and one was from the *Geometry & Measurement for All Shapes & Sizes* Unit. One observation only included feedback from the participants rather than an observation of a lesson. The data for analysis also included two administrators' interviews, seven teachers' focus groups, and 16 teachers' logs in which teachers responded to questions about all three of the What Works Mathematics Curricula. See Appendix A for the administrators' interview and teacher focus group protocols. See Appendix C for the data source key. The following research question guided the analysis of the data from Cohort I:

> How do teachers and administrators respond to their access to pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis?

Administrators' Interviews

Two administrators' interviews were conducted with liaisons from Cohort I during the first year of the study. One interview included Ms. Winchester who served as the administrator liaison from four schools in a Midwestern state. The second interview which took place in a Southeastern state, included Ms. Warren, Ms. McDowell, and Ms. Stark. The administrators in this group had an opportunity to write anonymous comment cards, which were also included in the analysis. The commonalities between the two interviews will be discussed.

Administrators from both school districts commented on the positive reactions from the teachers to the What Works Mathematics Curricula. Ms. Winchester explained that teachers constantly wished the What Works Mathematics Curricula were part of their own curriculum (AI, Pleasant View, MW). Ms. Warner expressed that participation in the study "ignited a passion" that had not been seen before at her school (AI, Rosewood Park, SE). Her teachers were excited and engaged while working "hard and diligently" planning lessons together.

Not only were the teachers' reactions positive, administrators commented on how the teachers changed and learned from participating in the study. One administrator wrote, "Participating in this study has generated enthusiasm among teachers and administrators and a willingness to embrace new ways of introducing math concepts to students" (AI comment card, Rosewood Park, SE). Another administrator from the focus group corroborated:

> Never before have I observed or experienced a transformation like this study. My teachers' passions have been reignited. Their students cheer aloud when it is time for math. They moan if the old textbook comes out. Teachers are talking, problem-solving, planning in a way they have NEVER done before. (AI, Rosewood Park, SE)

A common concern in both school districts was how to ensure that students were prepared for the statewide tests. Ms. Winchester recognized the effort that was required of her teachers to implement the new math curriculum and to learn how it would "mesh with grade level content and objectives." The teachers in her district needed to "make sure they are doing everything to meet standards" (AI, Pleasant View, MW). Administrators from Rosewood Park noted that their teachers addressed concerns by carefully reviewing and comparing the standards in the What Works Mathematics Curricula with those for their state tests (AI, SE). Implementing this math curriculum made a difference in the teachers' confidence, and after struggling to learn the process of teaching math in new ways, their level of understanding and excitement increased.

Administrators noted that the instructional climate within the classrooms also changed as a result of the math intervention. In both school districts, the administrators commented on the students' enjoyment of the units and their growth in understanding math. Ms. Winchester noted students were benefiting from the study, stating "[students used] their brains a lot more; excited about math; ties in with grade 3 science; teaching ecology and math at same time; learning a lot; having fun" (AI, Pleasant View, MW). The nature of the units also attributed to the positive responses to the changes in the instructional climate, especially the incorporation of differentiation practices into the lessons such as grouping students, hands-on learning, use of talk moves, and inquiry based learning. Ms. Stark commented, "We look at our kids and they see themselves as mathematicians" (AI, Rosewood Park, SE). Additionally, Ms. Warner had the opportunity to observe the students' math conversations and said it "blows me away" and our "jaws were hanging as students were talking" while students demonstrated a level of understanding of math concepts and an ability to communicate mathematically that was not typical of their school (AI, Rosewood Park, SE).

Overall, administrators' perceptions of the teachers' and students' responses to the What Works Mathematics Curricula were positive. The administrators discussed changes in teachers' knowledge about instruction in addition to changes noted in the instructional climate of the classrooms. Concerns recognized by the administrators included meeting the objectives of the statewide tests in conjunction with those in the What Works Mathematics Curricula.

Classroom Observations

The 14 Cohort I treatment classroom observations were analyzed inductively to identify common themes about how teachers responded to the What Works Mathematics Curricula. The following themes emerged from the data: (a) real-world learning, (b) discourse, (c) grouping, and (d) challenge. Each theme will be discussed below.

Real-world Learning

Twelve of the 14 treatment teachers who were observed taught lessons from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. Classroom observations were conducted around the time that many teachers were implementing this unit, which explains why the majority of the lessons were from the graphing unit. The unit was intentionally designed to incorporate real-world learning experiences making the connections between mathematical concepts and real-world applications explicit for students. In the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, students engaged in a school wide recycling project while learning how to design an intervention and how to collect, graph, and analyze data.

The observations revealed that teachers actively engaged their students as practicing mathematicians. For instance, in Ms. Morrison's class students eagerly shared their progress on a recycling project from the unit with the observer and reported that they had collected 56 pounds of plastic, cardboard, and paper. The observer noted, "Ms. Morrison commented that the students were very serious about how they conducted the observations with their note taking and their awareness of what possible changes could be made to increase recycling in their school" (TO, Lakeshore, MW). Similarly, in Mr. Chapman's class, students, called collectors, were responsible for collecting data from other classrooms, and they typically returned with an enormous bag of recyclables (TO, Pleasant View, MW). His students cheered for the success for their schoolmates and for the successful intervention.

In many of the observed classrooms, it was noted that proceeds from collecting cans would go towards charity or purchasing an item that was needed for the school. Students in Ms. Newman, Ms. Nelson, and Ms. Carter's classes collected broken crayons to donate to a company that would melt them down to create new ones to raise money for children with cancer (TO, Stone Mill, NE). Ms. Newman and Ms. Nelson's students were actively engaged as they completed projects in the form of posters, speeches, and decorated buckets; while Ms. Carter's class made posters, letters to teachers, buckets, and announcements to increase participation in the school wide recycling project.

Discourse

The majority of the Cohort I treatment classroom observations showed evidence of discourse between students and the teacher or just between students. Participants engaged in discourse in whole group or small group formats. Data were coded as discourse when the students explained, wrote, stated, challenged, related, defended a point of view, elaborated, predicted, conjectured, generalized, questioned, answered, or asked for clarification about mathematical problems. Discourse also included teacher participation through the use of questioning strategies such as Talk Moves (Chapin, O'Connor, & Anderson, 2009) and convergent and divergent questions. Divergent questioning was defined as instances where open-ended questions were asked by teachers to elicit multiple or a range of acceptable responses from students. Convergent questions are those with one expected response.

About half of the teachers observed showed evidence of using both convergent and divergent questions to facilitate classroom discourse. For example, Mr. Chapman taught a lesson on line graphs and asked students both divergent and convergent questions to guide them in understanding how to interpret graphs as well as label them properly. Divergent questions included: "Do you think the interventions worked?," "How can you predict what is going to happen with the data?," and "What are some possible explanations for why the data looks this way?" (TO, Pleasant View, MW). Students shared multiple responses to each of these questions. An example of a convergent question was when Mr. Chapman asked students to tell him what label was missing from the graph.

The discourse among the students and between the teacher and students appeared to have different functions. These various functions included promoting participation, brainstorming ideas, reviewing of prior lessons, introducing new concepts through guided exploration, sharing solutions and problem solving strategies, or reflecting upon students' mistakes. Some of these functions overlapped. For example, Ms. McCoy encouraged participation through brainstorming by asking a divergent question to the students: "What are some things you could reuse?" (TO, Lakeshore, MW). Students shared a variety of items that could be reused. The function of the discourse then shifted toward more direct instruction as the teacher used convergent questions to introduce the concept of line plots. After Ms. McCoy's students had an opportunity to work independently with line plots, she engaged students in discourse to review the material and then encouraged students to reflect upon their mistakes during the independent work activities.

The observers also made note of teachers' emphasis of mathematical terminology during classroom discourse with students. Ms. Hopkins began a lesson from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit by reviewing the terms vertical, horizontal, x-axis, and y-axis (TO, Lakeshore, MW). The students used their arms to demonstrate the meaning of each of the concepts. In another school, Ms. Carter not only reviewed the terms mode, median, mean, hypothesis, research questions, and range before starting her lesson; she also discussed the importance of the terminology. The observer noted, "[Ms. Carter] explained that math has a language of its own and that good mathematicians are able to speak that language" (TO, Stone Mill, NE). The students were able to define and give examples of each term. Students in her class enjoyed "most of the real and challenging vocabulary." One student in particular mentioned he or she "enjoyed learning the names of 9- and 10-sided figures."

Based upon the observations, many students in different treatment classrooms had opportunities to share their solutions and problem solving strategies with their classmates after working collaboratively on different activities from the What Works Mathematics Curricula. In Ms. Cooper's class, students were very excited to demonstrate the angles song in which they described perpendicular lines, intersections and parallel lines (TO, Historic Cove, NE).

A few observers noted the teachers' discourse was at times didactic in nature and restricted students' participation. For example, the observer in Ms. Oliver's class noted:

> In this particular classroom, the instructor demonstrated good classroom control and behavior management skills. However, there was little evidence of an interactive discussion between the teacher and students. Rather, the nature of the conversation was didactic; the teacher would raise a question and then answer it for the students, or the teacher would use declarative statements, providing students with knowledge or skills required to complete the task. (TO, Pleasant View, MW)

Grouping

In 11 of the 14 treatment classroom observations, teachers used grouping as an instructional strategy. Students either worked in homogeneous or heterogeneous pairs or small groups. At times, students worked in small heterogeneous groups and then further separated into pairs. In these instances, teachers typically assigned students to groups, although, in Ms. Nelson's class students were given the choice of selecting their own groups (TO, Stone Mill, NE).

Of the 11 teachers that showed evidence of grouping, 6 were observed grouping students homogeneously to work on the tiered activities provided in the What Works Mathematics Curricula. Students worked collaboratively and were engaged while working on the differentiated activities. In Ms. Morrison's class, students completed either the Babbage or Galileo student pages from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. The observer commented on how Ms. Morrison's students worked in their groups:

> Students responded to each other's questions as the desks were arranged in groups of four. They supported each other with the information required for the graphs, guided students to make sure they were using information from their previous notes' and created their bar graphs. They proceeded to respond to questions in the book about the meaning of the information on the graph and offered accurate conclusions about the data. (TO, Lakeshore, MW)

Many students shared their opinions about working with their peers in groups with the research team observers. A student in Mr. Chapman's class enjoyed group work because if you get stuck you can "ask a partner" (TO, Pleasant View, MW). Ms.

Stewart's students stated, "I like having other people to work with" and "When you work with other people, you get to hear their ideas" (TO, Historic Cove, NE).

Student Challenge and Struggle

The analysis of the observations also revealed that in about half of the classroom observations students experienced challenges and struggles with the What Works Mathematics Curricula. Both the teachers and students shared their perceptions of the challenging nature of the units.

Teachers mentioned that the lessons from the What Works Mathematics Curricula were at times challenging for certain students. Ms. Montgomery explained to the observer that her class struggled the most with the congruent triangle homework from the *Geometry & Measurement for All Shapes & Sizes* Unit (TO, West Valley, MW). She felt it was a very tough lesson that took more time but that it was very valuable. She also noticed that although students struggled with some aspects of the units, they showed huge improvements in writing complete answers in other subjects. In a different school, Ms. Hopkins commented, "The program is good in that students have problem solving opportunities" (TO, Lakeshore, MW). She also mentioned, "I liked that it urges students to justify or explain answers. I only wish my students grasped it and transferred their knowledge. I really needed to scaffold. Great challenge with my high/gifted student."

Some of the students who offered comments mentioned that the units were challenging, difficult, or hard. Ms. Morgan's students felt they were very challenged by the *Awesome Algebra* Unit (TO, Historic Cove, NE). They also pointed out that it was difficult to work with others and the amount of writing involved could be frustrating at times. While some students responded negatively to being challenged, others shared how they overcame these challenges. A student in Mr. Chapman's class described his thoughts when he first tried to write as a "huge web—I got confused" (TO, Pleasant View, MW). But when he went back, he realized he missed a word and then "got it." Still other students responded positively to being challenged. A student in Ms. McCoy's class shared how he or she "liked all books—challenging—makes us smarter each time; figure out new things" (TO, Lakeshore, MW).

Based upon the analysis of the treatment classroom observations, teachers engaged students in taking active roles as mathematicians in real-world learning experiences. Students had opportunities to participate in whole group and small group discourse. In addition, teachers perceived the What Works Mathematics Curricula to be challenging.

Teachers' Focus Groups

Seven out of the 10 school districts from Cohort I participated in the teachers' focus groups to discuss their reactions to implementing the What Works Mathematics Curricula. The total number of focus group participants in each group ranged from 3 to

10 teachers (see Appendix C). From the inductive analysis of the research team's field notes from the focus groups, three major themes emerged representative of the teachers' reactions to the curriculum which included: (a) positive teacher responses, (b) teacher change and learning, and (c) teacher concerns. These major themes will be discussed in the following section.

Positive Teacher Responses

Overall, teachers offered positive responses about their participation in the study. Teachers appreciated the structure of the units and the built-in differentiation components within the lessons. Teachers also reflected upon students' positive reactions to the What Works Mathematics Curricula.

Structure of the What Works Units

Teachers preferred the UConn What Works Mathematics Curricula over their traditional textbook because the units were exploratory in nature—incorporating hands-on learning, real-world connections, and integration of other subject objectives. A teacher explained, "These units are more than pages in the book; they show that math is everywhere around us" (FG 4, SE). Another teacher in this same group explained that her traditional textbook did not "provide real world" (FG 4, SE). A teacher in Focus Group 7 described the units as "investigation oriented—hands on" (FG 7, SE).

In addition, teachers expressed appreciation for the organization and format of the lessons. One participant explained, "The lessons are really well written" (FG 2, NE). Another teacher in this group remarked, "I feel very comfortable with the Geometry. It is so kid friendly. It is very easy to teach" (NE). Some teachers, however, were not as comfortable utilizing the units, stating that the teacher guide was hard to follow due to having to read paragraphs as opposed to bullet points (FG 6, MW).

Embedded Differentiation Components

Many teachers in the focus group discussions expressed positive reactions to the built-in differentiation components of the What Works Mathematics Curricula, such as the grouping guides for the differentiated instructional groups (see Appendix D for an example) and accessible tiered activities. One teacher expressed, "I am delighted to have the opportunity to teach math concepts with the modifications at my fingertips" (FG 2, NE). A teacher from Focus Group 5 corroborated as the researcher noted, "The teachers are happy to have the differentiation done for them already" (MW). The teachers in this group further explained that with their traditional program, tracking down materials made it hard and impractical to differentiate. Another shared, "We don't have to sit and rack our brains; to come up with all the lessons alone would be a nightmare" (FG 4, SE).

Teachers' Perceptions of Student Reactions

Part of the teachers' positive reactions to the study stemmed from their perceptions of the students' reactions, which were also positive. Teachers in Focus Group 6 said, "Students like how the book looks and feels; makes them feel more important" (FG 6, MW).

In particular, many teachers discussed students' responses to the collaborative components of the curriculum. An observer documented, "The teachers appreciate the teamwork component of the math—students first ask their peers any questions. They work together . . . they loved the game . . . they loved sharing their ideas . . . they definitely like the teamwork" (FG 5, MW). Similarly, teachers in Focus Group 4 shared that students "were excited to collaborate with peers during activities" (FG 4, SE). Another teacher noted that, "It made me slow down. It was a think tank of mathematicians. They liked the group activities. They supported each other" (FG 3, NE).

Teachers also reflected on the effect of the challenging nature of the curriculum on student growth and learning. "My students have grown as mathematical thinkers, sharing a rich variety of math strategies, skills, and techniques in a team-learning approach. All learners benefit from the strong activities and lessons in this unit" (FG 5, MW). Students were not always receptive to the challenge; however, the teachers recognized how challenge could develop perseverance. A teacher explained, "Although the 'Thinking Deeply' may 'hurt' their brains, it moves them forward in how they answer and approach questions to a higher level. This carries over into other subjects" (FG 5, MW). This teacher was referring to the Thinking Deeply Questions from the What Works Mathematics Curricula that the students could work on as an added challenge. One observer noted:

> It was the first time they were challenged. Prior to this everything was simple for them. [The] teacher presented this as a positive opportunity and students' reactions changed. She commented that it's not easy but students "stay in the struggle" and are more persistent in their problem solving. (FG 4, SE)

Some teachers made suggestions for possible revisions based on students' reactions to the preassessments. For instance, one of the teachers stated "One of the children said, 'this problem is really hard.' She wrote her feelings about the problem rather than solving the problem" (FG 1, SE). Teachers suggested adding in a few items to build students' confidence.

Teacher Change and Learning

Teachers across the focus groups reflected upon what they had learned about their own instructional practices and how they changed as a result of participating in the study. Some comments were related to differentiating for students: "Differentiation has become easier to do and has occurred more frequently in my classroom because of the strategies embedded in the program" (FG 2, NE). A teacher shared that "Differentiation [is] not as complex as I thought it was . . .” (FG 7, SE). A teacher explained how preassessments were used as a baseline for grouping students flexibly. Another teacher corroborated, “It made me much more aware of grouping and how you need to change these groups. Groups don’t stay the same for long. Flexible, I like that” (FG 2, NE).

Teachers also shared how they learned math content from the units as well as new instructional practices:

> I have learned a lot. I love the fish lesson . . . Deep Sea Café. It kind of validated myself. I wish I would have had that in algebra. I struggled with algebra. Now I understand what it really means. It has allowed me to think more like a child, so I don’t just throw out formulas. (FG 1, SE)

Another teacher in this group shared, “So many times as teachers, we teach the algorithms and this teaches us to show different ways. It allows students to complete problems in different ways” (FG 1, SE). Teachers’ expectations for students also changed:

> I am a controlling person. I was able to let them go a little bit and explore a little bit on themselves. It gave me an opportunity to see the dependence of the students. It was very interesting to see how much they could do by themselves. The low group could do more than what I was expecting. (FG 3, NE)

Another teacher admitted that she didn’t realize one of her students with dyslexia was such a good thinker. The teacher placed her in the top group and she stayed in that group the whole time (FG 5, MW). Interestingly, teachers in Focus Group 7 (SE) shared that they were either “giving students too much credit” or “not giving students quite enough credit.”

Concerns

While the majority of comments were positive, some teachers expressed the challenges of teaching and learning the content in the units and adjusting to a new curriculum. “One teacher said his comfort level with algebra was high, but he still had to sit down with his student teacher to figure out one of the equations in the unit.” Another teacher said “she needed to work very concretely to understand strategies/answers first. Some answers were more difficult to get/understand, especially in algebra” (FG 5, MW). Teachers in Focus Group 6 also found the algebra unit to be challenging.

Teachers also mentioned that the curriculum was time intensive, and they were concerned about ensuring that all the state standards were covered in time for the state tests. For instance, a teacher said that a “teachable moment can’t last more than 5 minutes” (FG 7, SE) due to the pressure of preparing for state tests. Some teachers expressed that the curriculum prepared students for their state tests. A teacher shared, “. . . if we can teach them to explain their thinking, it really prepares them for the [state test]. They should do really well on the open-ended questions” (FG 1, SE).

The teachers who participated in the focus groups were positive in their reactions to implementing the What Works curriculum. They appreciated the embedded differentiation components and preferred the What Works Mathematics Curricula instead of their traditional textbooks. Many teachers indicated how they advanced their learning and changed their instructional practices as a result of participation in the study. While most comments were positive, some teachers expressed concerns such as the time to complete the lessons from the units while simultaneously preparing students for the state exams.

Teachers' Logs

Of the 10 participating school districts and 32 treatment teachers in Cohort I, a total of 16 teachers returned a teachers' log from all three What Works Mathematics Curricula. A total of 25 teachers returned a teachers' log in response to the *Awesome Algebra* Unit and the *Geometry & Measurement for All Shapes & Sizes* Unit, while 19 teachers returned logs for the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. Teachers responded to different closed and open-ended questions for each of the units (see Appendix A). Three major themes emerged from the analysis of the teachers' responses across all three What Works Mathematics Curricula: (a) challenge and learning, (b) motivation and engagement, and (c) responses to differentiated instruction components of the units. Each theme will be discussed below.

Challenge and Learning

Cohort I treatment teachers perceived that students at all levels were challenged as a result of participating in the lessons from the What Works Mathematics Curricula. In the *Awesome Algebra* and *Geometry & Measurement for All Shapes & Sizes* teachers' logs, many of the teachers' comments were positive indicating that the units challenged students at all levels. Ms. Mendoza shared, "They all rose to the challenge and were very engaged" (*Awesome Algebra*, TL, New Horizon, SE). Ms. McKinney (*Geometry & Measurement for All Shapes & Sizes*, TL, Rosewood Park, SE) stated, "All students were appropriately challenged."

Some teachers expressed concerns that special education and low-achieving students struggled with the challenging nature of the lessons. Ms. Cooper shared, "Some of my Special Ed students were lost and the lowest of tiers did *not* meet their needs" (*Awesome Algebra*, TL, Historic Cove, NE). Still, others had mixed reactions. Ms. Morrison wrote, "The students who were able to understand the concept, the vocabulary, and activities seemed to really like the challenges presented with their lessons. Those students who really struggled in math found this also to be a struggle" (*Awesome Algebra*, TL, Lakeshore, MW). Ms. Tanner also had a mixed reaction, "The upper level students were challenged and had to think. The lower students had a very hard time" (*Geometry & Measurement for All Shapes & Sizes*, TL, Vista, SE).

In the *Greening Up With Graphing: Recycle, Reduce, & Reuse* teachers' logs, teachers indicated mixed responses with regards to the extent to which they agreed that their students demonstrated a greater capacity to tackle challenging problems that required analysis and problem solving skills as a result of the unit. For example, Ms. Coleman wrote, "The problems did challenge my students. They were able to get information from the graphs but when it came time to create a graph, it challenged them" (TL, Rosewood Park, SE). Others indicated that students still needed teacher support to work on challenging problems. As Ms. Montgomery shared, "They better understand the steps that go into doing an experiment with one intervention at a time. They also seem to have a better idea of what to do [with] that data once it's done. Developmentally they still need support—but for now they are doing well" (TL, West Valley, NW).

Motivation and Engagement

A second major theme that emerged from the analysis of the teachers' logs was that the students were motivated and engaged during lessons from all three of the What Works Mathematics Curricula. In the *Awesome Algebra* teachers' logs, the majority of teachers indicated that the lesson activities engaged most of the students. Many teachers also shared positive instances of how students at different ability levels were engaged by the lessons. Ms. Carter shared that her students responded "very well on all levels!" (TL, Stone Mill, NE).

In the *Geometry & Measurement for All Shapes & Sizes* teachers' logs, the majority of teachers indicated that the effect of the unit on student motivation in math was either somewhat positive or very positive. Teachers described a variety of components that the students enjoyed. For example, Ms. Bennett wrote:

> This unit provides many hands-on activities that are thoroughly fun and engaging for the students. Also, as with the Algebra unit, the kids really enjoy working in teams/partnerships, and looked forward daily to finding out who/which team they would be working with. (TL, Rosewood Park, SE)

After implementing the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, teachers were asked to compare the unit to their regular curriculum with regard to student interest and growth. Most of the teachers commented that students were motivated and enjoyed the unit. Ms. Parsley stated, "Students had fun doing the lessons" (TL, Vista, SE).

For some students, there was a relationship between the level of difficulty of the lessons and the degree to which they were motivated and engaged. Ms. Chapman explained, "Regardless of ability, the engagement level was high except a few students with low ability were lost (low engagement). The only students that *(sic)* were not engaged were in the lowest math group (Fib.)" (*Awesome Algebra*, TL, Pleasant View, MW).

Responses to Differentiated Instruction Components of the Units

The teachers' responses to some of the built-in differentiated components of the What Works Mathematics Curricula varied depending upon the unit. It should be noted that teachers' choices to comment on different components were a result of the content of the questions in the logs for the three units.

In the *Awesome Algebra* teachers' logs, many of the teachers shared that they used the Talk Moves during their lessons. Ms. LeBlanc explained that the Talk Moves were utilized to "enhance student understanding" and pointed out that "These were helpful in trying to get the students to think more and to think 'outside the box' " (TL, Vista, SE). Ms. Little shared, "I used the agree/disagree quite frequently" and "Depending on how the students responded it led us to a lot of classroom discussion" (TL, Lakeshore, MW).

The *Geometry & Measurement for All Shapes & Sizes* teachers' logs revealed that teachers generally held positive views about differentiated instruction. Ms. Michaels shared her response to differentiated instruction, "I will use it all the time. It helps the kids understand everything on their level and feel more confident on the new more challenging concepts" (TL, Heritage, SE). Some teachers remarked that they had already incorporated elements of differentiated instruction prior to participating in the research study. Ms. Stewart stated, "I often use a 3 tiered approach to instruction so this fit nicely with my classroom" (TL, Historic Cove, NE). Teachers also responded positively to the hands-on components of the *Geometry & Measurement for All Shapes & Sizes* Unit. This was reflected by Mr. Underwoods's statement, "The most successful experiences were when the students were able to create things" (TL, Crowder Point, NE).

Teachers' reactions to the differentiated instruction components embedded within the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit were generally positive as indicated in the teachers' logs. The majority of teachers who returned the teachers' log for this unit provided positive responses about how the unit addressed students' varied learning styles. Ms. Oliver expressed, "It provided many opportunities for hands-on learning" (TL, Pleasant View, MW). Ms. Carter was also positive stating, "The differentiation in each lesson enabled all to be successful and the 'hands-on' was very helpful" (TL, Stone Mill, NE). Some teachers expressed a concern that the units were time consuming and that additional differentiation was needed to support their students' needs. Ms. Hansen shared, "My students all scored poorly on the pretest so remained in the same group. They were quickly frustrated by the lack of scaffolding in harder sections" (TL, Lakeshore, MW).

When asked what knowledge, skills, or strategies they would apply from these units to other units of study, Ms. Coleman said, "Throughout this unit, I have acquired several strategies (pre/post test, differentiated instruction, groupings) that I use throughout all disciplines" (*Greening Up With Graphing: Recycle, Reduce, & Reuse*, TL, Rosewood Park, SE). Other teachers mentioned that they would make real-world connections in other subject areas, as Ms. Clayton stated, "Integration of other subject areas [such as] science with math—students were very engaged in these lessons and activities" (TL, Crowder Point, NE).

Cohort I Discussion

The purpose of this study was to investigate how teachers and administrators respond to their access to pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis. Overall, both teachers and administrators responded positively to participating in the study. Teachers preferred the investigative design of the What Works Mathematics Curricula to their traditional textbooks. They appreciated the embedded differentiation components, which allowed them to match the curriculum to students' readiness levels, and thus, offer more appropriate challenges to all students. Many teachers also perceived that students were engaged as practicing mathematicians and enjoyed working collaboratively within groups. In addition, administrators' perceptions of the teachers' responses to the What Works Mathematics Curricula were positive, noting their enthusiasm and willingness to try new approaches to teaching mathematics.

Teachers Challenged Students

The design of the What Works Mathematics Curricula intentionally integrated conceptual understandings, problem-solving tasks, opportunities for communication of mathematical ideas, and collaborative endeavors within the investigations of many of the lessons, as emphasized in the NCTM (2000) standards, to advance the mathematical development of students at all levels of readiness. From the analysis of the teachers' logs and classroom observations, teachers were able to implement the curriculum so that students at all levels were challenged and engaged by the What Works Mathematics Curricula. For example, based upon the analysis of the treatment classroom observations, teachers engaged their students in taking active roles as mathematicians in real-world learning experiences. Students worked on differentiated tiered assignments commensurate with their readiness levels. These findings are especially promising in light of previous research indicating that elementary teachers often only make minor modifications to differentiate for higher achieving students (Archambault et al., 1993).

Classroom observations also revealed that teachers emphasized mathematical terminology to promote more effective mathematical communication and utilized interactive discourse between themselves and students that went beyond simple question and answer sessions. This is promising as the units were designed with Kaplan's Depth and Complexity (2009) in mind, which consists of strategies that promote questioning and problem solving. Furthermore, according to the Communication Standards for School Mathematics, all students are to:

- Organize and consolidate their mathematical thinking through communication;

- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely. (NCTM, 2000)

Struggle Versus Challenge

While teachers and administrators perceived that all students were challenged by the What Works Mathematics Curricula, there did appear to be a connection between student readiness levels and the degree of engagement during the lessons. Higher achieving students appeared to be more enthusiastic about working on challenging tasks in comparison to other students. According to some of the teachers' comments in the teachers' logs, teachers shared concerns that their low students and special education students struggled with the lessons and needed further teacher support. Teachers' concerns about students' academic progress support Diezmann and Watters' (2011) perspective about recognizing the delicate nature of determining students' readiness levels, matching the tasks to student capability, and keeping students motivated to complete challenging tasks. Keeping students motivated to "stay in the struggle" while working on challenging tasks certainly presents a challenge for teachers.

Concerns Over State Standards

Another concern expressed by both teachers and administrators was related to preparing students to meet the objectives on the statewide tests in conjunction with implementing a new curriculum. The pressure to perform well on high-stakes assessments is a reality in most schools that must be addressed especially when implementing new curricular changes. Administrators in this study noted that teachers worked collaboratively with other teachers to ensure that state standards were met in conjunction with those of the What Works Mathematics Curricula. This also points to the need for a supportive administration when schools implement new curriculum and instructional initiatives, especially during the first year of a research intervention.

Teacher Learning and Change

As the What Works Mathematics Curricula were intentionally designed as educative curricula (Davis & Krajcik, 2005), or written in a way to support and develop teachers' pedagogical content knowledge by focusing on research-based practices to promote conceptual understandings, the responses from the teachers and administrators demonstrate the effectiveness of this approach. Analysis of the administrators' interviews and teacher focus groups revealed that as a result of participating in the study teachers were open to changing their instructional practices, learning and applying mathematical content knowledge, and implementing differentiated instructional practices. This is promising in light of research indicating that student achievement is attributable to teacher effectiveness and knowledge (Hill et al., 2005; Nye et al., 2004). Yet, pre-service elementary teachers are not necessarily prepared adequately for teaching conceptual understandings of mathematics (Ball, 1990; Sriraman, 2003).

It is also important to point out that in general teachers in the current study responded positively to the What Works Mathematics Curricula and were willing and open to learning. As found in Remillard's (2000) case studies of teachers implementing a new mathematics textbook, teachers who participated in implementing the What Works Mathematics Curricula adjusted their classroom practices. The outcome, though, may not have been the same for more resistant teachers. Another important finding was teachers' expectations of their students also changed as a result of students' engagement with the rigorous units. This reveals that when students are provided with an appropriately challenging curricula, they can show teachers what they are really capable of accomplishing.

CHAPTER 6: Cohort II Quantitative Results

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School-level Data

During the 2009-2010 school year, 17 schools representing five states participated in the second year of a study to determine the impact of implementing three pre-differentiated and enriched curricular units with grade 3 mathematics students of all ability levels. Fourteen of these 17 Cohort II schools had also participated the previous year in the pilot study of the curriculum intervention. Four new schools were recruited to join Cohort II. The 17 Cohort II schools were assigned to treatment or control status by school. Eight of the 17 participating schools were assigned to receive the curriculum intervention for 16 weeks of the school year, while the other nine schools were assigned to be in the control condition, implementing their regular "business as usual" grade 3 mathematics curricula for the entire school year.

A descriptive table of these schools' enrollments, proportions of underrepresented minority students, proportions of students eligible for free/reduced-priced meals through the NSLP, and student to teacher ratios during the 2009-2010 school year is provided in Table 6.1. The term underrepresented minority refers to the research literature (see Gentry et al., 2008 for a summary), which has documented the underrepresentation of African Americans, Hispanic/Latino students, and Native American students in gifted education programs in the United States. The National Research Center on the Gifted and Talented has focused its research efforts on providing enriched education opportunities for students from these groups to promote both excellence and equity for students of all races and ethnicities.

The schools participating in Cohort II of the What Works in Gifted Education Mathematics Study had a mean enrollment of 394.9 with a standard deviation of 146.2, and a median enrollment of 394. These schools' mean percentage of underrepresented minority students was 15.4 with a standard deviation of 10.1, and a median percentage of 15.1. This is approximately half the percentage of students identified as being in racial groups other than White and Asian in the nationally representative American Community Survey of 2006-2010 (NCES, 2012), which estimates this number as approximately 30% of American public school children.

In terms of socioeconomic status, the mean percentage of students from Cohort II schools eligible for free/reduced-priced meals was 28.6 with a standard deviation of 19.6, and a median percentage of 28.4. Nationally, approximately 62% of students meet eligibility criteria for free/reduced lunch status during the 2008-2009 school year (National School Lunch Program, 2012). Therefore the Cohort II sample of schools contained more students of higher socioeconomic backgrounds than in the nation overall. Additionally, the mean student to teacher ratio for the Cohort II schools was 16.6 to one with a standard deviation of 2.0 and a median ratio of 16.1 to one.

Table 6.1
Demographics[†] *for Cohort II Schools Participating in the What Works in Gifted Education Mathematics Study* (*N*=17)

School Pseudonym	Total Enrollment	White/Asian (%)	Other Ethnicities (%)	Free/reduced lunch (%)	Student Teacher Ratio
Acadia	356	96.9	3.1	36.2	16.1
Beaumont	411	90.8	9.2	4.9	14.7
Canyon Ridge	377	96.3	3.7	9.5	20.8
Cove	278	75.2	24.8	28.4	14.9
Crowder Point	430	84.4	15.6	11.4	15.0
Heritage	167	89.8	10.2	59.9	19.8
Historic Cove	444	83.1	16.9	12.8	16.2
Lakeshore	469	84.9	15.1	10.2	15.4
Lighthouse	394	83.3	16.7	14.7	15.5
North Point	413	94.0	6.0	30.0	19.8
Pleasant View	281	91.5	8.5	49.1	18.6
Riverside	311	73.6	26.4	27.3	16.4
Rosewood Park	760	81.1	18.9	37.2	15.5
Sidewind	675	89.2	10.8	5.3	17.2
Stone Mill	401	72.0	28.0	29.2	14.6
West Valley	232	93.1	6.9	56.5	15.4
Woodbridge	314	58.9	41.1	64.3	16.9

† Source: School Digger website: http://www.schooldigger.com (2009-2010 data).

Teacher- and Classroom-level Data

Teacher Demographics

The teachers in the sample of schools comprising Cohort II totaled 45, with 20 implementing the curriculum in the treatment condition and 25 implementing the regular mathematics curriculum in the control condition. Demographics information describing these teacher participants was collected and is summarized in Table 6.2. The treatment teachers in Cohort II were almost entirely female, as were the control teachers. Only one Cohort II treatment teacher identified as belonging to a race other than White, while all the control teachers who responded identified as being White. Both treatment and control teachers for Cohort II were generally highly experienced, with approximately half of each possessing 15 years or more total teaching experience. Two treatment and one control teacher had fewer than 5 years prior teaching experience. In terms of grade 3 teaching experience, the Cohort II treatment teachers had a fairly uniform distribution of grade 3 teaching experience, while the control teachers were less experienced with approximately three-quarters having less than 10 years experience teaching grade 3. Similar percentages of the Cohort II treatment and control teachers held a Master's degree or Sixth Year certificate, with a substantial minority holding Bachelor's degrees for both conditions.

Table 6.2
Cohort II Treatment and Control Group Teacher Demographics (*N*=45)

	Treatment Group (*n*=20) Frequency (%)	Control Group (*n*=25) Frequency (%)
Gender		
Male	2 (10)	2 (8)
Female	18 (90)	23 (92)
Ethnicity		
Black/African American	1 (5)	0
White	19 (95)	24 (96)
No Response	0	1 (4)
Total Years Teaching Experience		
0-4	2 (10)	1 (4)
5-9	3 (15)	5 (20)
10-14	6 (30)	5 (20)
15+	9 (45)	13 (52)
No Response	0	1 (4)
Years Teaching Grade Three		
0-4	5 (25)	10 (40)
5-9	5 (25)	9 (36)
10-14	6 (30)	3 (12)
15+	4 (20)	2 (8)
No Response	0	1 (4)
Educational Background		
BA/BS	4 (20)	4 (16)
MA/MS	13 (65)	18 (72)
Sixth Year/Ed. Specialist	3 (15)	2 (8)
Ph.D./Ed.D.	0	0
Professional Diploma	0	0
No Response	0	1 (4)

Teachers' Log Data

Teachers who participated in the What Works in Gifted Education Mathematics Study in 2009-2010 completed teachers' logs at the end of each curricular unit to record their responses to the implementation. These logs consisted mainly of close-ended Likert-type questions aimed at determining teachers' perceptions of how the intent of the curricula matched with the reality of how it functioned in their classrooms. Three additional open-ended questions gauged teacher reactions and positives and negative aspects of the implementation. A summary of the Cohort II teachers' responses to each of the units is found in Table 6.3. The response options presented were 1) Strongly Disagree, 2) Disagree, 3) Agree, and 4) Strongly Agree. Teachers' agreement rates were generally quite high for all items, with no items having a mean rating less than 3, or basic agreement.

Fidelity of Implementation

Researchers from the University of Connecticut conducted classroom observations to determine the degree to which teachers and students were interacting with the curricula in the manner intended by its authors. The research team observed 19 of the 20 treatment classrooms for Cohort II. Table 6.4 describes the high levels of fidelity of implementation that observers found in nearly all Cohort II treatment classrooms. Although the size of the research team precluded the triangulation of classroom observations, Cohort II classrooms appeared to be fully embracing the underlying instructional philosophies, strategies, and techniques that were included in the three mathematics curricular units and accompanying professional development sessions.

Table 6.3
Summary of Cohort II Teachers' Responses to Curricular Units

Unit	Item	Mean	Standard Deviation
Awesome Algebra	The preassessment helped me place students into readiness groups.	4.35	0.59
	I have noticed a positive difference in my students' writing abilities in math and other subjects because of this curriculum.	3.80	0.70
	The lessons in *Awesome Algebra* challenged all of my students.	4.45	0.51
	I found the additional resources (CDs, DVDs, and website) very helpful.	3.40	0.75
	My students seem more excited about math with this curriculum.	4.15	0.75
	The ability level of my students was higher than I had expected.	3.75	0.85
	My students are now better at discussing mathematical concepts with their peers and adults.	4.15	0.59
	Implementing this curriculum has improved my abilities to differentiate.	4.00	0.65
	The culminating project was helpful to gauge what my students had learned in *Awesome Algebra*.	3.45	0.69
	The teacher's manual was easy to comprehend and implement.	3.65	1.18
	I enjoyed teaching this unit.	4.20	0.70
Geometry & Measurement for all Shapes and Sizes	The preassessment helped me place students in readiness groups.	4.15	0.75
	My students looked forward to math class when we were working on this unit.	4.20	0.77
	This unit challenged all of my students.	4.30	0.47
	My students were engaged with the lesson in this unit.	4.20	0.41
	This unit helped me think about some geometry and measurement concepts in a new or unique way.	4.35	0.81

Table 6.3 (continued)
Summary of Cohort II Teachers' Responses to Curricular Units

Unit	Item	Mean	Standard Deviation
Geometry & Measurement for all Shapes and Sizes (continued)	I witnessed my students making considerable conceptual growth throughout this unit.	4.15	0.59
	My students benefitted from working with other students in their assigned groups.	4.35	0.49
	The teacher's manual was easy to comprehend and implement.	3.68	1.06
	My students were able to demonstrate their learning through the culminating project.	3.85	0.88
	I enjoyed teaching this unit.	4.25	0.64
Greening up With Graphing: Reduce, Reuse, & Recycle	My students' ability to communicate mathematical concepts in their written work improved as a result of this unit.	3.90	0.79
	My students have demonstrated a greater capacity to approach and tackle challenging problems using analysis and problem solving skills as a result of this unit.	3.75	0.91
	This unit addressed my students' varied learning styles.	4.05	0.76
	My students were able to understand and answer questions in the Student Journal.	4.05	0.83
	My students are better able to draw conclusions from data as a result of this unit.	4.10	0.55
	This unit added depth and complexity to the way graphing is usually taught in our third grade curriculum.	4.45	0.76
	My students exhibit a greater command and use of mathematical language in small group and whole class discussions as a result of this unit.	4.10	0.72
	I enjoyed teaching this unit.	3.95	1.10
	The teacher's manual was easy to comprehend and implement.	4.30	0.66

Table 6.4
Summary of Fidelity of Implementation Results for Cohort II Treatment Classrooms (n=19 observed classrooms)

Fidelity Item		Number Observed (Percent Observed)
1. Students understand the "Big Idea" of the lesson		
To a great extent	(4)	13 (68.4)
	(3)	5 (26.3)
	(2)	0
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	0
2. The materials provided for lesson implementation are used appropriately.		
To a great extent	(4)	17 (89.5)
	(3)	1 (5.3)
	(2)	0
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	0
3. Appropriate mathematical language is used.		
To a great extent	(4)	17 (89.5)
	(3)	0
	(2)	1 (5.3)
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	0
4. Students have a clear understanding of directions for the lesson activities.		
To a great extent	(4)	17 (89.5)
	(3)	1 (5.3)
	(2)	1 (5.3)
To a lesser extent	(1)	0
Not applicable	(NA)	0
5. The students and/or teacher make(s) connections to prior concepts.		
To a great extent	(4)	14 (73.7)
	(3)	3 (15.8)
	(2)	1 (5.3)
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	0

Table 6.4 (continued)
Summary of Fidelity of Implementation Results for Cohort II Treatment Classrooms (n=19 observed classrooms)

Fidelity Item		Number Observed (Percent Observed)
6. Student learning is assessed through observation, listening, and/or information gathering.		
To a great extent	(4)	15 (78.9)
	(3)	3 (15.8)
	(2)	0
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	0
7. Students are grouped according to suggested levels of differentiation.		
To a great extent	(4)	11 (57.9)
	(3)	1 (5.3)
	(2)	0
To a lesser extent	(1)	2 (10.5)
Not applicable	(NA)	5 (26.3)
8. Students are presented with challenging content.		
To a great extent	(4)	16 (84.2)
	(3)	1 (5.3)
	(2)	1 (5.3)
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	0
9. Discourse (whole group, small group, peer) about mathematical problems occurs.		
To a great extent	(4)	15 (78.9)
	(3)	2 (10.5)
	(2)	0
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	1 (5.3)
10. Students are invited to find multiple strategies or solutions to the mathematical problem.		
To a great extent	(4)	14 (73.7)
	(3)	3 (15.8)
	(2)	0
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	1 (5.3)

Table 6.4 (continued)
Summary of Fidelity of Implementation Results for Cohort II Treatment Classrooms (n=19 observed classrooms)

Fidelity Item		Number Observed (Percent Observed)
11. Students are encouraged to explain or justify their responses.		
To a great extent	(4)	17 (89.5)
	(3)	0
	(2)	1 (5.3)
To a lesser extent	(1)	1 (5.3)
Not applicable	(NA)	0
12. Students are engaged in the lesson.		
To a great extent	(4)	16 (84.2)
	(3)	2 (10.5)
	(2)	1 (5.3)
To a lesser extent	(1)	0
Not applicable	(NA)	0
13. Overall fidelity to the curriculum.		
Greater fidelity	(4)	16 (84.2)
	(3)	0
	(2)	1 (5.3)
Lesser fidelity	(1)	1 (5.3)
No Rating		1 (5.3)

Student-level Data

A total of 844 students from Cohort II of the What Works in Gifted Education Mathematics Study had valid posttest scores and were included in the quantitative analysis. Of these students, 463 were from schools assigned to the control condition and 381 were from schools assigned to the treatment condition. Table 6.5 contains a summary of these students' gender and ethnicity during the 2009-2010 school year. This sample of students contained a slightly larger percentage of males (approximately 53%) than females (approximately 46%). Similar to the school demographic information presented above, a large majority of Cohort II study participants (approximately 83%) identified as either White or Asian.

Table 6.5
Cohort II Treatment and Control Group Student Demographics (*N*=844)

Characteristic	Treatment Group (*n*=381) Frequency (%)	Control Group (*n*=463) Frequency (%)	Total (*N*=844)
Gender			
Male	217 (57.0)	228 (49.2)	445
Female	164 (43.0)	231 (49.9)	395
Gender not indicated	0	4 (0.9)	4
Total	381	463	844
Ethnicity			
American Indian/Alaskan Native	11 (2.9)	3 (0.6)	14
Asian/Hawaiian/Pacific Islander	24 (6.3)	26 (5.6)	50
Black/African American	24 (6.3)	22 (4.8)	46
Hispanic/Latino(a)	19 (5.0)	24 (5.2)	43
White	275 (72.2)	373 (80.6)	648
Multiple Ethnicities/Other	25 (6.6)	11 (2.4)	36
Ethnicity not indicated	3 (0.8)	4 (0.9)	7
Total	381	463	844

When conducting randomized experiments, it is important to verify that treatment and control participants have comparable characteristics prior to the intervention. This reduces selection bias concerns for causal inferences. Although random assignment should minimize pretest differences between the groups, empirical pretest data can also support this equivalence. Table 6.6 contains information on differences between treatment and control students for a number of pretest ability and achievement assessments, as well as teacher rating scales and demographic information.

Cohort II treatment and control students differed significantly on average age at pretest, with the treatment students somewhat older than control students. Control students scored higher than treatment students on the verbal subtest of the CogAT. For the pretest achievement measures, control students who took the MAP also had a significantly higher average mathematics pretest score than their treatment counterparts who also took the MAP. However, this achievement test was only taken by about one-fifth of the Cohort II students, and there were no significant differences among the achievement pretest scores for the remainder of the treatment and control students.

Gender distribution was also different between treatment and control students, with males disproportionately high in the treatment group and gender split more evenly in the control group. Students from the treatment condition also attended schools with significantly higher percentages of students eligible for free/reduced-priced meals than the schools control students attended. Finally, students in the control group were more likely to have been nominated as having “high learning potential” by their second grade teachers than were treatment students. The only variable that was significantly different between treatment and control students at the $p<.01$ level was the school-level percentage of students eligible for free/reduced-priced meals, which was influenced by the small sample size of schools (N=17) participating in Cohort II.

Table 6.6
Cohort II Students' Group Equivalence on Pretest Measures for Treatment and Control Groups

Variable	Condition	*M*	*SD*	*t*(*df*)	95% confidence interval for difference: Lower bound	95% confidence interval for difference: Upper bound
Student age at pretest (in months)*	Control	96.84	4.23	-2.24(738)	-1.39	-0.09
	Treatment	97.58	4.73			
CogAT age score (verbal)*	Control	106.19	12.90	2.07(722)	0.10	3.85
	Treatment	104.21	12.69			
CogAT age score (quant)	Control	106.66	14.11	1.12(715)	-0.88	3.24
	Treatment	105.49	13.82			
CogAT age score (nonverbal)	Control	106.77	13.62	1.26(719)	-0.72	3.30
	Treatment	105.48	13.77			
CogAT age score (composite)	Control	106.56	11.92	1.65(711)	-0.28	3.19
	Treatment	105.10	11.51			
ITBS Level 8 Math	Control	177.76	20.47	0.72(559)	-2.09	4.51
	Treatment	176.56	19.02			
MAP RIT Mathematics*	Control	198.63	9.58	2.37(164)	0.60	6.62
	Treatment	195.01	9.94			
SRBCSS Learning scale	Control	55.10	7.56	-0.67(171)	-3.00	1.47
	Treatment	55.87	6.73			
SRBCSS Motivation scale	Control	55.60	9.00	-0.75(170)	-3.74	1.67
	Treatment	56.64	8.28			
SRBCSS Creativity scale	Control	43.54	6.76	-0.11(171)	-2.31	2.07
	Treatment	43.66	7.65			
Gender (dummy code)*	Control	0.50	0.50	-2.11(838)	-0.14	-0.01
	Treatment	0.57	0.50			
Percentage non-White/ non-Asian students	Control	14.22	10.29	-0.40(842)	-1.41	0.94
	Treatment	14.46	3.72			
Percentage FRL† eligibility**	Control	19.72	16.52	-5.99(842)	-9.17	-4.64
	Treatment	26.63	16.85			
Proportion of students receiving teacher nominations*	Control	0.23	0.42	2.00(842)	0.00	0.11
	Treatment	0.17	0.38			

† FRL indicates free/reduced priced school meals
* $p<.05$.
** $p<.01$.

Table 6.7 contains the simple bivariate correlations between seven pairs of student-level and contextual variables for students in Cohort II. Pretest achievement, pretest ability, and posttest achievement scores are highly inter-correlated. For the aggregate contextual variables, school average ability and school average achievement were also quite highly correlated. Percentage of underrepresented minorities and percentage of students eligible for free/reduced-priced lunches were also somewhat correlated. Higher school-level percentages of underrepresented minority students and lower-SES students were substantially negatively correlated with school aggregate measures of ability and achievement, but less so with student-level ability and achievement measures. All of the variables were significantly correlated.

Table 6.7
Cohort II Students' Summary of Bivariate Correlations Between Variables in the What Works in Gifted Education Mathematics Study† *Scores*

Variable	Pretest Achievement Score	Pretest Ability Score	School % Non-White and Non-Asian	School % FRL Eligibility	School Mean Pretest Score	School Mean Ability Score	Posttest Achievement Score
Pretest Achievement Score	1	0.69	-0.10	-0.19	0.28	0.22	0.69
Pretest Ability Score		1	-0.15	-0.19	0.26	0.32	0.65
School % Non-White and Non-Asian			1	0.31	-0.30	-0.44	-0.13
School % FRL Eligibility				1	-0.68	-0.59	-0.22
School Mean Pre Ach. Score					1	0.80	0.28
School Mean Ability Score						1	0.24
Posttest Achievement Score							1

† All correlations are significant at the $p<.001$ level.

In addition to the nationally normed standardized achievement tests that were taken by both treatment and control students, treatment students completed researcher-developed unit tests before and after instruction of each of the three curricular units.

Table 6.8 presents pretest and posttest scores that have been disaggregated based on three categories of school-level socioeconomic status to enable comparisons of different groups' scores on these measures of algebra, geometry and measurement, and graphing and data analysis achievement. Student gains from pretest to posttest are also presented for the three school-SES categories, as well as Cohen's *d* effect size measures for these gain scores.

Table 6.8
Pretest Achievement, Unit Test Scores, and Difference Scores of Students in Cohort II Schools Under Different Categories of Student Eligibility for Free/Reduced Lunch (FRL)

Measure	Schools With<15% Student FRL Eligibility	Schools With 15%-40% Student FRL Eligibility	Schools With>40% Student FRL Eligibility	All Schools
National Percentile Rank on Achievement Pretest				
N	213	256	92	561
M	66.16	60.04	52.90	61.19
SD	26.07	27.28	28.78	27.42
Algebra Pretest				
N	180	116	74	370
M	*8.49*	*5.24*	*5.34*	*6.84*
SD	4.87	4.42	4.04	4.84
Algebra Posttest				
N	183	116	73	372
M	*15*.39	15.03	15.18	15.23
SD	5.47	5.79	5.74	5.61
Geometry and Measurement Pretest				
N	181	115	75	371
M	11.89	11.57	12.03	11.82
SD	4.64	4.52	4.57	4.58
Geometry and Measurement Posttest				
N	183	114	77	374
M	22.68	22.65	24.49	23.04
SD	4.62	4.48	3.86	4.48
Graphing and Data Analysis Pretest				
N	181	115	76	372
M	5.12	5.32	4.28	5.01
SD	2.80	2.44	2.11	2.59

Table 6.8 (continued)
Pretest Achievement, Unit Test Scores, and Difference Scores of Students in Cohort II Schools Under Different Categories of Student Eligibility for Free/Reduced Lunch (FRL)

Measure	Schools With<15% Student FRL Eligibility	Schools With 15%-40% Student FRL Eligibility	Schools With>40% Student FRL Eligibility	All Schools
Graphing and Data Analysis				
Posttest				
N	179	114	79	372
M	8.55	10.25	11.04	9.60
SD	3.09	3.06	2.94	3.21
Algebra Difference Score				
N	180	116	71	367
M	6.93	9.78	9.63	8.35
SD	4.11	4.83	5.51	4.84
Cohen's *d*	1.34	1.92	1.97	1.60
Geometry/Measurement				
Difference Score				
N	181	114	75	370
M	10.78	11.05	12.42	11.19
SD	4.18	4.93	4.74	4.57
Cohen's *d*	2.33	2.46	2.95	2.47
Graphing/Data Analysis				
Difference Score				
N	178	113	76	367
M	3.40	4.92	6.73	4.56
SD	2.60	2.76	2.76	2.97
Cohen's *d*	1.15	1.79	2.67	1.57

Cohort II: Quantitative Results

For Cohort II of the What Works in Gifted Education Mathematics Study, the research team examined the impact of the pre-differentiated and enriched curricula on student achievement using a cluster randomized design. Schools were randomly assigned to treatment or control (*N*=17) and a total of 45 teacher and 846 students participated.

The Model

Two-level hierarchical models provided an appropriate method to analyze the data as students were nested within schools. Students' post achievement in mathematics on the ITBS was predicted by both student- and school-level variables.

At the student level, the model controlled for student initial achievement in mathematics and examined the impact of a student being in the top 10% in ability within a school on mathematics achievement. Student initial achievement in mathematics was measured differently at some of the schools. The majority of the schools used the ITBS (n=16). One of the schools used the MAP. To retain this school all of the students' scores were rescaled onto the same achievement scale. To do this, national percentile ranks for each student on the specific achievement test that was given at the student's school were combined as one variable. Because percentile scores are not on an interval scale, the national percentile ranks were converted into a *z* score by using an inverse cumulative density function based on a normal distribution. To determine the highest ability students at each school, the mean ability of all of the students in a particular school and the distribution of ability within a school was calculated. Students performing within the top 10% of students at his or her school on the CogAT composite were defined as gifted.

At the school level, the statistical models controlled for initial school mean mathematics achievement and examined whether treatment impacted student post-achievement in mathematics. The initial school achievement variable was the mean *z* score of student achievement at each school. Treatment and gifted status indicators were dummy coded (0-control, 1-treatment). Given that the student and school achievement variables were already in *z*-score units, they were entered into the models uncentered.

The Results

First, the model predicted scores for students who were of average initial achievement in mathematics and were not defined as gifted. For students who attended control schools that had average initial aggregate achievement, the predicted posttest achievement score was 188.22 (γ_{00}=188.22, SE=1.96, p<.001). Students from schools assigned to treatment did not have statistically different outcomes from students in control schools after accounting for student and school pretest math achievement (γ_{01}=1.83, SE=1.74, p=.31). However, schools that had higher mean pretest math scores also had statistically significantly higher posttest mathematics scores (γ_{02}=14.06, SE=3.45, p<.001), even after controlling for students' pretest scores. After controlling for students' pretest math achievement, schools with initial mathematics achievement scores that were one standard deviation above the average achievement were expected to score approximately 14 points higher on the post-ITBS.

Next, the results showed the impact that initial mathematics achievement had on the post-ITBS. Students who were one standard deviation above the mean in initial mathematics achievement were expected to score approximately 15 points higher (γ_{10}=15.04, SE=0.74, p<.001) on the post-achievement test, after controlling for all other variables in the model. Neither treatment nor school pre-achievement level moderated the relationship between pretest mathematics achievement and posttest mathematics achievement in Cohort II.

Finally, the model estimated the effect of being in the top 10% in ability in a school on posttest math achievement, after controlling for pretest math achievement and school pretest math achievement. Those students who were classified as gifted (who were in the top 10% of their school in terms of their ability) were expected to score about 5 points higher (γ_{200}=5.44, SE=2.43, p=.026) on the post-ITBS, after controlling for all other variables in the model.

The results would seem to indicate that the treatment had no effect. However, there were a couple of complications that may have masked the impact. First, 10% of the students received the highest score possible on the post-ITBS. Thus, the true growth of these students was likely not measured accurately and introduced measurement error into the model. Second, the skills that were taught in the treatment were not reflected on the post-ITBS. Many of these skills were above grade level and not captured by the grade level ITBS. Both of the problems could be solved by administering above-grade level post assessments.

National Assessment of Educational Progress Results

The National Assessment of Educational Progress (NAEP) has provided a national benchmark of students' academic achievement in the United States for over 40 years. As part of participation in the second year of the NRC/GT mathematics study, grade 3 students who received the pre-differentiated and enriched curricula were administered 14 mathematics items from the NAEP. The items were chosen for their similarity to the conceptual knowledge contained in the University of Connecticut's curricula.

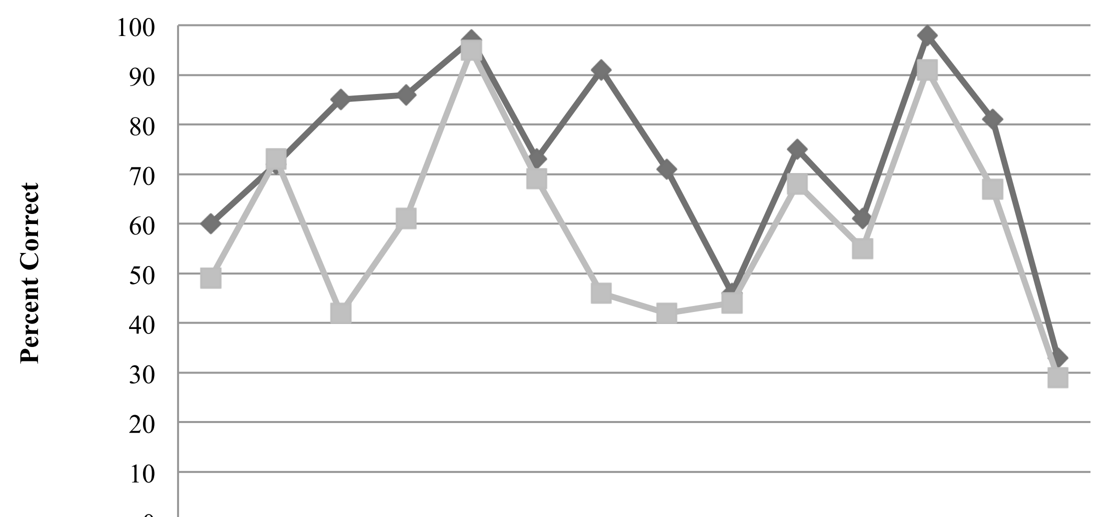
Although students involved in the study were in grade 3 during the curriculum implementation, the items on which they were tested were drawn from the grade 4 NAEP item bank. "Above-grade level" testing targets student achievement above the levels that could be measured by "at-grade level" items. For each item tested, the percentage of students who responded correctly to the item is shown compared to the national grade 4 percentages. With the exception of items 10 and 13, more than 50% of the grade 3 students mastered the grade 4 items (see Figure 6.1). These results represent the 393 treatment students who participated in the second year of the study.

When the content of the items most closely matched the units, a higher percentage of students responded correctly, as these concepts were covered in multiple ways throughout the units. This provides further evidence of how students were able to learn when presented with challenging and differentiated curricula.

Mathematics Curriculum Unit Test Results

The What Works in Gifted Education Mathematics Study provided challenging pre-differentiated and enriched curricula based on the well-known models of gifted and talented education by Sandra N. Kaplan, Carol A. Tomlinson, Joseph S. Renzulli and Sally M. Reis. Regular education grade 3 classrooms participated in three units entitled

Awesome Algebra, Geometry & Measurement for All Shapes & Sizes, and *Greening Up With Graphing: Recycle, Reduce, & Reuse* during the 2009-2010 school year. Each unit was developed in line with the NCTM's (2000) Principles and Standards for School Mathematics, which provides guidance for educational decision makers in grades Pre-K through 12. The development of these units assumes that students in grade 3 possess the prior knowledge indicated by the standards for grades Pre-K-2 and extends this knowledge by focusing on the standards for grades 3-5. In addition, the authors of these units relied on the NCTM focal points, which provide additional specificity of content for grade 3.



	Q1	Q2	Q3	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
Study	60	72	85	86	97	73	91	71	46	75	61	98	81	33
Nation	49	73	42	61	95	69	46	42	44	68	55	91	67	29

Graphing & Data Analysis Geometry & Measurement Algebra

Question 4 was not used in the analysis due to problems with the item.

Figure 6.1. Comparison of percentage of correct responses on NAEP items by Cohort II treatment students and national sample.

Each unit had challenging content and a unit test with a high ceiling so all students could demonstrate growth. Despite the challenging content, across the sites all students showed significant gains from pretest to posttest in all three units (see Figure 6.2). These results represent the following number of students who participated in the second year of the study, Algebra (*n*=392); Geometry & Measurement (*n*=393); and Graphing & Data Analysis (*n*=387). These results demonstrate that all students can learn when presented with challenging and pre-differentiated and enriched curricula.

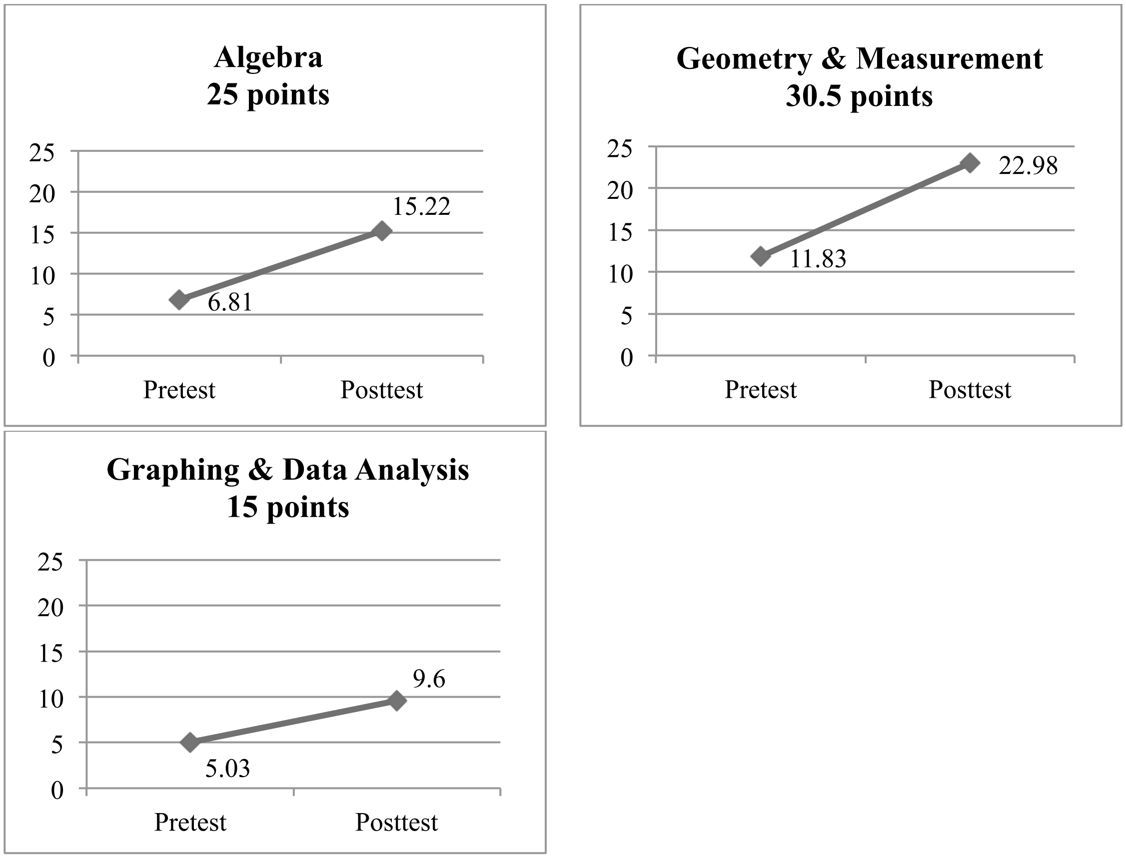


Figure 6.2. Gains on unit tests across all sites.

Since each test had a different total score, effect sizes provided a method for comparing change across different measures. Researchers hold different opinions about the appropriateness of specific formulas to calculate effect sizes for paired *t*-tests. Cohen's *d* using the pooled standard deviation as the standardizer was selected for the unit test analyses, which is the most conservative approach to computing effect sizes with repeated measures data. Cohen (1988) defined a Cohen's *d* of 0.2 as small, 0.5 as medium, and 0.8 or above as large. The growth of students on each test was considerably greater than Cohen's definition of large (see Figure 6.3). This further demonstrated that all students can make practically significant gains when presented with challenging pre-differentiated and enriched curricula based on gifted and talented pedagogy.

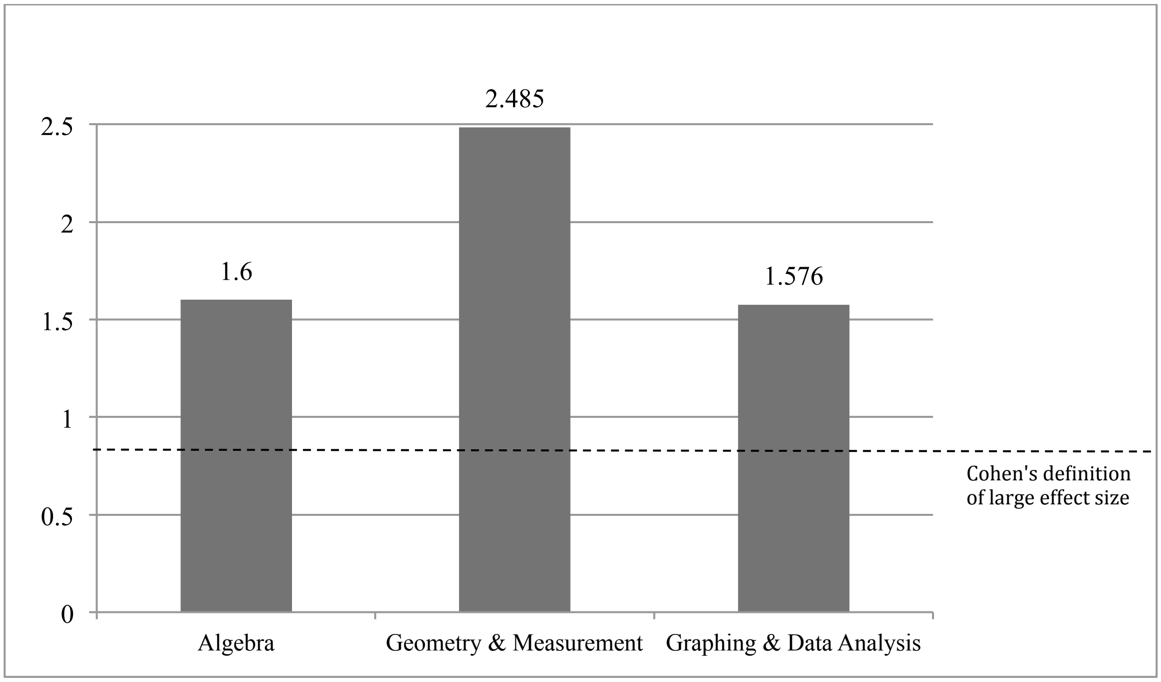


Figure 6.3. Effect sizes of Cohort II student gains.

Pretest and posttest unit data indicated that students successfully learned and applied the challenging content and concepts presented in the algebra, geometry and measurement, and graphing and data analysis units. Students' academic needs were met by the teachers' implementation of the pre-differentiated and enriched curricula based on the pedagogical models of gifted and talented education. The designers of each unit used several strategies proposed by these models' authors as they designed lessons to engage both students and their teachers in teaching mathematics.

CHAPTER 7: Cohort II Qualitative Results

Micah N. Bruce-Davis

The data sources for the qualitative analysis of Cohort II included treatment teacher classroom observations, treatment teacher focus groups, administrators' interviews, and treatment teachers' logs. Most teachers in this cohort were continuing participants from Cohort I. Eight schools from Cohort I continued into Cohort II. The data set comprised 18 treatment teacher observations, two administrators' interviews, five teacher focus groups, and 19 completed teachers' logs. See Appendix A for the administrators' interview and teacher focus group protocols. See Appendix C for the data source key. The following research question guided the analysis of the data from Cohort II:

> How do teachers and administrators respond to their access to pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis?

Administrators' Interviews

Two administrators from the Midwest responded to the administrators' interview questions. Each interview will be discussed below. Administrators commented on the reactions teachers had to the units, the effects the curriculum had on students, how they hoped the curriculum would inform teachers' future pedagogical practices, and their general impressions of participating in a research study.

Ms. York from Lakeshore Elementary stated that teachers had various reactions to the curricular units "really mixed, depending on comfort level with the material." However, she explained that the units did benefit the students because "It is about meeting the kids where they are. The whole thing is laid out beautifully. [Teachers] are able to meet students where they are" (AI, MW). She believed that teachers were supported throughout the study. The preassessments were described as helpful. Ms. York hoped that teachers would develop tiered assignments in other subject areas, but she noted that teachers may have a hard time developing the tiered assignments on their own. Ms. York hoped that the teachers will continue to use flexible grouping.

Ms. Winchester responded for West Valley Elementary and Pleasant View Elementary. She stated that teachers "love the units of instruction" (AI , MW). When asked about how the math curriculum benefited the students, she stated, "Kids like math! They enjoy the challenge and are rising to the challenge. These differentiated units are good for all kids" (AI, MW). She hopes that the teachers continue to differentiate. She also appreciated how the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit engaged students in a recycling project that raised money to purchase a revolving electric sign to advertise birthdays.

Observations

Eighteen of the 20 Cohort II teachers were observed by members of the research team. Three teachers were observed teaching a lesson from the *Awesome Algebra* Unit*,* eight teachers taught a lesson from the *Geometry and Measurement for All Shapes & Sizes* Unit, and seven teachers were observed teaching a lesson from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit*.* After an inductive analysis the following categories emerged: (a) discourse, (b) collaboration, and (c) student engagement.

Discourse

During the qualitative analysis two major categories of classroom discourse were evident: (a) discourse that focused on students' ability to explain their thinking, and (b) and discourse focused on eliciting multiple responses. Discourse included interactions among multiple students or discourse that emerged from teacher-led discussions. It should be noted that the pre-differentiated and enriched curricula had many embedded opportunities to promote discourse.

In 16 of the 18 observed classes, significant portions of the observations were coded in the discourse category indicating that mathematical conversations were occurring often among the students and/or between the students and the teacher. This category included discourse that focused on students' ability to explain their thinking, and conversations where the teacher emphasized eliciting multiple responses from students. A frequently observed example was a lesson on creating a graph based on data from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. Teachers also engaged students in discourse designed to elicit multiple responses.

Explain Your Thinking

Several of the lessons observed were from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. In these lessons teachers and students were frequently engaged in whole group lessons where teachers had students dictate and discuss the steps needed to create various graphs.

In Ms. Manning's class, the students worked on the *Line Graphs (Part 2)* lesson, which focused on how to determine where numbers should be placed on line graphs. Students worked on identifying where to plot various temperatures on the graphs at their desks, and then in a whole group setting, Ms. Manning asked the students to describe how they determined where to place the temperatures on the graph. Ms. Manning used the mathematical language, such as "axis," while students directed her actions on the board.

> T: OK, where does the number go here on the graph? I am going to take one of my fingers and slide up the line that corresponds to 8 a.m., now you said 56 degrees, so what do I do? There's no 56 on the axis, what do I do?
> S: You go in the middle of 50 and 60. (TTO, Crowder Point, NE)

She utilized mathematical language during the whole group lesson. After the class discussion, students worked independently to develop graphs, the teacher walked around the room and looked at students' work, and then four students demonstrated how they decided where to place numbers on the graph displayed on the board.

Evidence of discourse involving students explaining their thinking was also demonstrated during another lesson from the *Awesome Algebra* Unit. One observer noted, "In group discussion students shared how they decomposed the number to get an 'easier' number, how they added on by counting in their heads or counting by twos, or by rounding numbers and adjusting the final answer" (Bennett, SE).

At Lakeshore, Mrs. McCoy guided students through the process of creating a graph. First, she asked the students to tell her what they should do first to create a graph. The students responded with the following:

S1: Put the numbers on the side of the graph.
T: Tell me more.
S1: There needs to be numbers on the vertical axis.
S2: There needs to be labels on both the horizontal and vertical axis. (TTO, Lakeshore, MW)

During this process, Ms. McCoy's class finished creating the graph. When students needed help, Ms. McCoy asked specific questions such as "What do the tally marks stand for?" and students continued to create elements of the graph such as axis labels.

Multiple Responses

Teachers engaged students in conversations that elicited multiple responses from students through brainstorming activities. Teachers also encouraged participation by asking, "Did anyone do it differently?"

Ms. Cooper utilized brainstorming strategies throughout the observation of a lesson from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. She asked, "What ways could we communicate to the school the need for an intervention such as recycling plastic bottles?" (TTO, Historic Cove, NE). Students responded with the following ideas: (a) making morning announcements, (b) creating posters, (c) bringing in colored bins to collect the bottles, (d) giving speeches to other classrooms, (e) handing out flyers, and (f) creating and performing a recycling play.

She continued to have students brainstorm ideas in response to the question: "How are you going to know if you are making a difference?" One student responded, "Look at the garbage before, and then make a graph." Ms. Cooper countered, "What do I have to do first before I make a graph?" After the student did not respond, she reminded the students about the tallying they discussed the other day and said they need to count the items that are thrown away or recycled. One student responded, "We could weigh all of it." Another student said that they could measure the height of the garbage in the can (TTO, Historic Cove, NE).

In addition to brainstorming, teachers elicited multiple responses from students by asking them to explain a different method to solve a problem. Examples of teachers asking students to explain different methods for solving problems were primarily observed during lessons from the *Awesome Algebra* Unit. Ms. Bennett elicited several responses by asking to students to explain how they completed the following problem: "36+9=?." The students explained with the following examples:

> S1: I started with 36 and added 9 to it.
> S2: I knew 9 was really 4+5 so I added 4 to 36 to get 40 and then added the 5.
> S3: 9 is close to 10 so I just added 1 to 9, then added 10 to 36 to get 46 and then subtracted the 1 back to get 45. (Bennett, TTO, Rosewood Park, SE)

Collaboration

Teachers asked students to solve problems or work on assignments collaboratively with partners, in groups, and during whole class situations. For example, Ms. Newman grouped students into pairs and instructed them to work on a tiered assignment from the *Awesome Algebra* Unit. All of the pairs were positioned together in one part of the room. The Kovalevsky group was on the carpet, and Ms. Newman sat down with the Fibonacci group for the majority of the work time to read the questions and go over possible answers. The observer commented, "[T]he different strategies [utilized] within the groups are very obvious. Once again, some students are using the number line to add while another student is estimating and adding in his head. He said for 72+72 is 75+75, which equals 150–6, which is 144" (Newman, TTO, Stone Mill, NE).

Ms. Montgomery was also observed utilizing the grouping structures from the units. She had students "split up into Fibonacci and Diophantus groups in various locales around the room so they could not see each other's work. Each group worked on the *Lunch at the Deep Sea Café* student page to develop a general explanation or an algorithm to describe a growing pattern. After 25 minutes in groups the class regrouped at the front of the classroom to recap what they had done in each group and explain how they got their answers" (TTO, West Valley, MW).

Students who were working in groups or with a partner would also support each other during the assignment. In Ms. Hopkins' class, the observer commented, "Students sought help from each other as needed. They seem to be very comfortable working in groups" (TTO, Lakeshore, MW).

Engagement Through Class Discussions

An analysis of the observations also revealed that students in these classes were engaged throughout the lesson. Examples of student engagement were seen during whole group discussions and while students were working in smaller groups. Students were able to answer questions and add to discussions, especially through both the teachers and the students connecting the content of the lesson to real-world examples.

In Mr. Chapman's class, the students participated in a reading of the "Rectangles Only Club" as part of the *Geometry and Measurement for All Shapes & Sizes* Unit. When the play was finished, he asked them, "What's going on mathematically at the end of the play?" Students talked about the question that asked about squares also being rectangles from lesson 4 of this unit. The teacher directed students' attention to the first question on the student page. He modeled a response using mathematical language on the overhead projector for the first question. As the students discussed the mathematical concepts introduced during the play, the observer noted, "The teacher is good at showing the students how to break down the big question into smaller ones" (TTO, Pleasant View, MW).

Ms. Morgan engaged students with a real-world connection during a lesson from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit that focused on developing interventions that the class could enact, with the focus of graphing the results from the intervention.

> T: Eli, what's your idea?
> S: Composting.
> T: Composting, my daughters' school composts and they grow flowers and vegetables to sell for fundraising.
> S: What about Dunkin' Donuts coffee cups? They're hard to break down in the ground?
> T: Okay, now pair and share about what you think might be good ideas, good to recycle in school. (TTO, Historic Cove, NE)

Students spent the last five minutes of class discussing which items the school should recycle with a partner.

Teacher Focus Groups

Ten teachers at four schools participated in the teachers' focus groups. Each focus group included comments from between one and four teachers. They responded to questions about students' reactions to the units, their own comfort level teaching math and teaching with the pre-differentiated and enriched curricula, and their own growth as a teacher during the study. Through an inductive analysis, the following categories emerged: (a) positive teacher responses, (b) teacher change and learning, and (c) concerns.

Positive Teacher Responses

Teachers at all four sites noted that teaching with the pre-differentiated and enriched units facilitated differentiation practices. Ms. Oliver commented that the curriculum “makes it [accommodating for diverse learners] easy by providing everything I needed. I did not have to dig somewhere for materials” (FG 9, MW). The pre-differentiated and enriched curricula allowed students to be challenged appropriately. A teacher stated, “Every student can explore math concepts at his/her comfort level and experience success” (FG 10, NE). Ms. Montgomery noted, “The lessons and activities met all of their [diverse students’] needs without pointing out that they were doing something different” (FG 11, MW).

In addition, teachers remarked on students’ reactions to the units. In one focus group a teacher commented that there was enough challenge to keep students engaged without causing too much frustration (FG 11, MW). Teachers also commented that students enjoyed working in groups. Mr. Chapman explained, “Students’ reactions to the math curriculum were very positive. The students enjoyed being active problems solvers rather than passive listeners” (FG 9, MW).

Teacher Change and Learning

Several teachers commented that they grew more comfortable teaching the units during their second year participating in the What Works study. One teacher explained, “I am much better this year. I feel better with both approach and concepts” (FG 8, MW). Teachers in Focus Group 10 also noted that using the units helped them with their own math comfort levels (FG 10, NE).

Teachers’ growing comfort with teaching with the units was often connected to their increased conceptual understanding of the content. Mr. Chapman commented, “In geometry after I truly understood key concepts my comfort level increased. Thus, increasing my ability and effectiveness to teach the concepts” (FG 9, MW). Ms. Newman mentioned, “Algebra made sense and geometry didn’t. Now I feel like I really understand it. I go back and really understand the concept. Since starting to teach third grade, I feel like I really understand the concepts” (FG 10, NE). Ms. Montgomery commented that teaching with the units helped develop her students’ and her own conceptual understanding of the topics covered. She became more comfortable with the units the second year, and noted that the *Awesome Algebra* Unit was where she struggled but grew the most. Her struggle to implement the *Awesome Algebra* Unit connected to her lack of comfort with algebra, but after teaching with the units, she planned to offer her students “the options of choice and freedom to explore in order to learn concepts” (FG 11, MW).

Teachers also commented on the development and growth of their pedagogical practices. One teacher commented that after using the units, “I now use more open-ended discussion—getting the kids more involved” (FG 8, MW). While teaching the units, another teacher in Focus Group 8 explained that she realized there is more than one way to solve problems. Other teachers started using more content-specific vocabulary. The Focus Group 10 teachers appreciated how the unit(s) reinforced best teaching practices and made them feel better about the kind of job they were doing (FG 10, NE). One teacher commented that she learned the following while teaching the What Works curriculum, “You can forget all the pieces. This kinda renews it and makes sure to include all the pieces so you can be successful. . . . You can get away from that. . . . It is good to be reminded. You are also told about all the things that you are doing right" (FG 10, NE).

Concerns

Very few concerns were mentioned during the teachers' focus groups. However, teachers in Focus Group 8 mentioned some concerns (FG 8, MW). Specifically, they mentioned that it was difficult to balance their state's standard curriculum and test requirements and material from the "regular" curriculum while teaching the pre-differentiated and enriched units with one teacher stating, "We are doing double duty. All of a sudden we haven't covered what we were doing before." She later mentioned, "algebra is not as applicable to the state test" (FG 8, MW).

Teachers' Logs

Teachers in Cohort II discussed several topics in their teachers' logs, frequently citing positive reactions to the units both by their students and themselves. Many of the teachers elaborated on the reasons for the positive responses, which included the students gaining new or deeper understandings or skills and the real-world nature of the units. Teachers also made suggestions on ways to improve the units or voiced concerns over the rigor of the units. Out of 20 teachers, 19 teachers wrote comments in their teachers' logs.

Positive Reactions to the Units

Quite frequently teachers mentioned positive reactions from their students (n=16). Ms. McKinney noted, "I enjoyed teaching the *Awesome Algebra* Unit. It's difficult in the beginning because students want you to tell them how to get the answers but when they discover that they can use what they know to 'uncover' their own methods they get excited. I love[d] seeing their faces light up" (TL, Rosewood Park, SE). Ms. Carson commented on her surprise that units developed for gifted students also reached her special needs students: "I was surprised that some of my children with special needs really enjoyed the lessons and were able to stay focused on the tasks" (TL, Crowder Point, NE).

Challenge, Enjoyment, and Growth

Teachers also discussed how the challenging nature of the units led to students increasing the depth of their understanding with mathematical concepts. Ms. McCoy commented, "I like the way the unit challenged my students to think beyond the lesson and make new connections" (TL, Lakeshore, MW). Ms. Coleman stated, "A few of my gifted kids were stunned that they were actually being challenged. They loved the unit" (TL, Rosewood Park, SE). Some of the teachers commented on a specific student. Ms. Newman related, "The [s]tudent was very impressed that we were doing algebra. She looks forward to math time and was willing to try each assignment or task, I saw increased confidence in her math ability" (TL, Stone Mill, NE).

Teacher Growth

Teachers also recognized their own growth. Ms. Montgomery discussed both her students and her own growth:

> My kids enjoyed and were very challenged by this unit. They were fun to watch as lightbulbs went on and they figured out equations/problems. They improved as mathematical reasoners. It was so much more smooth the second time around. I felt more comfortable and the kids caught on to more concepts. (TL, West Valley, MW)

Grouping, Real-world Projects, and Engagement

In connection with the challenging nature of the units, some teachers noted that the students supported each other through the rigorous assignments. Ms. Nelson stated, "I was a bit surprised to see that although a task was challenging they persisted at it when working with partners/group, without losing focus" (TL, Stone Mill, NE).

Teachers also commented on how the real-world nature of the units seemed to increase investment in completing the work, especially in the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. Mr. Chapman stated, "The students were very invested in their learning due to the recycling project" (TL, Pleasant View, MW). Also, Ms. Howard discussed her students' engagement with graphing, "Students enjoyed implementing interventions to see how that would cause change in the graph" (TL, Historic Cove, NE).

Concerns

Occasionally, teachers discussed how students struggled with the units. They cited that students either struggled with the amount of reading and writing required to complete the lessons or the students struggled to understand the mathematical concepts in depth. Ms. Manning explained, "[It is] just too hard conceptually. I liked being able to teach line graphs and line plots for a change—but the students often got these mixed up because the names are so similar. Also, I don't think they truly understood the purpose of these two graphs (data over time vs. frequency). Conceptually I think this is still too advanced for their age" (TL, Crowder Point, NE).

Other concerns centered on the teacher's manual. While Ms. Cooper thought that one unit was "good," she also stated there is "[t]oo much info in Teacher's Edition. [The] lessons [are] too lengthy." (TL, Historic Cove, NE).

Discussion

One of the purposes of the What Works in Gifted Education Mathematics Study was to develop an understanding of how teachers and administrators respond to their access to pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis. Teachers in this study generally had very positive responses to the units often acknowledging growth in pedagogical practices and content knowledge. Administrators' comments from two schools supported teachers' perceptions of pedagogical growth. Continued use of the units over a 2-year period increased teachers' comfort with utilizing the units. However, explicitly connecting the state standards and state assessments to the units continued to be a concern for some teachers.

Teachers Challenged Students

Teachers commented on student reactions to the pre-differentiated and enriched curricula and indicated that students perceived the curriculum as challenging and engaging. They noted that students of varied readiness levels persisted through challenging work with the support of classmates, and because of the real-world nature of the problems presented. Many teachers expressed enthusiasm about their ability to challenge students and to see what students are truly capable of in class. Considering that previous studies have found that in many schools, gifted students receive only minimal modifications to the curriculum and instruction (Archambault et al., 1993; Moon, Tomlinson, & Callahan, 1995; Reis et al., 2004; Westberg, Archambault, & Brown, 1997; Westberg, Archambault, Dobyns, & Salvin, 1993), this finding is encouraging, and demonstrates that teachers can and will differentiate instruction when given access to pre-differentiated and enriched curricula. Even more encouraging is that the teachers perceived the material to be appropriately challenging for students of all readiness levels. As new curriculum is developed, curriculum developers should consider pre-differentiating material with a focus on high level learning as a means to challenge all students.

Concerns

The few concerns that were raised dealt with the teacher's manual and state testing. Some teachers found the teacher's manual to be too cumbersome. Other teachers were concerned that the curricular objectives would not necessarily align with those on their state assessments. In addition, they worried about having the time to cover both the state standards and those from the research study. In addition, a few of the teachers felt the curriculum was too advanced for students in their class.

In light of these concerns, careful consideration should be paid to state standards when creating educative curriculum. In addition, when participating in a research study or completing new interventions, teachers need administrative support (e.g., time to collaborate, professional development related to state standards) to fully implement research curriculum, especially if the content consists of material that goes beyond the state standards for that particular grade level. Furthermore, one-on-one support may be needed to help teachers adjust the curriculum to meet the needs of all learners in their class when teachers first begin to differentiate and enrich lessons.

Teacher Pedagogical and Conceptual Growth

Teachers in Cohort II voiced positive reactions to utilizing the pre-differentiated and enriched curricula and to participating in the research study. In addition to the educative curriculum, teachers in Cohort II utilized on-going professional development opportunities, including two on-site in-services, and continuous e-mail communication with the research team.

In connection to Davis and Krajcik's (2005) definition of educative curriculum, the mathematics curricula was designed to develop teachers' pedagogical content knowledge and content knowledge. Many Cohort II teachers in the study noted growth in these areas. Teachers appreciated the pre-differentiated and enriched curricula, and they also felt the instructional strategies included in the texts supported practices such as grouping and utilizing discourse in the class. They shared how they grew in their understanding of the concepts presented in the units. Teachers also took an active role in the development of their conceptual understandings and the development of new pedagogical practices. The teachers' assertions that they utilized instructional strategies such as discourse were supported in the observations.

The teachers' recognition of their continued growth during the year and overall increase of comfort with the implementation of the units the second year indicates that when utilizing new interventions teachers may need extensive time to feel confident implementing the intervention. The Cohort II treatment teachers' perceptions of growth connect to other studies of professional development where teachers felt that their ability to apply their new understandings in the classroom context provided a more effective format for professional development (Garet et al., 2001; Remillard, 2000). As schools select future professional development experiences, opportunities where teachers utilize new strategies or implement new content in their own classroom setting should be included. These experiences allow teachers to deeply internalize new content and practices.

CHAPTER 8: Cohort III Quantitative Results

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The results of the analyses of the quantitative data from the Cohort III treatment and control participants are described below. The data sources included results from pretest and posttest achievement measures and pretest aptitude measures from treatment and control group students. In addition, pretest and posttest unit tests and posttest only NAEP sample items on algebra, geometry and measurement, and graphing and data analysis are summarized.

Research Questions and Hypotheses

The primary research questions for Cohort III involved the efficacy of the model-based mathematics curriculum to a broad spectrum of students. The quantitative analysis focused on academic outcomes of treatment and control group students on a norm-referenced achievement test. Treatment group students were also assessed for their specific knowledge of content, concepts, and skills directly related to the criterion-referenced unit tests in algebra, geometry and measurement, and graphing and data analysis and for their mastery of selected grade 4 items from the National Assessment of Educational Progress.

The research team hypothesized that students involved in the treatment condition would outperform students in the control condition on the norm-referenced achievement test, namely the ITBS—Math Problem Solving and Data Interpretation subtest. The research questions and corresponding hypotheses follow.

Research Question 1
What is the impact of creating pre-differentiated and enhanced curricula in algebra, geometry and measurement, and graphing and data analysis on the achievement of grade 3 students, after controlling for pretest achievement scores?

Hypothesis 1
Treatment group students involved in pre-differentiated and enhanced curricula in algebra, geometry and measurement, and graphing and data analysis will outperform control group students on the Iowa Tests of Basic Skills—Math Problem Solving and Data Interpretation subtest, after controlling for pretest achievement scores.

Research Question 2
Are there practically meaningful gains between the pretest and posttest criterion-referenced unit test scores in algebra, geometry and measurement, and graphing and data analysis for the treatment group students?

Hypothesis 2
There will be large gains in the treatment group students' performance criterion-referenced unit tests between pretest and posttest.

Research Question 3
Will at least 50% of the grade 3 treatment group students involved in the mathematics curriculum master the content, concepts, and skills typically addressed by grade 4 students?

Hypothesis 3
At least 50% of the treatment group students will master each item in algebra, geometry and measurement, and graphing and data analysis from the grade 4 National Assessment of Educational Progress.

These quantitative research questions and hypotheses were addressed by multiple assessments to determine the extent to which grade 3 students can master challenging, differentiated mathematics curricula.

School-level Data

Table 8.1 includes a description of the sample of the schools recruited for the What Works in Gifted Education Mathematics Study, obtained from publicly available data from the Great Schools website (GreatSchools, 2011), which provides NCES statistics reported across all 50 states for public schools. Forty-two of the schools were public schools, and one school was private.

Table 8.1
Demographics[†] *for Cohort III Schools Participating in the What Works in Gifted Education Mathematics Study* (*N*=43)

School Pseudonym	Total Enrollment	White/ Asian (%)	Other Ethnicities (%)	Free/Reduced Lunch (%)	Student/ Teacher Ratio
Apple Tree	647	91	9	5	14
Bald Eagle	387	95	5	33	16
Calder	470	81	19	47	22[‡]
Casini	238	98	2	3	12
Cedar Brook	98	85	15	—[*]	7
Centurion	435	96	4	8	13
Cortana	363	94	6	15	16
Deer Creek	377	88	12	21	12
East Halsey	508	90	10	5	16
East Point	609	80	20	20	17
Evergreen Street	585	99	1	46	22
Farnsworth	244	76	24	68	13
First Sun	346	97	3	8	13
Forge Hill	684	98	2	9	16
Franklin Bridge	572	93	7	13	20
George Washington	636	98	2	9	17
Governor's Park	607	93	7	2	18
Grand Arch	509	54	46	43	14
Halcyon	368	90	10	25	12
Haverbrook	589	64	36	61	13
Lucasville	402	84	16	49	25[‡]
Morrowind	381	100	0	3	12
Mustang Ranch	321	98	2	32	15
Newton	243	98	2	29	12
Northwest	610	90	10	10	15
Old Toll Road	655	79	21	25	16
Oyster Harbor	649	88	12	12	16
Pegasus	518	73	27	31	10
Savannah	176	100	0	64	18
Seabreeze	576	90	10	6	17
Shade Rock	605	77	23	13	16
Shady River	449	57	43	27	14

† Source: Great Schools website: www.greatschools.org (2008-2009 data).
‡ State reports average class size rather than student to teacher ratio.
* Private school is not eligible for free/reduced lunch.

Table 8.1 (continued)
Demographics for Schools Participating in the What Works in Gifted Education Mathematics Study (*N*=43)

School Pseudonym	Total Enrollment	White/ Asian (%)	Other Ethnicities (%)	Free/Reduced Lunch (%)	Student/ Teacher Ratio
Shelbyfield	783	91	9	18	16
Skinner	563	95	5	2	14
Smithton	770	91	9	9	16
Solsbury Valley	515	89	11	7	17
Southeastern	834	80	20	23	15
Springville	725	59	41	63	14
Staten Ridge	503	85	15	32	23‡
Sunnyside	739	49	51	51	15
Sycamore	646	92	8	5	16
Terracotta	266	97	3	4	11
Vermillion	278	99	1	51	19

The schools in the sample were located in 12 states throughout the nation. The largest number of schools (15) was located in the Northeast region, with nine schools from the Midwest, seven from the Southeast, six schools from the Southwest, and six schools from the Mid-Atlantic region. The schools represented a broad cross-section of the nation's schools. In addition, the schools were diverse not just geographically but also in terms of urbanicity. While a majority of the schools recruited were from rural and suburban settings, three schools were located within a large city. The mean school enrollment for the sample of schools was 515 with a standard deviation of 177. Most of the schools included more than 551 students, as illustrated in Figure 8.1. The variability in the school enrollment is attributable to the variety of grade structures of the schools in the sample. Some of the schools with smaller enrollments contained only grades K-3 or 3-5, while the larger enrollment schools generally comprised grades K-5 or K-6.

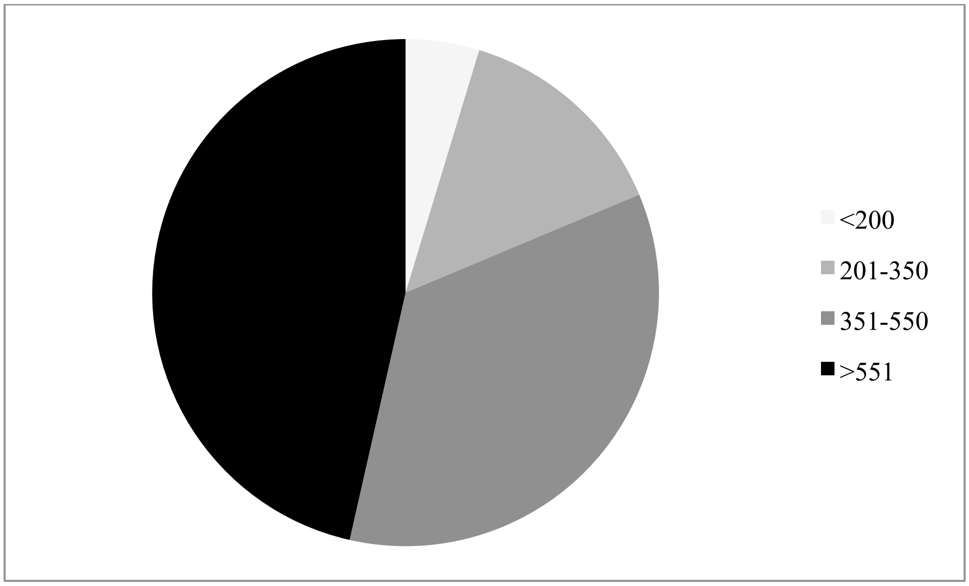


Figure 8.1. School enrollment of Cohort III schools.

Abundant research (see Gentry et al., 2008 for a summary) has documented the under-representation of specific ethnic and racial groups in gifted education programs in the United States. The National Research Center on the Gifted and Talented has focused its research efforts on providing enriched education opportunities for students from historically under-represented ethnic groups, which have generally been those groups other than White and Asian students.

Despite the desire to include students from schools with more significant proportions of traditionally under-represented students, schools recruited for the 2009-2010 year of the study had a student composition that contained more White and Asian students than in the nation as a whole. The sample of schools in Table 8.1 had an average of 86.5% White and Asian students, which is considerably higher than the national average of close to 70% of public school children listing “White only” or “Asian only” as their racial designation as measured by the NCES American Community Survey (NCES, 2012). The comparison of racial and ethnic composition is somewhat complex due to the different categories used in the national survey and the What Works in Gifted Education Mathematics Study. Specifically, Hispanic/Latino(a) was among one of the options for ethnicity listed for the What Works study information form, while the NCES study categorizes students by Hispanic/non-Hispanic status and separately by race. Therefore it is possible to be White and Hispanic, Black and Hispanic, etc. under the NCES categories, but these were listed as mutually exclusive categories when collected for the present study. Because of the difference in the measurement of race and ethnicity between NCES data and the present study, a direct comparison of racial and ethnic diversity between a nationally representative sample and the present study’s sample would not be appropriate. Descriptively, the What Works in Gifted Education—Mathematics Study sample of schools contained nine schools with more than 20% non-

White/non-Asian student enrollment and five schools with more than 30% non-White/non-Asian student enrollment.

Socioeconomically, schools participating in the mathematics research study had a wide range of student compositions. Although free/reduced lunch status is certainly not a perfect indicator of students' individual socioeconomic background, it is commonly used as an indicator of socio-economic status (SES) in educational studies in which more detailed information about students' home lives is not available. The schools in the mathematics research study included a range of very affluent schools (those with 0% and 2% of students eligible for free/reduced lunch) to those with a majority of eligible students (a maximum of 68% eligible students) (see Figure 8.2). The average percentage of students eligible for free/reduced lunches from the sample of schools was 24.0, indicating that this group of schools was more affluent than the national average: nationally, approximately 62% of students meet eligibility criteria for free/reduced lunch status (National School Lunch Program, 2012).

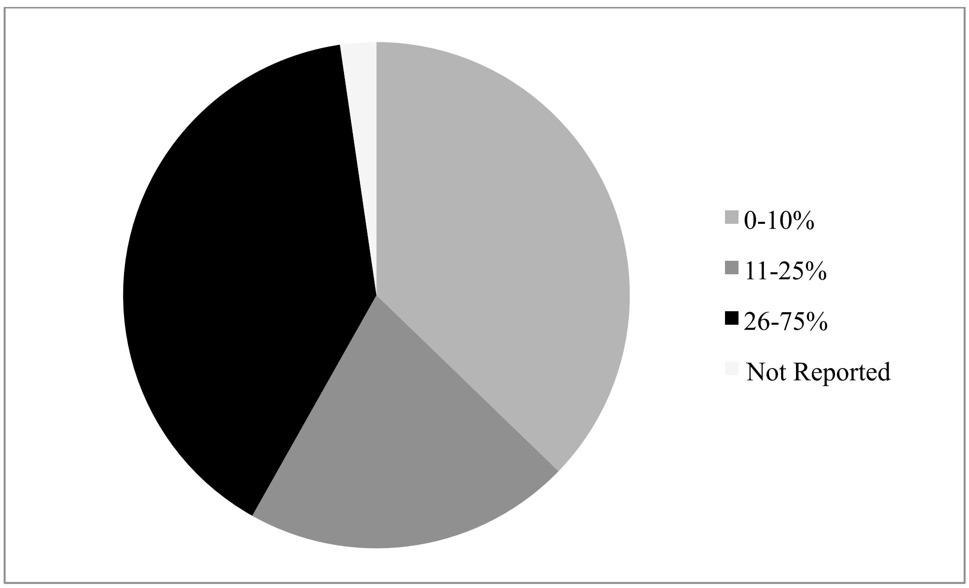


Figure 8.2. Percentage of students receiving free/reduced lunch.

Teacher and Classroom Data

Table 8.2 presents the treatment and control group teacher demographics. In both groups, teachers were predominantly female and White. Teachers had several years of teaching experience. Of the 84 treatment teachers, 58% had 10 or more years of experience. Of the 57 control teachers, 62% had 10 or more years of experience. Table 8.2 indicates that the majority of teachers had less than 10 years of experience with grade 3 students, with 74% of the treatment group teachers and 67% of the control group teachers documenting 0-9 years of teaching grade 3 students. As experienced elementary school teachers, both groups were familiar with the math curriculum for their schools, and they had experience in working with grade 3 students who were the focus of the research curricula. They also had advanced degrees with 56% of the treatment group teachers and 67% of the control group teachers earning Master's of Arts (MA) or Master's of Science (MS) degrees.

Table 8.2
Cohort III Treatment and Control Group Teacher Demographics (*N*=141)

	Treatment Group (*n*=84) Frequency (%)	Control Group (*n*=57) Frequency (%)
Gender		
Male	4 (5)	1 (2)
Female	80 (95)	56 (98)
Ethnicity		
American Indian/Native American	0	0
Asian/Hawaiian/Pacific Islander	4 (5)	0
Black/African American	0	0
Hispanic/Latino(a)	1 (1)	0
White	79 (94)	57 (100)
Multiple Ethnicities/Other	0	0
Years Teaching Experience		
0-4	9 (11)	6 (11)
5-9	25 (30)	16 (28)
10-14	23 (27)	9 (16)
15+	26 (31)	26 (46)
No Response	1 (1)	0
Years Teaching Grade 3		
0-4	37 (44)	20 (35)
5-9	25 (30)	18 (32)
10-14	14 (17)	14 (25)
15+	8 (9)	5 (9)
Highest Degree Earned		
BA/BS	34 (40)	18 (32)
MA/MS	47 (56)	38 (67)
Sixth Year/Ed. Specialist	1 (1)	0
Ph.D./Ed.D.	0	1 (2)
Professional Diploma	1 (1)	0
Other	0	0
No Response	1 (1)	0

Student-level Data

Of the students in the analytic sample, a similar percentage of males (50%) and females (49%) comprised the treatment and control groups across all schools (see Table 8.3). Over 80% of students in the treatment and control groups were White, with fewer than 20% representing other racial/ethnic groups.

Table 8.3
Cohort III Treatment and Control Group Student Demographics (*N*=2,290)

Characteristic	Treatment Group (*n*=1,391) Frequency (%)	Control Group (*n*=899) Frequency (%)	Total (*N*=2,290)
Gender			
Male	704 (50.6)	458 (50.9)	1,162
Female	683 (49.1)	441 (49.1)	1,124
Gender not indicated	4 (0.3)	0	4
Total	1,391	899	2,290
Race/Ethnicity			
American Indian/Native American	15 (1.1)	8 (0.9)	23
Asian/Hawaiian/Pacific Islander	67 (4.8)	51 (5.7)	118
Black/African American	51 (3.7)	36 (4.0)	87
Hispanic/Latino(a)	93 (6.7)	60 (6.7)	153
White	1,147 (82.5)	725 (80.6)	1,872
Multiple Ethnicities/Other	17 (1.2)	19 (2.1)	36
Ethnicity not indicated	1 (0.1)	0	1
Total	1,391	899	2,290

Treatment and Control Group Comparisons Prior to Intervention

When conducting randomized experiments, it is important to verify that treatment and control participants have comparable characteristics prior to the intervention. This reduces selection bias concerns for causal inferences. Although random assignment should minimize pretest differences between the groups, empirical pretest data can also support this equivalence.

Table 8.4 illustrates a comparison of treatment and control students on a variety of characteristics prior to the intervention. Treatment and control groups were generally very comparable. Treatment students had significantly higher scores than control students on the verbal subtest of the CogAT and on the ITBS mathematics pretest. Treatment schools also had a significantly higher proportion of students eligible for free/reduced priced meals than control schools. Otherwise, there were no measured pre-existing differences between treatment and control group students in Cohort III.

Table 8.4
Cohort III Students' Group Equivalence on Pretest Measures for Treatment and Control Groups

Variable	Condition	*M*	*SD*	*t*(*df*)	95% confidence interval for difference Lower bound	Upper bound
Student age at pretest (in months)*	Control	96.84	4.23	-2.24(738)	-1.39	-0.09
	Treatment	97.58	4.73			
Student age at pretest	Control	100.09	5.42	-1.49(2195)	-2.03	0.31
	Treatment	100.39	5.40			
CogAT age score (verbal)**	Control	106.62	13.39	-2.69(2235)	-2.78	-0.44
	Treatment	108.23	13.99			
CogAT age score (quant)	Control	107.00	13.87	-0.92(2235)	-1.75	0.61
	Treatment	107.56	13.86			
CogAT age score (nonverbal)	Control	106.49	13.40	0.00(2232)	-1.18	1.18
	Treatment	106.49	14.19			
CogAT age score (composite)	Control	107.48	13.28	-1.44(2235)	-2.02	0.31
	Treatment	108.33	14.08			
OLSAT age score (total)	Control	93.54	18.18	-0.04(51)	-9.57	9.24
	Treatment	93.70	15.87			
ITBS Level 8 Math*	Control	183.28	21.88	-1.99(1896)	-3.99	-0.03
	Treatment	185.29	21.17			
TerraNova Math Scale Score	Control	597.24	37.42	1.53(297)	-1.97	15.84
	Treatment	590.31	38.85			
Stanford Achievement (math)	Control	597.77	42.39	0.42(51)	-17.74	27.21
	Treatment	575.04	39.09			
MAP RIT Score (math)	Control	198.55	7.75	-0.18(38)	-7.57	6.32
	Treatment	199.17	10.29			
SRBCSS Learning scale	Control	54.33	7.65	-0.42(525)	-1.56	1.01
	Treatment	54.61	7.09			
SRBCSS Motivation scale	Control	55.81	8.55	0.86(524)	-0.81	2.06
	Treatment	55.19	7.90			
SRBCSS Creativity scale	Control	43.56	6.80	0.21(524)	-1.02	1.26
	Treatment	43.44	6.29			
SRBCSS Math scale	Control	49.64	7.86	0.59(523)	-0.92	1.71
	Treatment	49.25	7.21			
Gender (dummy code)	Control	0.51	0.50	0.09(2284)	-0.04	0.04
	Treatment	0.51	0.50			
Proportion non-White/ non-Asian students	Control	12.32	11.61	1.27(2288)	-0.35	1.64
	Treatment	11.67	12.07			
Proportion of FRL† eligibility*	Control	18.88	17.46	-2.79(2288)	-3.58	-0.63
	Treatment	20.99	17.72			
Proportion of students receiving teacher nominations	Control	0.23	0.42	-0.29(2288)	-0.04	0.02
	Treatment	0.23	0.42			

† FRL indicates free/reduced priced school meals
* *p*<.05.
** *p*<.01.

Reasoning Ability

The CogAT assesses students' developed abilities in reasoning and problem solving using verbal, quantitative and nonverbal symbols. The 6th edition of CogAT was co-normed with the ITBS. Verbal, quantitative, and nonverbal reasoning scores are available for 2 subtests for grades K-2, and 3 subtests for grades 3-12. Primary grade (K-2) items do not require any reading and are paced by the teacher. Tests for grades 3-12 require some reading and the total testing time is 90 minutes for the 3 subtests, plus 15 minutes for distributing and collecting forms. The test was standardized on approximately 150,000 students in grades K-8 and 31,000 students in grades 9-12 from all 50 states. The sample of students represented various types of communities, ethnicity, race, and socioeconomic status. Internal consistency estimates ranged between .85 and .98 on the composite score. The test was subjected to content validity and construct validity and it correlates with achievement and other measures of ability. Factor analytic and bias studies were implemented. Criterion validity is .54-.87 (Lohman & Hagan, 2002).

Students participating in the study completed the three subtests (verbal, quantitative, and nonverbal) from the CogAT (Level 2) at the end of their second grade year. The CogAT aims to measure general "learned reasoning abilities" (Lohman & Hagen, 2001) in the three domains. The scaling of the CogAT age scores sets a mean of 100 and a standard deviation of 16. These three scores are combined to form an age score composite, which may be thought of as a domain general measure of reasoning ability. As shown in Table 8.4, the mean pretest CogAT composite score for the treatment group was 108.42 (*SD*=14.11) and 107.61 (*SD*=13.29) for the control group. These scores are somewhat higher than for the national CogAT norm group, indicating a sample of students with above average reasoning ability. Fifty-three students completed the Otis-Lennon School Abilities Test instead of the CogAT. Their scores were rescaled to be comparable with the CogAT scores in the analysis.

Mathematics Achievement

The ITBS provide a comprehensive assessment of student progress in the basic skills. They consist of a Complete Battery (reading, language arts, mathematics, social studies, and science), a Core Battery (reading, language, and mathematics), and a Survey Battery (shortened version of Core Battery).

Test content is aligned with the most current content standards, curriculum frameworks, and instructional materials. The test was standardized on a national sample of students K-9, with approximately 3,000 students per level per form completing the tests. Internal consistency estimates using KR 20 varied between .79 and .98 (Hoover et al, 2003). Students in the standardization sample represented various types of communities, ethnicity, race, and socioeconomic status. The standardization sample included public, parochial, and non-parochial schools. Schools in the standardization were further stratified by socioeconomic status. Data from these sources were used to develop special norms for a variety of groups (e.g., race/ethnicity, public school).

Mathematics Pretest

The ITBS Level 8 Math Problems subtest was administered prior to the curricular intervention to obtain information on students' achievement in mathematics. The Level 8 ITBS subtest had 30 items. A small proportion of students completed other mathematics achievement pretests (the TerraNova, the MAP, or the SAT). Because the achievement tests were on different scales, *z* scores for the scores on each of the four achievement tests were calculated so that students' pretest achievement could be compared across tests.

Mathematics Posttest

All students in the Cohort III sample analyzed in the current study took the 25-item Level 9 Math Problem Solving and Data Interpretation subtest of the ITBS. The mean posttest score for treatment students across schools was 206.3 (*SD*=22.9) and 205.6 (*SD*=22.0) for control students. A substantial test ceiling was present for both treatment and control students on this measure, indicating that higher achieving students' actual achievement was not adequately measured. With censored data, a Tobit model can attempt to estimate what would have occurred in the absence of the test ceiling and should provide less biased parameter estimates for regression models with censored data.

Assessments for Treatment Group Students Only

Several assessments were selected to monitor the impact of implementing pre-differentiated and enriched curricula with all grade 3 students. Only the treatment group teachers administered the pretest and posttest criterion-referenced units tests in algebra, geometry and measurement, and graphing and data analysis, as well as a subset of items from the National Assessment of Educational Progress on these same topics, which were administered after the curriculum intervention.

Criterion-referenced Unit Tests

Criterion-referenced unit tests were created to reflect the content, skills, and application of the mathematical concepts. The unit tests consisted of polytomously scored items with multiple subparts, which were scored by the treatment teachers according to rubrics developed by the curriculum authors. Some item subparts required closed-ended answers while others required more extended responses. The algebra unit test was comprised of five items, each of which contained between two and four subparts. The maximum score possible for the algebra unit test was 25 points. Internal consistency for the algebra pretest was .71 and .78 for the posttest.

The geometry and measurement unit test included five items, each of which contained between one and eight subparts. The individual items varied in maximum score from 3 to 11.5 points for a maximum test score of 30.5. Internal consistency for the geometry pretest was .60 and .72 for the posttest. The graphing and data analysis unit test consisted of three items, each of which contained between two and four subparts, with a total maximum score of 15 points. Internal consistencies for the graphing pretest and posttest were .69 and .76, respectively.

National Assessment of Educational Progress Released Items

Released items from the National Assessment of Educational Progress were selected to match the emphasis on algebra, geometry and measurement, and graphing and data analyses. The selected items required application of learned concepts that are typically designed for students completing grade 4. Five algebra, five geometry and measurement, and five graphing and data analysis items comprised the content of the post-only assessment. Of the 15 items, one item had to be eliminated due to a printing error. Of the 14 remaining items, the overall internal consistency reliability was .63.

Methodology for Analyzing ITBS Achievement Test Results

For Cohort III, a multisite cluster randomized design was implemented to answer research question one. In other words, classrooms (teachers) within each school were randomly assigned to the treatment or control groups. In total, 2,290 students and 141 teachers in 43 schools had sufficient data to be included in the Cohort III multilevel analysis.

The intervention was delivered to grade 3 students in randomly assigned classrooms. In the spring of grade 2, students in the participating schools completed the ITBS Math Problem Solving and Data Interpretation subtest or another nationally standardized achievement test. In addition, students completed the CogAT or another standardized group measure of ability. Most schools used the CogAT and the ITBS; however 10 schools used alternative achievement measures (Terra Nova (n=6); MAP (n=2); and the Stanford Achievement Test (n=1)). All pretest measures of ability and achievement were aligned using the equipercentile method. Because the pretest scores were on different scales, the newly created pretest score is a z score, where 0 means that the student scored at the national mean on the math pretest, 1 indicates that the student scored 1 standard deviation above the national mean on the math pretest, etc. Pretest math achievement scores were used as a covariate in the resulting analyses. Table 8.5 contains the school mean pretest z scores, as well as the posttest ITBS scores for each of the 43 schools in the study. Pretest ability scores were used to create a “gifted” variable. The “gifted” variable was based on a student’s relative standing within his or her school: A student was designated as gifted if his or her score on the ability assessment placed him or her in the top 10% of his or her school. “Gifted” status was coded as 1 and “non-gifted” was coded as 0. The dependent variable was ITBS math scores at the end of grade 3.

Table 8.5
Cohort III Descriptive Statistics of Pretest and Posttest Scores by School

	z score pretest			Posttest (ITBS)		
School	*N*	Mean	*SD*	*N*	Mean	*SD*
Apple Tree	14	1.00	0.69	14	217.50	20.74
Bald Eagle	49	0.67	0.82	50	202.56	21.72
Calder	28	1.01	0.70	33	216.58	19.77
Casini	19	0.97	0.76	19	215.68	25.67
Cedar Brook	11	1.90	0.41	14	232.64	9.68
Centurion	48	0.70	0.93	52	216.02	23.42
Cortana	62	0.34	0.86	62	211.08	21.01
Deer Creek	36	0.16	0.97	39	198.13	21.56
East Halsey	79	0.89	0.84	90	211.73	22.58
East Point	68	0.18	0.98	78	200.18	22.81
Evergreen Street	72	0.38	0.83	79	199.67	21.49
Farnsworth	30	-0.18	1.11	30	190.57	23.75
First Sun	87	0.88	0.97	90	209.50	20.83
Forge Hill	84	0.74	1.04	90	211.53	22.29
Franklin Bridge	112	0.58	0.96	126	206.22	20.16
George Washington	49	0.70	0.92	53	213.30	21.14
Governor’s Park	66	1.10	0.95	69	216.20	19.20
Grand Arch	45	0.18	0.94	54	195.20	24.07
Halcyon	69	0.31	0.90	69	200.10	22.60
Haverbrook	33	0.92	0.97	34	206.97	21.32
Lucasville	26	0.95	0.67	32	212.44	15.60
Morrowind	90	1.16	0.83	98	214.12	22.05
Mustang Ranch	40	0.70	0.85	40	203.35	18.25
Newton	37	0.74	0.91	40	203.65	25.44
Northwest	45	0.86	1.03	72	214.25	22.20
Old Toll Road	20	0.32	1.05	28	197.18	25.69
Oyster Harbor	54	0.27	0.97	79	197.20	20.97
Pegasus	59	0.33	0.96	63	200.97	24.87
Savannah	20	0.65	0.76	20	203.55	19.33
Seabreeze	61	0.91	0.82	67	204.91	20.30
Shade Rock	40	0.47	0.52	52	188.54	18.30
Shady River	29	0.33	0.93	30	196.57	25.41
Shelbyfield	120	0.48	0.98	120	203.08	23.11
Skinner	126	0.83	0.83	129	209.50	20.57
Smithton	99	0.95	1.01	103	209.86	21.36

Table 8.5 (continued)
Cohort III Descriptive Statistics of Pretest and Posttest Scores by School

	z score pretest			Posttest (ITBS)		
School	*N*	Mean	*SD*	*N*	Mean	*SD*
Solsbury Valley	55	0.48	0.83	66	200.18	21.94
Southeastern	73	0.47	0.89	88	207.98	22.77
Springville	66	0.01	0.79	69	191.86	20.01
Staten Ridge	64	0.75	0.86	66	211.83	20.16
Sunnyside	47	0.14	0.88	62	181.27	17.36
Sycamore	38	0.63	0.75	44	200.00	24.04
Terracotta	56	1.24	0.80	56	211.80	20.03
Vermilion	34	-0.14	0.61	37	200.19	21.49
TOTAL	2,360	0.62	0.95	2,606	205.36	22.88

Analysis

To examine the effects of the pre-differentiated and enriched curricula, analyses consisted of a series of 3-level multilevel models (students were nested within classrooms, which were nested within schools). Level 1 predictors included pre-ITBS score, which was grand mean centered, and “gifted” status. Treatment status which was dummy coded, was entered as a predictor at level 2. At level 3, the effect of school mean achievement was modeled by creating an aggregate of each school’s pre-intervention math ITBS score.[1] School aggregate math score was also a *z* score. Table 8.6 contains the correlations among the variables of interest in the study.

[1] The models were unable to include both the percentage of free/reduced lunch status and the prior achievement level of the school because of collinearity issues. These two variables were correlated above 0.70. Either variable essentially serves as a proxy for the other. Therefore, mean prior achievement was included due to the more complete and reliable data on that variable.

Table 8.6
Cohort III Students' Summary of Bivariate Correlations Between Variables in the What Works in Gifted Education Mathematics Study† Scores

Variable	Pretest Achievement *z* Score	Pretest Ability Scale Score	School % Non-White and Non-Asian	School % FRL Eligibility	School Mean Pretest Score	School Mean Ability Score	Posttest Achievement Scale Score
Pretest Achievement *z* Score	1	0.71	-0.17	-0.21	0.36	0.29	0.68
Pretest Ability Scale Score		1	-0.19	-0.25	0.32	0.40	0.66
School % Non-White and Non-Asian			1	0.58	-0.49	-0.48	-0.20
School % FRL Eligibility				1	-0.59	-0.64	-0.20
School Mean Pretest Score					1	0.81	0.28
School Mean Ability Score						1	0.27
Posttest Achievement Scale Score							1

† All correlations are significant at the p<.001 level.

The full 3-level model appears below.

Level 1:

$$y_{ijk} = \pi_{0jk} + \pi_{1jk}(student\_Pretest_{ijk}) + \pi_{2jk}(gifted\_status_{ijk}) + e_{ijk}$$

Level 2:

$$\pi_{0jk} = \beta_{00k} + \beta_{01k}(treatment_{jk}) + r_{0jk}$$
$$\pi_{1jk} = \beta_{10k} + \beta_{11k}(treatment_{jk})$$
$$\pi_{2jk} = \beta_{20k}$$

Level 3:

$$\beta_{00k} = \gamma_{000} + \gamma_{001}(school\_Pretest_k) + u_{00k}$$
$$\beta_{01k} = \gamma_{010} + \gamma_{011}(school\_Pretest_k)$$
$$\beta_{10k} = \gamma_{100} + \gamma_{101}(school\_Pretest_k) + u_{10k}$$
$$\beta_{11k} = \gamma_{110} + \gamma_{111}(school\_Pretest_k)$$
$$\beta_{20k} = \gamma_{200}$$

Results

First, the unconditional means model (Model 1) estimated what proportion of the variance lay at each of the three levels. Approximately 7% of the variance was between schools, 12% of the variance was between students within schools, and 81% of the variance was between students within classes. The level-1 model (Model 2), which included pretest achievement (as measured using math ITBS) and “gifted” status explained 42% of the variance between students within classes, 75% of the variance between classes within schools, and 57% of the between-school variance. Adding treatment at level 2 (Model 3), did not reduce the residual between-class variance in the intercept and only reduced the between-schools variance in the intercept by 4%. However, adding the treatment at level 2 did reduce the between-schools variance in the pre-ITBS slope by 16.4%. Finally, adding school aggregate pre-ITBS scores at level 3 (Model 4) resulted in a 13.7% reduction in the between-schools variance in the intercept and a 44% reduction in the between-schools variance in the pre-ITBS slope. Tables 8.5 and 8.6 contain descriptive statistics for the 43 schools. Table 8.7 contains the results of the four HLM models.

The final model failed to show a main effect for treatment, but did uncover interesting cross-level interaction effects. Examining Model 3, although there was no statistically significant difference between treatment and control groups when school aggregate pre-ITBS was held constant, there was a statistically significant effect of treatment on the pre-ITBS slope, that is, on the effect of pre-ITBS on post-ITBS. The effect of pre-ITBS on post-ITBS was stronger in treatment classes than in control classes. In other words, the pre-ITBS slope was steeper in treatment classes, indicating that the treatment appeared to have a differentiating effect on students. However, the picture is even more complex. The school aggregate pre-ITBS score moderated the cross-level interaction between treatment and pretest score. In schools with lower pre-ITBS scores, the treatment slope was steeper than the control slope. However, in high aggregate pre-ITBS schools, this effect was reversed. These three-way interaction effects are most easily understood graphically. Therefore, Figures 8.3, 8.4, and 8.5 illustrate the relationship between pre-ITBS and post-ITBS scores in low aggregate pre-ITBS schools, high aggregate pre-ITBS schools, and average aggregate pre-ITBS schools. In average aggregate pre-ITBS schools, there appears to be no discernible treatment effect, based on the final HLM models. In low pre-ITBS schools, students with higher pretest scores do better in the treatment group, and students with lower pretest scores do better in the control group. In high pre-ITBS schools, students with lower pretest scores do better in the treatment group, and students with high pre-ITBS scores appear to do equally well in either group.

Table 8.7
HLM Results

	Null Model Coefficient (*SE*)	Model 1 Coefficient (*SE*)	Model 2 Coefficient (*SE*)	Model 3 Coefficient (*SE*)
Model for Intercept of post test score (β_{00j})				
Intercept				
Intercept (γ_{000})	205.75**	205.48**	205.37**	205.19**
	(1.26)	(0.81)	1.00	(0.99)
Mean school pre test (γ_{001})				3.78
				(2.86)
Treatment				
Intercept (γ_{010})			0.13	0.70
			(1.01)	(1.04)
Mean school pre test (γ_{011})				2.03
				(3.08)
Model for student achievement slope				
Intercept				
Intercept (γ_{100})		14.91**	13.74**	13.63**
		(0.46)	(0.63)	(0.63)
Mean school pre test (γ_{101})				2.17
				(1.80)
Treatment				
Intercept (γ_{110})			1.99*	1.91*
			(0.80)	(0.80)
Mean school pre test (γ_{111})				-6.14**
				(2.30)
Model for gifted effect				
Intercept				
Intercept (γ_{200})		6.76**	6.58**	6.79**
		(0.86)	(1.19)	(1.20)
Variance				
Level 1 (between students)	416.75**	243.02**	242.39**	241.66**
Var(e_{ijk})	(12.7)	(7.58)	(7.56)	(7.54)
Level 2 (between teachers)	68.00**	16.90**	17.34**	17.48**
Var(r_{0jk})	(13.57)	(4.63)	(4.69)	(4.70)
Level 3 (between schools)	34.79**	14.85**	14.20**	12.26**
Var(u_{00k})	(14.88)	(5.61)	(5.51)	(5.13)
Var(e_{10k})		2.19**	1.83*	1.03
		(2.49)	(2.44)	(2.32)
Goodness of fit				
AIC	20531.5	19243.6	19241.0	19236.6
BIC	20537.0	19257.7	19258.6	19261.3
Deviance	20523.5	19227.6	19221.0	19208.6
Parameters	4	8	10	14

* p<.05.
** p<.01.

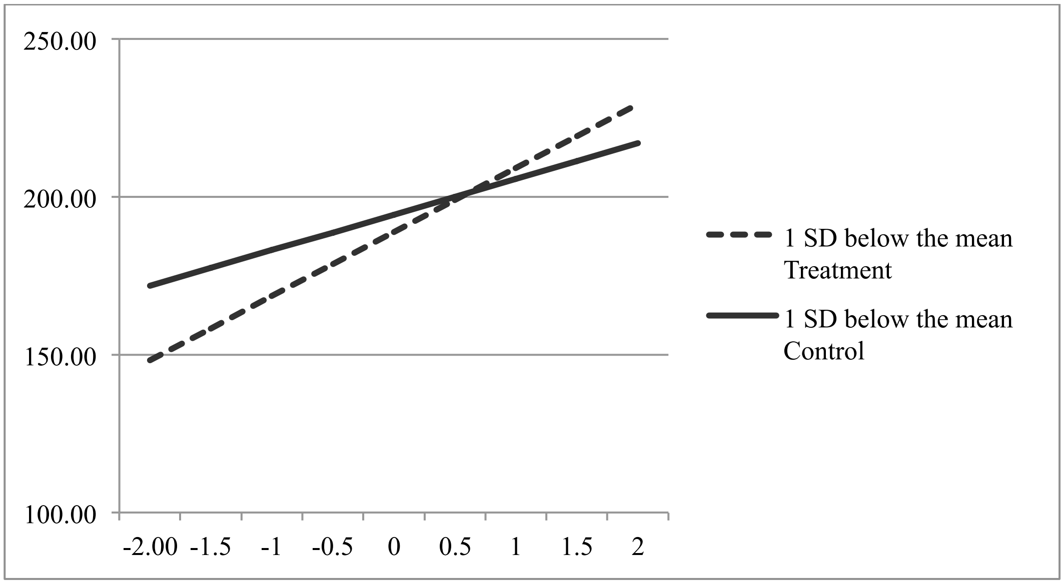


Figure 8.3. Predicted values for students with a given math pretest score (X-axis) on final math posttest score (Y-axis) in schools that scored one standard deviation below the sample mean on pre math achievement.

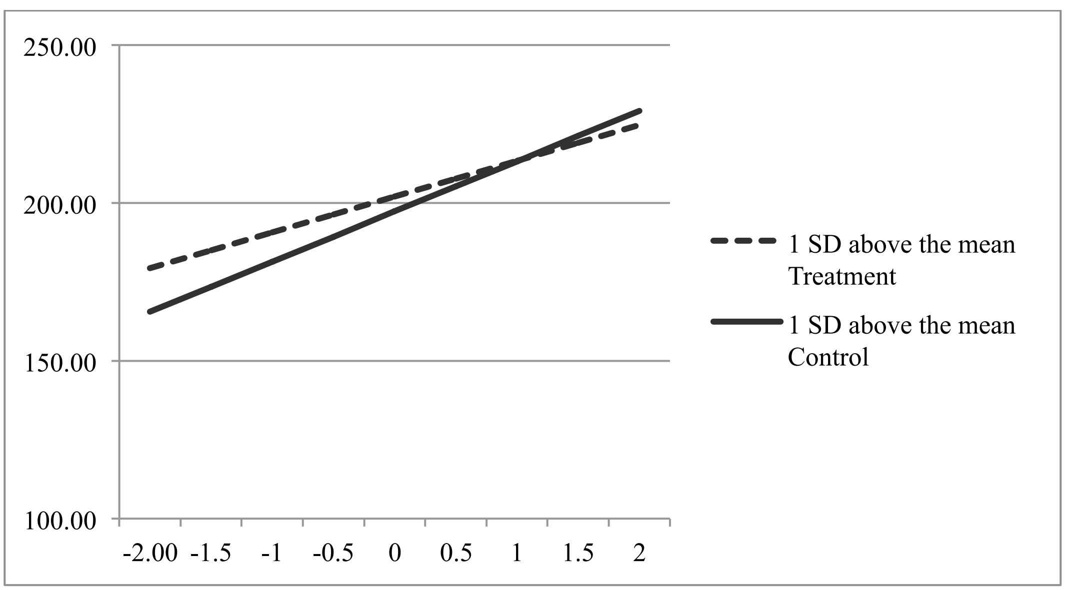


Figure 8.4. Predicted values for students with a given math pretest score (X-axis) on final math posttest score (Y-axis) in schools that scored one standard deviation above the sample mean on pre math achievement.

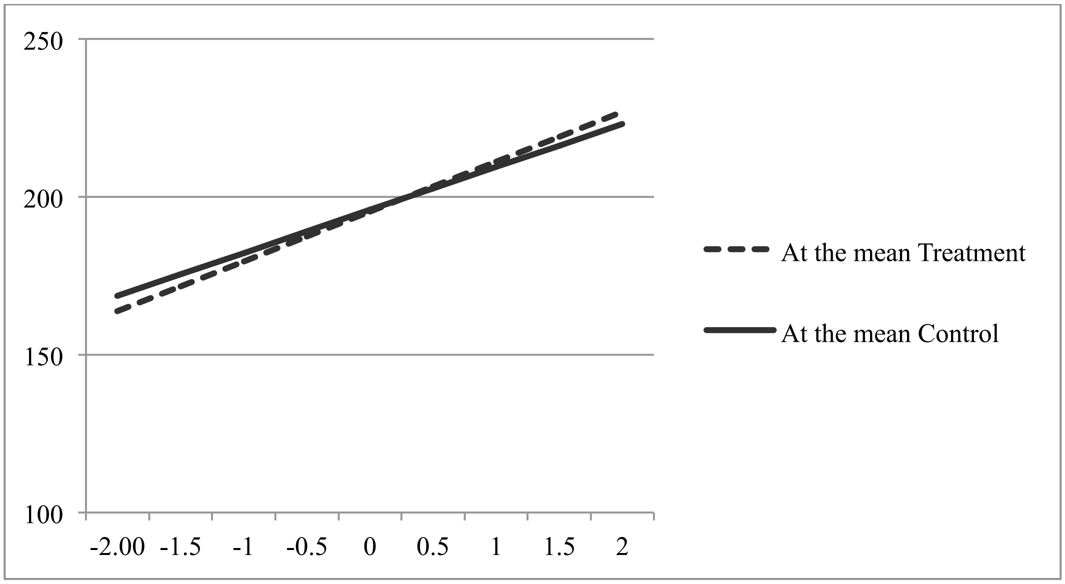


Figure 8.5. Predicted values for students with a given math pretest score (X-axis) on final math posttest score (Y-axis) in schools that scored at the sample mean on pre math achievement.

A substantial ceiling effect was present in Cohort III's outcome data. Over 12% of the students scored at the ceiling on the post ITBS at the end of third grade. In the control group, approximately 11% of the students scored at the ceiling, whereas in the treatment group, 14% of the students scored at the ceiling. This difference in the proportion of students who scored at the ceiling was statistically significant (p=.026), favoring the treatment group. In other words, students from the treatment group were slightly more likely to score at the ceiling of the test than students from control classrooms.

The presence of ceiling effects is troubling because only partial information is available about students who scored at the ceiling of the test. The ceiling effects prevent the study's capacity to distinguish among these students in terms of their abilities and to determine the students' true abilities. Scores that hit the floor or ceiling of an assessment are referred to as censored outcomes. Censored outcomes lead to biased parameter estimates in most statistical models (McBee, 2010; McDonald & Moffitt, 1980; Muthen, 1990). Given the considerable ceiling effect in the data, a multilevel Tobit model was employed. Unfortunately, MPLUS 6 did not allow for the estimation of 3-level Tobit models. Therefore, a 2-level Tobit model that explicitly modeled the student and classroom levels was specified using the TYPE=COMPLEX syntax to invoke the robust standard errors for the clustering at the school level in MPLUS. Table 8.8 compares the results of the 3-level multilevel model, which failed to account for censored outcomes to a 2-level multilevel model with corrected standard errors, which also failed to account for censoring, and finally a 2-level multilevel model with corrected standard errors that did account for the censored nature of the data.

Table 8.8

Results of Multilevel Analyses of the Treatment Effect Using a Tobit Model

	HLM Model Coefficient (*SE*)	MPLUS Non-censored	MPLUS Censored
Model for Intercept of post test score (β_{00j})			
Intercept			
Intercept (γ_{000})	204.17***	205.32***	206.09***
	(1.08)	(0.92)	(0.99)
Mean school pretest (γ_{001})	2.90	3.30	4.43
	(3.11)	(3.06)	(3.34)
Treatment			
Intercept (γ_{010})	0.81	0.51	0.87
	(1.10)	(0.96)	(1.08)
Mean school pretest (γ_{011})	1.12	2.44	1.76
	(3.25)	(2.80)	(3.23)
Model for student achievement slope			
Intercept			
Intercept (γ_{100})	13.46***	13.31	14.33
	(0.64)	(0.64)	(0.72)
Mean school pretest (γ_{101})	2.39	2.01	4.01*
	(1.81)	(1.78)	(1.82)
Treatment			
Intercept (γ_{110})	2.28	2.26**	2.79**
	(0.82)**	(0.75)	(0.83)
Mean school pretest (γ_{111})	-6.92**	-6.11**	-6.77**
	(2.36)	(2.16)	(2.29)
Model for gifted effect			
Intercept			
Intercept (γ_{200})	6.88***	6.67***	9.89***
	(1.27)	(0.91)	(1.24)
Variance			
Level 1 (between students)			
Var(e_{ijk})	245.89	242.31	303.04
	(7.84)	(7.56)	(12.62)
Level 2 (between teachers)			
Var(r_{0jk})=$\tau\beta$	17.07***	28.96	34.93
	(4.88)	(7.23)	(8.83)
Level 3 (between schools)			
Var(u_{00k})	14.85***		
	(6.07)		
Goodness of fit			
AIC	17596.6	19073.7	17580.8
BIC	17665.4	19136.8	17643.8
Deviance	17572.6	19051.7	17558.8
Parameters	12	11	11

Table 8.8 (continued)

Results of Multilevel Analyses of the Treatment Effect Using a Tobit Model

	HLM Model Coefficient (*SE*)	MPLUS Non-censored	MPLUS Censored
Model for Intercept of post test score (β_{00j})			
Intercept			
Intercept (γ_{000})	204.17***	205.32***	206.09***
	(1.08)	(0.92)	(0.99)
Mean school pretest (γ_{001})	2.90	3.30	4.43
	(3.11)	(3.06)	(3.34)
Treatment			
Intercept (γ_{010})	0.81	0.51	0.87
	(1.10)	(0.96)	(1.08)
Mean school pretest (γ_{011})	1.12	2.44	1.76
	(3.25)	(2.80)	(3.23)
Model for student achievement slope			
Intercept			
Intercept (γ_{100})	13.46***	13.31	14.33
	(0.64)	(0.64)	
Mean school pretest (γ_{101})	2.39	2.01	4.01*
	(1.81)	(1.78)	(1.82)
Treatment			
Intercept (γ_{110})	2.28	2.26**	2.79**
	(0.82)**	(0.75)	(0.83)
Mean school pretest (γ_{111})	-6.92**	-6.11**	-6.77**
	(2.36)	(2.16)	(2.29)
Model for gifted effect			
Intercept			
Intercept (γ_{200})	6.88***	6.67***	9.89***
	(1.27)	(0.91)	(1.24)
Variance			
Level 1 (between students)			
Var(e_{ijk})	245.89	242.31	303.04
	(7.84)	(7.56)	(12.62)
Level 2 (between teachers)			
Var(r_{0jk})=$\tau\beta$	17.07***	28.96	34.93
	(4.88)	(7.23)	(8.83)
Level 3 (between schools)			
Var(u_{00k})	14.85***		
	(6.07)		
Goodness of fit			
AIC	17596.6	19073.7	17580.8
BIC	17665.4	19136.8	17643.8
Deviance	17572.6	19051.7	17558.8
Parameters	12	11	11

* $p<.05$.
** $p<.01$.
*** $p<.001$.

When comparing the results from the 2-level Tobit model to the results from the 3-level linear model, the results are very similar. There is a tendency for the aggregate school pretest to be more strongly related to the posttest ITBS score, for the aggregate pretest achievement slope to be more strongly related to the outcome, for the aggregate school pretest to be even more strongly related to pretest-posttest achievement slope, and for the difference between "gifted" and "non-gifted" students to be slightly larger after controlling for all of the other variables in the model; however, otherwise the results remain remarkably similar. The magnitude of the treatment effect on the posttest ITBS was almost identical between the two models, and the effect of the treatment on the relationship between the pretest and the posttest ITBS scores was only slightly larger in the Tobit model than it was in the linear model (see Table 8.8). The consistency of the results from the multilevel Tobit model and the 3-level linear model indicate that accounting for the ceiling effect observed in the data does not appreciably alter our findings. Neither the Tobit model nor the standard regression model provides evidence of a statistically significant treatment effect.

Descriptive Analyses of the ITBS Data

Although the results of the main analyses found no statistically significant main effect for the math intervention, the cross-level interactions described above indicated differential treatment effects for different subpopulations of students. To better understand the nuanced pattern of treatment effects, a series of follow-up descriptive analyses revealed for whom the math intervention appeared to have the largest effects. First, the descriptive analyses examined differences in the treatment effect for students of different relative ability levels. Because the intervention had its roots in the pedagogy of gifted education, it seemed plausible that high achieving students might benefit more from the treatment than average or low achieving students.

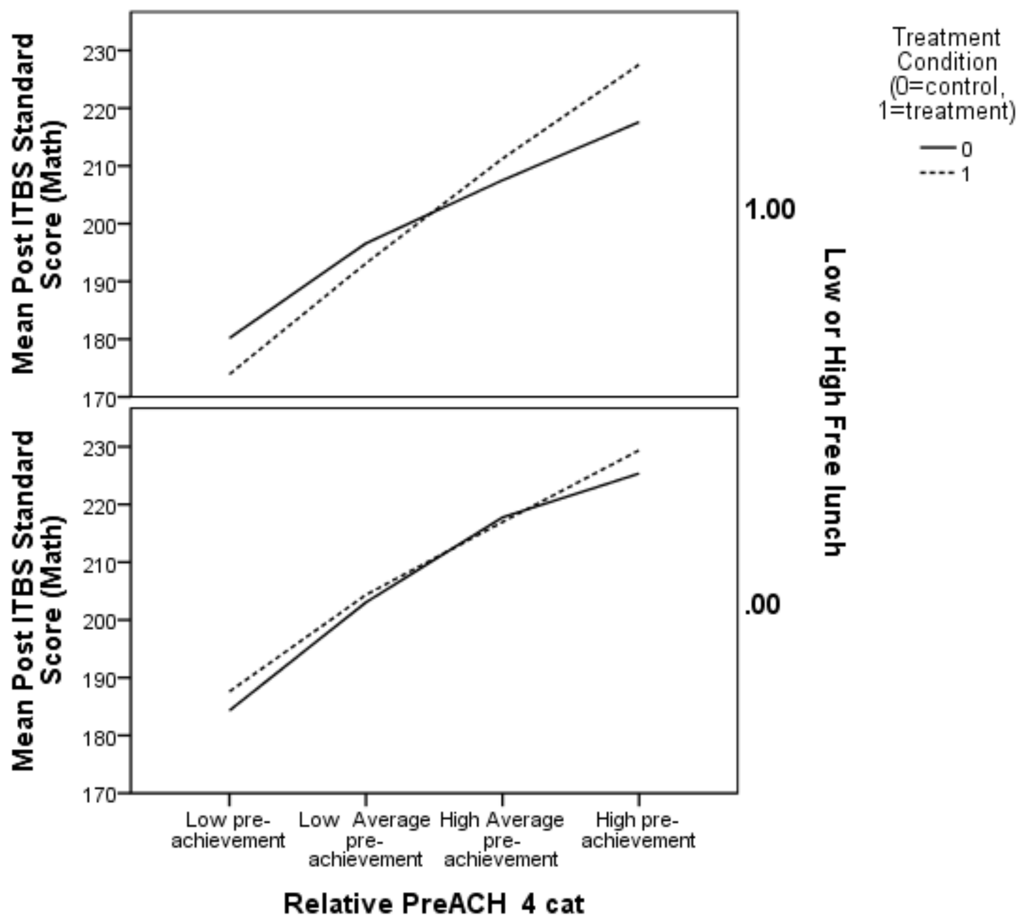
To examine this hypothesis, students were classified into four subgroups: students who scored at least one standard deviation below their school's mean on the ITBS pretest (low achievers), students who scored between one standard deviation below their school's mean and their school's mean (low average achievers), students who scored between the school mean and one standard deviation above the school mean (high average achievers), and students who scored at least one standard deviation above their school's mean on the ITBS pretest. Table 8.9 contains the results of these analyses. For the low achievers, the low average achievers, and the high average achievers, there was essentially no treatment effect. The Cohen's *d* effect sizes for these three achievement groups ranged from -0.09 to +0.04. However, for the highest achieving students, the results were quite different. For students with pretest achievement scores that were at least one standard deviation higher than their schools' means, the treatment did appear to provide some benefit. Treatment group students with high relative achievement scored six points higher than their control counterparts. This translates into an effect size of 0.41 standard deviation units, which is a fairly moderate effect size. The students with the highest pre-achievement scores relative to their classmates scored better in the treatment condition than in the control condition, and this advantage was observed regardless of the socio-economic composition of the school. In fact, in the schools with higher numbers of free lunch eligible students, the treatment effect for the high achieving students was even larger. Whereas the treatment advantage was approximately four points in schools with fewer than 15% free lunch eligible students, the treatment advantage was more than 10 points in schools with more than 15% free lunch eligible students. Figure 8.6 graphically depicts the treatment effect for high achieving students in schools with low, moderate, and high numbers of free/reduced lunch eligible students.

Table 8.9

Mean Posttest Achievement of Treatment and Control Students in Four Categories of Pretest Achievement

Experimental Condition	Pretest Achievement				
	Low Pretest	Low-Average Pretest	High-Average Pretest	High Pretest	Total
Control					
Group *N*	133	301	338	127	899
Mean	182.66	200.4	213.41	222.60	205.80
SD	15.99	19.13	19.09	16.46	22.03
Treatment					
Group *N*	168	523	495	205	1391
Mean	181.34	198.66	214.18	228.61	206.51
SD	17.37	19.54	19.00	13.69	22.98
Effect Size					
Cohen's *d*	-0.08	-0.09	0.04	0.41	0.03

What about the low achievers? Overall, the low achievers performed equally well in the treatment and control conditions. However, in the moderate and high free lunch schools, the control group students did outperform the treatment group students by over six points, which translates into a Cohen's *d* of nearly 0.40 standard deviation units. However, in the low poverty schools, low achievers in the treatment group outperform comparison students by over three points. Therefore, the findings for the lowest achieving students are less consistent than the findings for the high achieving students.



Note: 1=High Free lunch schools 0=Low Free lunch schools

Figure 8.6. School mean posttest achievement interactions between treatment and control groups across four levels of school mean pretest achievement, disaggregated to low and moderate/high free/reduced lunch eligibility.

Discussion of ITBS Achievement Test Results With Treatment and Control Group Students

This study revealed a fascinating three-way interaction between school pre-ITBS score, treatment condition, and the pre-ITBS/post-ITBS mathematics achievement slope. Although the analyses failed to support a main effect for treatment, the treatment moderated the pre-ITBS math achievement slope such that, holding school characteristics constant, treatment classes exhibited more positive pre-ITBS math achievement slopes than control classrooms. However, this effect itself was moderated by school aggregate pre-math achievement. In schools with higher than average initial math achievement, the 3-level interaction term exerted a negative influence on the pre-ITBS math achievement slope, counteracting and in the highest achieving schools, reversing the positive effect of the treatment on the pre-ITBS math achievement slope. In contrast, in the lower than average pretest schools, the positive effect of the treatment on the pre-ITBS math achievement slope was magnified. Further, the treatment had a universally positive effect on the highest achieving students, regardless of the context of the school, suggesting that this intervention is appropriate for gifted and high achieving students. The treatment effect was less clear for average and low achieving students. Whereas low achieving treatment students in more affluent schools outperformed their control counterparts on the ITBS, the opposite pattern was observed for low achievers at moderate/high free/reduced price lunch schools.

The results of this study underscore the importance of examining treatment effects for differentiated instruction using nuanced comparisons among treatment effects. Whereas the treatment appeared to benefit high pre-ITBS students in the low aggregate pre-ITBS schools, it appeared to benefit low pre-ITBS students in the high aggregate pre-ITBS schools. Further, although the treatment appeared to universally benefit the highest achieving students, the magnitude of this benefit differed according to the context of the school. The highest achieving students in the most affluent schools received less of a benefit than the highest achieving students in the less affluent schools. There are several possible reasons for these findings. First, it is possible that the differentiated curriculum provided more challenge to the top students in the lower achieving or less affluent schools, allowing them to work more to their potential. However, the lowest achieving students in the less affluent, lower achieving schools seemed to do slightly better with the control curriculum. Perhaps they received more support, more repetition, and more remedial instruction in the general education curriculum, which may have served them better. In contrast, in the higher achieving more affluent schools, the lower achieving students appeared to do better in the treatment group. However, the lower achieving students in the high aggregate pre-ITBS schools had stronger math skills than those in the lower achieving schools; therefore, the additional challenge, enrichment, and differentiation seemed to serve them well. The highest achieving students in the most affluent, highest achieving schools seemed to do very well with either the treatment curriculum or the control curriculum, although the highest achieving students did do slightly better in the treatment condition. Perhaps the "business as usual" curriculum at the highest achieving schools was more challenging than the math curriculum at the lower achieving schools. In fact, it is possible that the "business as usual" condition in the highest achieving schools was more similar to the treatment than the "business as usual" condition was in the lower achieving schools. Also, given the ceiling effects observed on the posttest, the math achievement of the highest students may not be adequately measured, even using the Tobit model for censored data. The highest students were working above grade level, and the on-grade level assessment did not adequately assess above-grade-level content.

National Assessment of Educational Progress Results

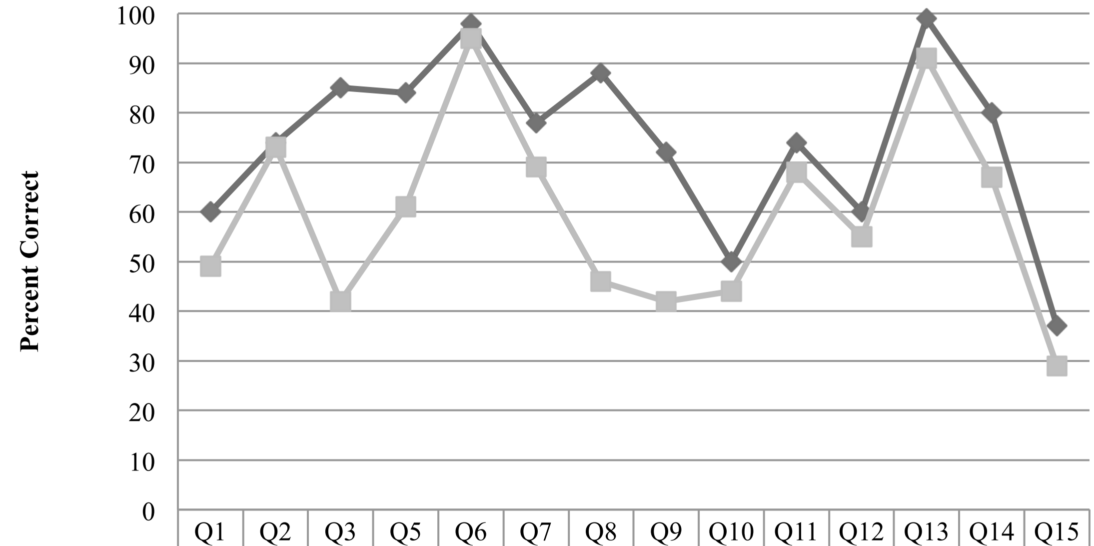
Due to the challenging content of the criterion-referenced unit tests and the above-grade level content from the National Assessment of Educational Progress, these two assessments were administered to treatment group students only. Treatment group students completed pretests and posttests before and after each of the units in algebra, geometry and measurement, and graphing and data analysis; they completed NAEP items after the intervention.

The composition of the treatment group students involved in the unit and NAEP tests was not a perfect match with the students involved in the ITBS pretests and posttests. Prior analyses described above involved treatment and control group students with matched achievement test scores and valid ability scores, while the criterion-referenced unit tests involved treatment group students only with matched unit test scores. ITBS scores may or may not have been available for these students. Descriptions of the results follow.

As part of participation in the NRC/GT mathematics study, grade 3 students who received the What Works Mathematics Curricula completed 15 mathematics items from the NAEP. (Note: Question 4 was not used in the analysis due to problems with the item.) The items were chosen for their similarity to the conceptual knowledge contained in the University of Connecticut's curriculum.

Although students involved in the study were in grade 3 during the curriculum implementation, the items on which they were tested were drawn from the grade 4 NAEP item bank. "Above-grade level" testing targets student achievement above the levels that could be measured by "at-grade level" items. For each item tested, the percentage of students who responded correctly to the item is shown. With the exception of question 15, 50-99% of the students responded to the item correctly (see Figure 8.7). These results represent the 1,615 treatment students who completed the NAEP items.

When the content of the items most closely matched the units, a higher percentage of students responded correctly, as these concepts were covered in multiple ways throughout the units. This provides further evidence of how students were able to learn when presented with challenging and differentiated curricula.



	Q1	Q2	Q3	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12	Q13	Q14	Q15
Study	60	74	85	84	98	78	88	72	50	74	60	99	80	37
Nation	49	73	42	61	95	69	46	42	44	68	55	91	67	29

Graphing & Data Analysis   Geometry & Measurement   Algebra

Figure 8.7. Comparison of percentage of correct responses on NAEP items by Cohort III treatment students and national sample.

Criterion-referenced Unit Test Results

The pre-differentiated and enriched curriculum units were developed in line with the NCTM's (2000) Principles and Standards for School Mathematics, which provides guidance for educational decision makers in grades Pre-K through 12. The development of these units assumed that students in grade 3 possess the prior knowledge indicated by the standards for grades Pre-K-2 and extends this knowledge by focusing on the standards for grades 3-5. In addition, the authors of these units relied on the NCTM focal points, which provided additional specificity of content for grade 3.

Each unit included challenging content and a unit test with a high ceiling so all students could demonstrate growth. Despite the challenging content, across the sites all students showed significant gains from pretest to posttest in all three units. These results demonstrate that all students can learn when presented with challenging and differentiated curricula (see Figure 8.8). These results represent the following number of students in treatment classes who participated in the research study, algebra (*n*=1,529); geometry and measurement (*n*=1,647); and graphing and data analysis (*n*=1,620).

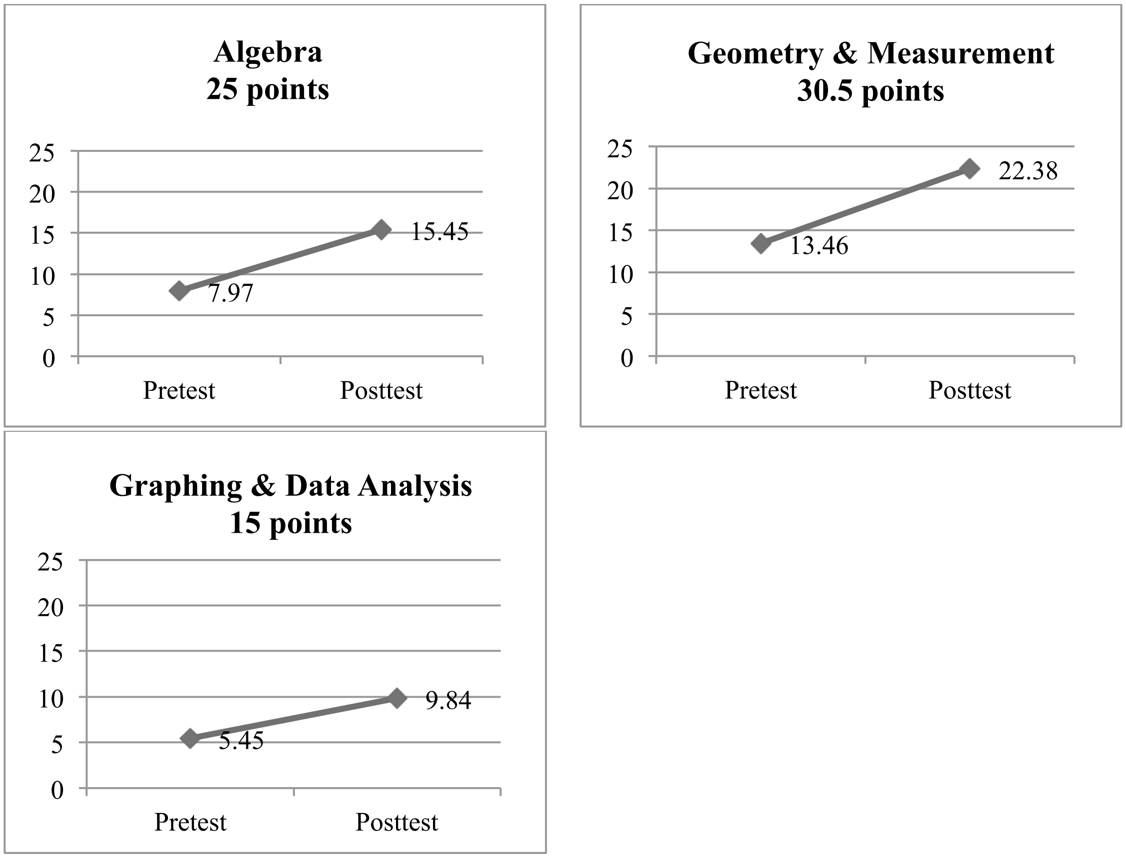


Figure 8.8. Gains on unit tests across all sites for Cohort III treatment students.

As each test had a different total score, effect sizes provided a method for comparing change across different measures. The unit test analyses utilized a Cohen's *d* measure of effect size with the pooled standard deviation as the standardizer. The gains of students on each test was considerably larger than one standard deviation unit (see Figure 8.9). This demonstrated that Cohort III students made practically significant gains when presented with challenging curricula based on gifted and talented pedagogy.

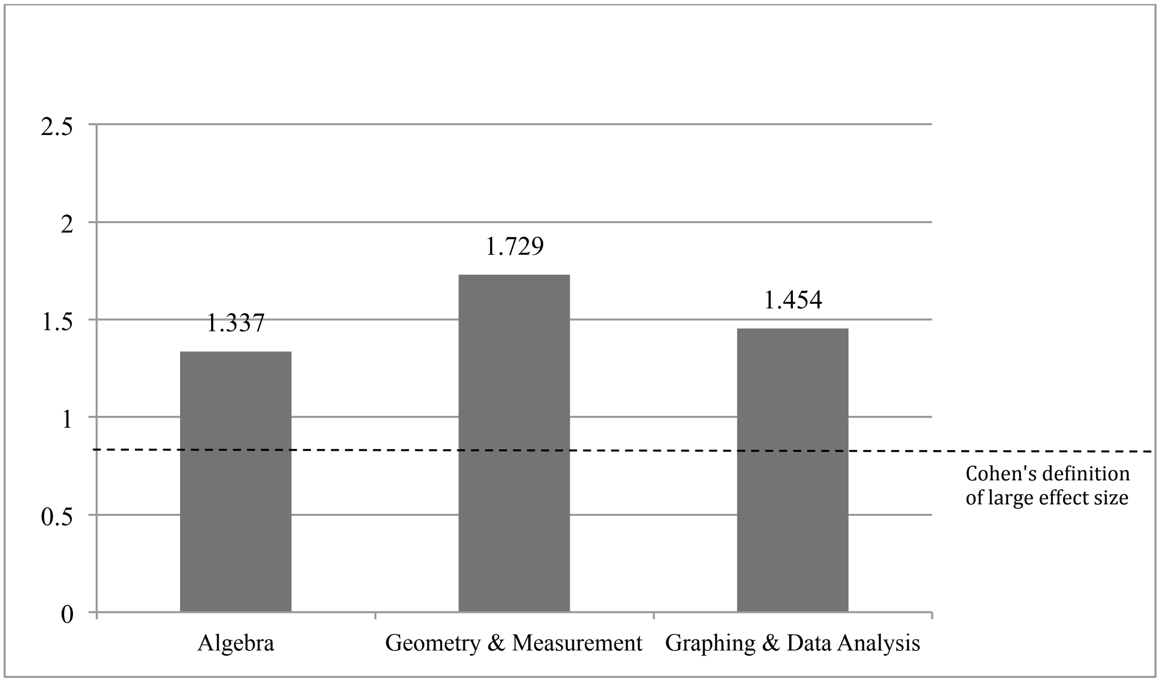


Figure 8.9. Effect sizes of Cohort III treatment student gains.

Pretest and posttest unit data indicated that students successfully learned and applied the challenging content and concepts presented in the algebra, geometry and measurement, and graphing and data analysis units. Students' academic needs were met by the teachers' implementation of the differentiated curriculum units based on the pedagogical models of gifted and talented education. The designers of each What Works Mathematics Curricular used several strategies proposed by these models' authors as they created lessons to engage both students and their teachers in teaching mathematics.

Discussion of Unit Tests and NAEP Test Results

The academic variability of students in general education programs is evident as teachers analyze students' progress in understanding and applying mathematical content, concepts, and skills. Oftentimes, teachers must use different level textbooks and multiple technology- and non-technology resources to find appropriate materials for students who need further practice with specific content, concepts, and skills and for those who have mastered them. They have to manage all of these resources and decide how to group or regroup students to ensure they are learning what they need to know rather than re-learning mastered content. To help teachers offer students challenging and meaningful mathematics content that will promote students' achievement of new learning goals, the research team designed three units in algebra, geometry and measurement, and graphing and data analysis based on widely adopted models in gifted and talented education. Curricular and instructional models by Tomlinson, Kaplan, and Reis and Renzulli place the teacher in the role of knowledge broker, facilitator and guide, emphasizing differentiation of curricula in general education classrooms as well as pullout and special classes. These models were analyzed for their similarities and differences to determine the curricular and instructional principles to guide the development of differentiated lessons for three mathematics units in algebra, geometry and measurement, and graphing and data analysis.

Using curricular and instructional strategies promoted by the models' authors, the team designed the mathematics units as replacements for the general mathematics curriculum. Implementation of the units required 16 weeks, which is more than 40% of the school year. Cohort III teachers embraced the demands of the curricula and the research study as 91% of the treatment classrooms completed the algebra unit, 95% and 96% completed the geometry and measurement and graphing and data analysis units, respectively.

Treatment group students experienced content, concepts, and skills in-depth as warranted by their presenting skills. Dependent *t*-tests were used to explore research question 2. The results documented statistically significant differences between the pretest and posttest criterion-referenced unit tests, and Cohen's *d* values verified large effect sizes, confirming that students learned and applied challenging content, concepts, and skills in algebra, geometry and measurement, and graphing and data analysis. Students were successful in achieving new learning goals. In addition, descriptive statistics for research question 3 revealed that at least 50% of the treatment group students mastered all but one of the grade 4 items in algebra, geometry and measurement, and graphing and data analysis from the National Assessment of Educational Progress.

Limitations

There were several limitations to the current study. The largest limitation concerns the chosen posttest, the ITBS Math Problem Solving and Data Interpretation subtest. Obviously, the ceiling effect on the test was a major problem. However, equally problematic was the alignment of content on the ITBS Math Problem Solving and Data Interpretation subtest with the curriculum that was taught in the differentiated and enriched math units. Unfortunately, the differentiated and enriched curricular units that were taught as part of the intervention did not align well with the ITBS subtests that were chosen to evaluate the effects of the intervention. There was no algebra nor geometry and measurement content on the ITBS posttest, and the three units that treatment teachers taught were algebra, geometry and measurement, and graphing and data analysis. Therefore, at least two-thirds of the content covered in the intervention did not appear on the posttest. This is a very unfortunate occurrence, and it makes it impossible to truly adequately assess the effects of the treatment. Perhaps there were no main effects of the treatment, or perhaps another assessment that was better aligned with the treatment curriculum would have detected effects that the ITBS did not. Unfortunately, it was not possible to disentangle this confound during the current study.

Another limitation is that the unit tests and the NAEP selected items, which were more aligned with our pre-differentiated and enriched curricula, were only administered to the treatment group students. There were three major reasons for this decision during the planning phase of the study.

1. School administrators expressed concerns about the amount of time required for testing.
2. The challenge level of the curricula was high; therefore, concern existed that some students might demonstrate limited understanding of the content, concepts, and skills at pretest administration, as their textbooks may not require in-depth study of the selected content areas. This response to pretests would not engender a perspective that pretests allow teachers to guide their students' new learning because the control students would not have access to our mathematics curricula. It was plausible that students, teachers, and parents would express concern about what their students did or did not learn and what steps would need to be taken to remedy the situation. Without access to our curricula and with the importance of maintaining "business as usual" in the control condition, the unit tests potentially would have frustrated educators and students alike. Therefore, the unit test assessments were given to treatment students only.
3. Items selected from the National Assessment of Educational Progress intended to determine the extent to which treatment group students would be able to demonstrate mastery on grade 4 items. The ceiling on these items needed to be high enough to determine if students could master challenging content based on their experiences with our mathematics curricula. Requiring control students to take yet another challenging assessment without opportunities to learn the high-level content, concepts, and skills was not deemed to be the most appropriate educational decision.

In retrospect, administering the criterion-referenced unit tests and the NAEP items to treatment and control students would have offered valuable alternate outcomes to assess the effectiveness of the What Works Mathematics Curricula intervention. Instead of presenting these assessments to administrators as tests (with the potential to induce student anxiety and distress), they could have been presented as opportunities to discover what students know, what they need to know, and what teachers can do to help them learn the content, concepts, and skills. An additional recommendation based on the present study's limitations would be that researchers seriously consider using above grade level assessments for research involving differentiation, especially if they are interested in capturing the growth and achievement of the highest achieving students.

In the future, the National Research Center on the Gifted and Talented team hopes to conduct another efficacy trial of the intervention using assessments that are more closely aligned to the curriculum and which have adequate coverage of above-grade-level material.

Conclusions and Recommendations

The What Works in Gifted Education Mathematics Study involved schools throughout the country. The theoretical framework was based on Renzulli and Reis's (1997) conception of the act of learning, which is central to the change process if educators want to ensure that all students are exposed to challenging, pre-differentiated and enriched curricula. Too many times, struggling students are faced with repetitive curricula that is not tailored to what they know and what they need to know to fully understand the content, concepts, and skills, while average achieving and high achieving students are not presented with appropriate levels of challenge that will foster continued growth in learning. These students may be waiting to learn new concepts or just re-learning prior concepts. The act of learning emphasizes the interaction between student, teachers, and curricula. Educators must consider:

> 1. present achievement levels . . . [in mathematics],
> 2. the learner's interest in particular topics and the ways in which they can enhance present interests or develop new interests, and
> 3. the preferred styles of learning that will improve the learner's motivation to pursue the material that is being studied. (Renzulli & Reis, 1997, p. 35)

The critical importance of the interactions between and among the teacher, student, and curricula was constantly reviewed while developing the algebra, geometry and measurement, and graphing and data analysis units using models developed by Tomlinson, Kaplan, and Reis and Renzulli. The pre-differentiated and enriched curricula were purposely designed to be challenging; therefore, lessons were scaffolded to provide more or less details to help students master the concepts and to offer curricular extensions when students demonstrated their knowledge and understanding of the mathematics.

In the curriculum development process, the What Works Mathematics Units were developed to provide in-depth lessons in the three areas of mathematics. The modified, pre-differentiated units focused on conceptual thinking, replacing 16 weeks of the general education grade 3 mathematics curriculum. Research question 1 hypothesized that treatment group students would outperform control group students on the ITBS Math Problem Solving and Data Interpretation subtest. Instead, the post-ITBS scores of students in the treatment group were equal to those in the control group. Several conclusions can be posited:

1. The ceiling on the norm-referenced test was not high enough to record students' true level of content, concepts, and skills mastered in problem solving and data interpretation.
2. The norm-referenced ITBS was not a good match to content in the algebra and geometry and measurement units.
3. The lack of a main effect illustrated that eliminating 16 weeks of the "business as usual" curricula for the treatment group students did not have a negative impact on students involved in the intervention.

4. The curricula benefited students differentially depending on the achievement status of their schools and their designation as treatment group or control group students.

A defensible interpretation of the results from research question 1 is to consider that treatment teachers were able to replace grade level curriculum with more challenging, pre-differentiated and enriched curriculum without negatively impacting standardized test scores. In the current age of increased accountability, teachers are often afraid to stray from the mainstream curriculum for fear of jeopardizing their state test scores. Assuming the ITBS posttest measures the typical grade 3 mathematics curricula, the current study provides some evidence that teachers can replace typical at-grade level curriculum with more challenging, pre-differentiated and enriched curriculum without suffering adverse consequences on standardized assessments. Viewed through this lens, the results of this study should encourage teachers to consider stepping out of the lock-step curriculum to differentiate their math curriculum.

The results of data analyses for research questions 2 and 3 focusing on the unit tests and NAEP test provided a more positive picture of the efficacy of using differentiated mathematics curricula with students of all abilities. This statement is made with full recognition that the more stringent research design of comparing the results of the treatment and control groups was not operative with these two research questions. Further study of the impact of the differentiated curricula is warranted.

The study produced several recommendations for future researchers. First, it is essential that the outcome measure adequately assesses the constructs that are taught as part of the curriculum. Standardized assessments may have very narrow content coverage. Second, seriously consider using out-of-level assessments, even when assessing the entire grade level, as ceiling effects are clearly evident for on-grade assessments. Third, consider administering researcher-developed measures to both the treatment and control conditions. Tremendous learning gains occurred between the unit tests administered to the treatment group students at pretest and posttest. Unfortunately, because those tests were not administered to the control group students, there is no way of comparing the growth of the treatment group students to the growth of the control group students. Future research will incorporate researcher-developed measures into the assessment plan for both treatment and control group students. As with all research, finding answers to initial hypotheses often results in myriad questions. Some answers provide guidance for future studies, while other questions remain.

CHAPTER 9: Cohort III Qualitative Results

Cindy M. Gilson
Micah N. Bruce-Davis

Researchers from the University of Connecticut's National Research Center on the Gifted and Talented conducted observations of treatment and control teachers who participated in the What Works in Gifted Education Mathematics Study. Treatment teachers were not instructed to teach a specific lesson from the What Works Mathematics Curricula. Therefore, the researchers observed a variety of lessons. Some treatment teachers taught lessons from the *Awesome Algebra, Greening Up With Graphing: Recycle, Reduce, & Reuse*, or the *Geometry & Measurement for All Shapes & Sizes* Units, which were developed at the University of Connecticut. Control teachers taught a mathematics lesson that was from their district or school mathematics program.

In both the control and treatment situations, the nature of the lessons observed varied. Observations may have included the whole mathematics lesson or only a portion of the mathematics lesson. Some lessons that were observed incorporated review in preparation for upcoming state testing. In some cases, treatment teachers were observed either introducing one of the What Works Mathematics Curricula or reviewing what had been learned from the mathematics unit. In other instances, treatment teachers scaffolded for students by teaching prerequisite skills before beginning one of the What Works Mathematics lessons.

The following summary presents qualitative findings based on observations of Cohort III teachers' classrooms. Cohort III teachers participated in the study during the 2009-2010 school year and were randomly assigned to treatment or control status by teacher.

The qualitative results are presented in response to the research question that guided the analysis of the Cohort III treatment and control classroom observations, teachers' logs, focus group interviews (see Appendix E for a list of Cohort III Focus group pseudonyms), and administrators' interviews (see Appendix A for the administrators' interview and teacher focus group protocols and Appendix C for the data source key):

> How do teachers and administrators respond to their access to pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis?

Five main themes were identified through inductive coding using QSR's NVivo 9 Qualitative Analysis Software. The major themes that emerged from the qualitative analyses include: (a) challenge, (b) questioning and discourse, (c) preassessments and grouping, (d) teacher and administrator reflections, and (e) concerns over state standards. Table 9.1 shows the specific data sources supportive of each category.

Table 9.1
Cohort III Data Sources for Qualitative Findings

Data Source/ Abbreviation	Challenge	Questioning & Discourse	Preassessments & Grouping	Teacher & Administrator Reflections	Concerns Over State Standards
Treatment Teacher Classroom Observation (TTO)	✔	✔	✔		
Control Teacher Classroom Observation (CTO)	✔	✔	✔		
Teachers' Logs (TL)	✔		✔		✔
Teachers' Focus Groups (FG)	✔		✔	✔	✔
Administrators' Interviews (AI)	✔		✔	✔	✔

Challenge

Recurring evidence of the degree to which teachers challenged students mathematically emerged across four data sources: classroom observation data, teachers' logs, teachers' focus groups, and administrators' interviews. For the purpose of coding the multiple data sources, challenge was defined as instances in which students were engaged in demanding or stimulating mathematics activities, assignments, or class discussions that involved multiple solutions or strategies, depth and complexity, higher level thinking, the need to explain or support one's thinking, or positive reactions as student mathematicians.

First, the findings related to challenge as evident from the classroom observation data are presented. Evidence of how teachers challenged students and their students' responses to challenge are further divided into four main subsections: (a) larger and more complex numbers (b) complex concepts or skills, (c) explaining one's thinking, and (d) struggle in response to mathematical curricula. Examples from both the treatment and control classroom observations for each of these four main subsections are shared.

Second, the findings related to teachers' and administrators' perceptions and responses to challenging students with the What Works Mathematics Curricula are presented. Data for this second section emerged from the analyses of the teachers' logs, teachers' focus groups, and administrators' interviews.

Classroom Observations of Challenge

As indicated by the Cohort III classroom observations, treatment teachers challenged students more often than control teachers. Forty-five percent (n=36) of the Cohort III treatment teacher observations showed evidence of students being challenged, while only 38% (n=19) of the control teacher observations showed evidence of challenge. When teachers in the Cohort III treatment and control classrooms attempted to challenge students, they both did so in a variety of ways.

Larger and More Complex Numbers

Both treatment and control teacher observations in Cohort III were coded as demonstrating evidence of challenging students if teachers extended student learning by the use of larger and more complex numbers. First, examples of how treatment teachers challenged students in this way are presented followed by instances from the control teacher observations.

Cohort III Treatment Teacher Observations

Treatment teachers challenged students in mathematics by using larger or more complex numbers. In one example, Ms. Snyder was working with students on a lesson from the *Awesome Algebra* Unit. The lesson involved calculating the total number of fish that could be seated around 7, 14, and then 22 under-sea "tables" that were aligned side-by-side. Square tiles were used to represent the tables. Ms. Snyder purposely gave students various numbers of tables to have students explore creating a formula based on a pattern. The researcher observed the following interaction between Ms. Snyder and her students as she challenged them using larger numbers:

> T: Ok, can any of you come up with a rule, which will work for ANY number of tables that I give you?
> S1: Tables times two plus two equals the number of fish.
> S2: Another student goes up to the Smart Board and writes: "(tx2)+2=f."
> T: What has [S2] done? Turn and talk to your neighbor and discuss what [S2] has written. (After a brief period) Ok, what about 3,421,143 tables?
> (In a short while, the students come up with the answer: 6,842,288). (TTO, Cortana, NE)

In a different lesson from the *Awesome Algebra* Unit, students learned how to predict a certain letter if a person's name were written repeatedly. The lesson was modeled previously by having students figure out what the 18th letter would be if the name Davis were written over and over. In differentiated groups, students then were challenged to determine the 42nd, 82nd, and the 95th letter that would be written. As an additional challenge when students finished their work, Ms. Zimmerman asked them to use their own names, predict the 50th letter that would be written, and explain how they arrived at the answer (TTO, Terracotta, NE).

Teachers also challenged students using larger numbers while encouraging them to make predictions after interpreting data displayed in a line graph. For example, during a lesson from the What Works *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, Ms. Simmons had students predict what would happen if they continued the graph for 100 days. The given line graph was filled in from 0 to 50 days only. She also had them explain their predictions (TTO, Smithton, SE).

In addition, treatment teachers used larger numbers to determine if students were able to apply what they had learned about patterns in a hundreds chart. Students in Ms. Powell's class first explored multiples of 2 using a hundreds chart that only went up to the number 100. The observer noted an instance in which Ms. Powell then challenged the students with larger numbers:

> T: What column the number 346 would be in?
> S: (hesitating) 6.
> T: Why did you pick 6?
> S: The 6 is in the ones position.
> T: What about 910?
> S: The last number is zero.
> T: You should call it the "ones column" instead. (TTO, Smithton, SE)

The teacher ends with the examples 555 and 111. The students answer correctly.

Cohort III Control Teacher Observations

As a means for challenging students, Cohort III control teachers also encouraged students to use larger numbers when applying mathematics concepts but to a lesser extent than the treatment observations. For instance, Ms. Banks encouraged her students to take on a challenge during a multiplication activity in which students created pictures and number stories to go along with a multiplication number model:

> If you want a little challenge, try using the numbers 50 and 40. You will probably not be able to draw a picture. It will take a long time to do. You will write a number model to solve and prove your answer. (CTO, Morrowind, NE)

One student accepted this challenge, and indeed the computational aspect of the number model was a challenge. The student was not sure how to multiply 50 and 40, but she eventually figured out how to use her fingers to arrive at the solution.

Similarly, other control teachers prepared students for a challenge by communicating to them ahead of time that the work will be challenging. In reference to comparing fractions with unlike denominators, Ms. Hunt mentioned, "Number 3 is the challenging one—the bottom numbers are all different" (CTO, Farnsworth, NE).

Complex Concepts or Skills

Another way in which challenge manifested itself was through complex concepts or skills that were introduced, taught, or reviewed by the teachers. Observations were coded as a complex concept or skill if they were more advanced in comparison to the mathematics standards as provided by the National Council of Teachers of Mathematics (NCTM, 2000).

Cohort III Treatment Teacher Observations

Observations of the Cohort III treatment classrooms revealed that students were challenged to analyze data, formulate rules, and explore patterns. Students were also challenged to think like mathematicians, explore and solve real-world problems, interpret data, find multiple solutions to problems, work with variables, and learn content not typically taught in grade 3 such as the hypotenuse of triangles or baselines for data collection.

In some instances, students were challenged to find multiple solutions to real-world problems while thinking like mathematicians. For example, students participating in the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit in Ms. Adam's class had the opportunity to explore multiple options to measure the effects of possible interventions to increase recycling in their school:

> The students brainstormed various options for tracking the amount of paper that was being recycled. They narrowed their options as they thought about the most efficient way to collect the data. Students actively participated in the discussion and offered their ideas to the teacher and to the other students. As noted by the observer, the level of engagement in the lesson was high throughout the classroom. (TTO, E. Halsey, SW)

Similarly, students participating in a lesson from the *Geometry & Measurement for All Shapes & Sizes* Unit were also challenged to find multiple unique solutions to a mathematics problem. As their teacher, Ms. Andrews, instructed:

> Pretend you are a landscaper. You have been given 30 feet of fence. Make a rectangle on the property using the 30 feet of fencing. Write the dimensions on each side of your rectangle. The perimeter must equal 30 feet. [Some students finish quickly; she asks them to find a different way to get a 30-foot perimeter] Try to do one nobody else has done. (TTO, Centurion, NE)

Ms. Andrews provided even more challenge for students who finished early by having them work on an extension assignment with a partner:

> As an extension for students who are already finished, the teacher assigns the next page (113). As an extra challenge for students who finish p. 113, the teacher has them partner up with one partner drawing a shape. The two partners then work to figure out the perimeter of that shape, and then the other partner is to draw a different shape with the same perimeter. Both partners check the second figure to make sure it has the same perimeter as the first.

Ms. Armstrong posed an open-ended, exploratory challenge for students working with the *Geometry & Measurement for All Shapes & Sizes* Unit. She explained:

> I'm going to hand out these envelopes to each group. In the envelope are paper clips, a rubber band, a pencil, and some string. I want you to work on making a perfect circle with these tools. Write down the way you did it. (TTO, Forge Hill, NE)

In Ms. Bloomstein's class, students were learning how to determine the possible values of missing variable(s) from a lesson out of the *Awesome Algebra* Unit. As noted by the observer, Ms. Bloomstein scaffolded the lesson before challenging students to determine the values of two missing variables:

> She gives examples of problems they have already done, such as 9+_____=10, and then demonstrates how the same problems look with variables instead of blanks: 9+a=10. She asks them, "If a+b=6, what could the variables *a* and *b* be? Turn to the person next to you and discuss what those variables could be." She tells them that she hears "good things" and that she can tell they understand. She allows the students to share what their partners came up with. (TTO, Terracotta, NE)

Interestingly, students who were observed participating in the *Awesome Algebra* Unit lessons demonstrated an ability to persevere while being challenged. In the following example, the students were trying to determine the total number of fish that would fit around a given number of square tables when pushed together in a row. Students were to record in a chart the total number of fish that corresponded to the given number of tables. In Ms. Horton's class the observer noted:

> They realized it increases by 2 so they just filled in the whole chart without getting the manipulatives. Then when they got the manipulatives they laid them out. One boy said, "Oh, I was wrong. James was right." Then they started to place the fish around the tables. So they counted by twos and added it on at the end. One student added the two 3s at the end and only using n - 2 tables times 2. This group came up with so many formulas without knowing about variables. (TTO, Morrowind, NE)

In another related lesson from the *Awesome Algebra* Unit, students continued to explore the concept of creating formulas for predicting numbers in a growing pattern. For example, after students learned how to figure out the rule to determine the total number of fish that could fit around any given number of square tables, Mr. Lucas challenged students by asking them to consider how changing the shape of the tables might change the rule:

> The teacher introduced the lesson using triangular shapes and guided the students as they thought about the number of fish that would be able to sit at the triangular tables as each table was added. Students were given foam triangles and started exploring the results. Students lined up 23 triangles and tried to determine the final number of fish. (TTO, Springville, SE)

Some of the students struggled with counting the fish, although a couple of the students recognized the rule that each additional triangular table would result in adding one extra fish to the total number of fish that could sit at the tables.

Cohort III Control Teacher Observations

Although there were many more instances of challenging students through complex concepts or skills in the treatment teacher observations, they were evident in some of the control teacher observations as well. For example, Ms. James challenged students to apply a complex skill of predicting the proper placement of parentheses in an equation:

> "Let's move onto a different problem: 32–6+4=30. How can you make this statement true? [Male student], why don't you come up to the board and put the parentheses where they belong?" He placed them in the correct place (32–6)+4=30. "Here is a trickier one for you: 3x6+13=57." (CTO, Shady River, MW)

Other teachers challenged students to arrive at a rule or formula. This is exemplified in one observation in which students were encouraged to create a formula for perimeter (Ms. Cruz, CTO, Cortana, NE). Students took turns guessing a formula, although most guessed the one for area. Finally a student suggested the following formula using addition: b+b+h+h=p. Ms. Cruz followed up this response by asking if anyone used multiplication in his or her formula. The observer noted that students seemed stuck on using the formula for area until one student suggested: "How about (bx2)+(hx2)=p?" After the student suggested the formula, Ms. Cruz led the students through a set of examples to test the formula.

Similarly, Ms. Scott (CTO, Grand Arch, NE) challenged students by having them create a rule for a number pattern. Students had read a fictional story about characters collecting different amounts of marbles each night. Next, the students calculated the total number of marbles that the characters collected, recorded the amounts in a table, created a rule to predict the next number of marbles in the pattern, analyzed the patterns to learn about exponential growth, and explained their thinking.

There were very few instances of control teachers specifically giving advanced students alternative mathematics assignments. In those few instances, students either worked with an enrichment specialist or they had extra work once they finished the regular curriculum. In one noteworthy instance, Ms. Lily did attempt to provide enrichment to challenge one particular student, but the student did not have the opportunity to actually start it during the observation lesson:

> [Student name] had a contract because he tested out of the geometry unit, but he still had to do problems from each section. He got to check his own answers. Once he finished the chapter he could do enrichment problems. (CTO, Pegasus, MA)

At the end of the lesson, this same student grumbled "I already know how to do that" when the teacher assigned homework that was almost identical to the assignment they had just completed in class.

Explaining One's Thinking

Another interesting finding was that teachers challenged students to explain out loud or in writing how they arrived at a solution to a mathematical problem. In some instances, students struggled at first to figure out how to verbally express their thought processes while working through a mathematics problem to find a solution.

Cohort III Treatment Teacher Observations

Having students explain their thinking was evident in about one-third of the Cohort III treatment teacher observations coded within the theme of challenge. One observer in Ms. Bloomstein's class noted that before students began working in their groups with partners on an *Awesome Algebra* Lab, "The teacher then goes over the questions for each group and stresses to them to explain their thinking and back it up with reason and proof" (TTO, Terracotta, NE).

This was also seen in the following example in which Ms. Walker reviewed an *Awesome Algebra* assignment where students identified the pattern in a sequence of numbers and had to make predictions about the 15th term in the sequence. The observer noted, "The teacher did ask students for multiple options in solving the problem. She also made students explain their methods thoroughly before accepting their answers" (TTO, Seabreeze, SW).

Treatment teachers also challenged students to explain their thinking in the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit lessons. For example, Ms. Wagner challenged students during a lesson on completing line graphs using data from a table:

> At the bottom there is a challenge part. If you know something about number lines, you will make a graph. Fill in time and temp on the line graph. Explain how you decided to put in the number of points. (TTO, Governor's Park, NE)

Cohort III Control Teacher Observations

Having students explain their thinking was also evident in some of the control teacher observations, but did not occur as often as was observed with the treatment teachers. It was noted in an observation that Ms. Pinkman, utilizing whole group instruction, “presses them to explain their thinking” while completing a worksheet on measurement and fractions (CTO, Southeastern, SE). In other instances, students attempted to explain their thinking but had some difficulty, or the student discovered that his or her thinking was inaccurate. This is exemplified in a conceptual division lesson in which Ms. Long had multiple students explain or demonstrate how to solve a word problem using pictures on the board:

> “Okay [Student name], explain what you did,” the teacher says. The student is explaining her thinking and gets confused. Teacher: “Does anyone want to help her out?” One student tries to explain what he thought the student was doing. (CTO, Skinner, NE)

Similarly, one student was challenged to explain his thinking in front of the class only to discover that his solution was not possible. The lesson involved discovering which shapes could easily be divided into equivalent fractional parts. This student tried to use a hexagon to show different fractions. The observer noted:

> [Student name] went up to show her/his 10 equal parts, then she said “Now show me an eleventh fraction.” He ended up admitting that it was not possible and not as easy to divide as a square or circle. (CTO, Forge Hill, NE)

Throughout the lesson, Ms. Burns challenged students to think of multiple solutions and explain their answers.

Struggle in Response to Mathematics Curricula

The final major recurring theme related to challenge was that of “student struggle.” There were instances in which students struggled with the content that they were presented. Students struggled with understanding concepts and skills such as improper fractions, identifying area, regrouping money, how to use a scale, creating graphs, or writing their own equations. The analysis of the classroom observational data revealed that 19% percent of the treatment teacher observations (n=15) showed evidence of students struggling. Additionally, 16% of the control teacher observations (n=8) showed evidence of students struggling.

Cohort III Treatment Teacher Observations

In many instances where students struggled with some part of the mathematics lessons, the end results were positive in that students were able to persevere to arrive at a solution. This can be seen in the following example from an *Awesome Algebra* Unit lesson in Ms. Franklin’s class:

> Other students did seem to struggle a little, but they all were able to build on and find the pattern. The discussion in the end was really interesting because a few groups really started to understand how to create a rule to figure out the pattern for n number of tables. (TTO, Halcyon, NE)

This is also illustrated in an example lesson from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. With encouragement and guidance from Ms. Wagner, students were able to successfully complete the assignment. Students were working in groups completing line graphs in which they had to plot more than one set of data points:

> Some students had trouble tracking the data to the appropriate intersection. The teacher demonstrated the process and students were able to practice it with some data. The lesson was implemented with high fidelity. The line graphs with temperatures from 2 or 4 cities were challenging for the respective groups. Students concentrated on their work and were able to create their line graphs successfully. (TTO, Governor's Park, NE)

Similarly, Ms. Simpson was observed encouraging students to stay focused even though students were challenged during a *Geometry & Measurement for All Shapes & Sizes* Unit lesson:

> "Now, you are going to tell me how many lines of symmetry are in the shapes I give you. You can talk to your neighbors." I overheard one student say, "This is hard!" Students were able to name each shape that they received. They were also super excited to keep the shapes. "Stay focused. Talk to your neighbor about how many lines of symmetry." (TTO, Sunnyside, SW)

When asked for opinions about the What Works Mathematics Curricula, a student in Ms. Baker's class responded, "This is challenging." As a follow-up, the observer inquired, "Is that a good thing?" Finally, the student replied, "It helps with my education" (TTO, Seabreeze, SW).

Cohort III Control Teacher Observations

An analysis of the control teacher observations also showed evidence of students struggling with the mathematics curriculum. For example, in a lesson in which students are guided through applying and learning the concept of area, Ms. Hall had students calculate the area of their mathematics book by covering it with foam tiles. The challenge was that the students did not have enough tiles and the mathematics book dimensions were not in whole units.

> The 8.5 inches of the textbook threw some of them off until [the teacher] told them to use a benchmark. This word here meant an educated estimate. She also reminded the students to think about the arrays they worked with. Then a few students realized you could multiply the length by the width to get the area. One

> student, who was getting it, leaned over to a struggling student and explained it to him. Another student thought about using 5's and 10's to count up the tiles to estimate an answer.
>
> At the end of the lesson, Ms. Hall asked them to explain the term "area." The observer noted that the students really struggled and stated that area was using squares, square units, the number of figures there are, and how big it is. Ms. Hall eventually just told them the answer. (CTO, Old Toll Road, SE)

In another observation, a group of students in Ms. Kim's class struggled with creating number sentences to solve a word problem during a lesson on interpreting remainders within the context of division word problems:

> With the students remaining in front of her, she draws a picture on the board representing one of the problems and asks, "What's my number sentence going to be?" The students don't seem to understand as they are calling out random number sentences that don't really go with the picture. She then asks, "What's the difference between the number sentence in the problem and the number sentence needed to solve the problem?" Students struggle answering. (CTO, First Sun, NE)

Conclusion

Based upon the analysis of the Cohort III treatment and control teacher observations, students were challenged in a variety of ways. Teachers demonstrated evidence of challenging students when they asked students to use larger and more complex numbers, taught complex concepts or skills, and required students to explain their mathematical thinking. In response to challenging mathematics curricula and teacher expectations, at times students responded positively as student mathematicians and other times struggled. If students struggled, they did so with higher-level mathematics concepts, skills, and explaining their thinking. The analysis of challenge indicated that a higher percentage of treatment classroom observations were coded with instances of students being challenged than in the control classrooms. In addition, there were more observations coded as involving students struggling in the treatment classrooms than in the control classrooms.

Teacher and Administrator Perceptions of Challenge

The analysis of the Cohort III treatment teachers' logs, teachers' focus groups, and administrators' interviews yielded multiple comments about the challenging nature of the What Works Mathematics Units. The participants also remarked on the hands-on lessons and real-world applications and how they contributed to engaging students and developing their conceptual understanding.

The Challenging Nature of the Units

Many of the Cohort III treatment teachers and administrators commented on the challenging nature of the units noting their own perceptions as well as the students' reactions. In fact, several teachers mentioned how the units challenged some students for the first time. During the *Awesome Algebra* Unit, Ms. Jordan noted, "[Student name] started with an attitude that he knew everything—until we began to be really challenged—at first he refused to try, then, working with his group, began to learn!" (TL, Shelbyfield, SE). Ms. Duncan shared, "The student who is usually an average student surprised me with her performance in this unit. She succeeded where others didn't. She probably would not have been offered challenges if it was not part of the curriculum" (TL, Skinner, NE). Ms. Decker, an administrator from Forge Hill School also commented on the challenging nature of the units stating, "Our students benefitted by having the UConn units bring them up to a higher level of thinking and problem solving. The units caused our students to look at familiar math concepts in new and different ways" (AI, Forge Hill, NE).

Teachers also remarked that the students were engaged, enjoyed the challenge, and grew in their ability to reason independently. A teacher participant in Focus Group 17 said that her students appreciated being able to be challenged and move on to the higher student page when they could (NE). Ms. Mosby commented, "I had a student say 'Today's lesson was really hard—but I really liked it anyway' " (*Geometry & Measurement for All Shapes & Sizes*, TL, Franklin Bridge, MW). Ms. Flores commented, "It was a great experience for me as a teacher and for the students. It built their confidence and took their thinking to a higher level" (*Awesome Algebra*, TL, Grand Arch, NE). Ms. Weaver noted:

> My students enjoyed the Initiate activities in the unit. They were always engaged in these activities. They were also able to create their own thinking in the Investigate sections. This helped them come up with thinking on their own rather than just being told what to think. (*Geometry & Measurement for All Shapes & Sizes*, TL, Farnsworth, NE)

Ms. Fletcher noted that her enjoyment was due to the complexity involved:

> I really loved this unit because it stretches the students' minds for mathematical problem solving and reasoning. The parents of my students were also very excited to see their kids learning "algebra" at such a young age. This unit also helped students make connections with other math concepts. For example: how to use multiplication to help you with "The Name Game." (*Awesome Algebra*, TL , Solsbury Valley, SW)

A teacher in Focus Group 21 shared her appreciation for the challenge level of the units stating, "The UConn curriculum really makes students think. It's something that they can dig into deeper. They have to sit and process and slow down. For kids who can't visualize well, these kinds of lessons help them to see the connections and the concepts"

(SW). Echoing this sentiment, Ms. Carrillo, an administrator, noted that the curriculum "Pushes the students to higher levels of thinking. . ." (AI, Oyster Harbor, SE). During a discussion comparing the What Works Mathematics Curricula to their schools' textbooks, teachers in Focus Group 19 explained that the approach to mathematics in the What Works Mathematics Curricula incorporated more exploration, discovery, and discussion. Furthermore, students were "uncovering content and concepts for themselves" (SE).

While many teachers enjoyed the challenge of the units, a few teachers expressed concerns about the appropriateness of the units for students who are English Language Learners or who have a documented writing or communication disorder. Ms. Carroll stated, "My classroom has many children who are on IEPs for writing and communication difficulties so this unit (as the others) were very difficult and frustrating for them. The other children found the unit enjoyable" (*Awesome Algebra*, TL, Skinner, SE). Similarly, Ms. Bridges said, "I spoke to the Special Education teacher (inclusion) who is working within the classroom. She said that some of the skills are difficult for her students to access" (AI, First Sun, NE). A teacher in Focus Group 21 mentioned how she realized that "talking math" and explaining thinking helps students understand the material. She said that students knew they had to tell the answer and explain it by writing it out. She thought the *Awesome Algebra* Unit would be great, but her students really struggled with explaining their thinking and writing it out. In addition, this teacher found the mathematical language difficult for the students.

The units in general did require students to explain their thinking in writing, which may have caused some frustration for students who were more comfortable with verbal communication skills. Teachers did mention that further differentiation was necessary to adapt lessons for these students.

Hands-on Activities Fostered Student Understanding and Engagement

One of the features of the units that may have helped students with the challenging nature of the units was the "hands-on" activities included in many of the lessons. Mr. Lange stated, "The students were excited about mathematics. They were thinking more like mathematicians as opposed to consumers of math units. The hands-on component really grabbed their attention and made them want to learn" (AI, Shady River and Apple Tree, MW). A teacher in Focus Group 25 said that having the manipulatives as a guide to help students at the beginning of concept instruction was helpful (SE).

While all of the units included the use of manipulatives, treatment classroom teachers commented on the helpfulness of the manipulatives to support learning goals most frequently in the *Geometry & Measurement for All Shapes & Sizes* Unit. Mr. Wheeler explained:

> My students really enjoyed the various activities that went along with this unit. The ability to apply what they were learning to hands-on activities was very

> beneficial to them and a big hit. They were learning and having fun at the same time. (TL, Cedar Brook, NE)

Ms. Daniels commented on a specific lesson, “The lesson with the [masking] tape was a great way to introduce fractions and numerator or denominator. Walking it out” (*Greening Up With Graphing: Recycle, Reduce, & Reuse*, MW). Another teacher in Focus Group 14 said, “. . . her students had fun during the graphing unit with the recycling. They collected bottles, and the students looked forward to advertising for the collections and collecting and counting bottles” (MA).

The use of manipulatives not only helped engage the students, but also helped facilitate discussions in groups. Ms. Simmons noted. “The children looked forward to doing this work, loved and wanted the patty paper. They were great about sharing ideas and re-explaining things to their classmates” (*Geometry & Measurement for All Shapes & Sizes*, TL, Smithton, SE).

Teachers commented that the use of manipulatives also helped struggling students gain conceptual understanding. Ms. Garcia stated:

> One student in particular who has struggled with our regular graphing curriculum did surprisingly well with this unit. I believe the use of manipulations and small group work helped him immensely. I loved the activities in this unit. They were very visual and this really helped many of my students. (*Greening Up With Graphing: Recycle, Reduce, & Reuse*, TL, Pegasus, MA)

In addition, Ms. Potter noted, “My ‘slowest’ math student was able to understand the concepts of congruence and similarity when he played with the tangram pieces and actually put one piece on top of another. He really ‘got it!’ ” (*Geometry & Measurement for All Shapes & Sizes,* TL, MA). A teacher in Focus Group 14 (MA) said that the hands-on nature of the curriculum helped “students who were lost” and that the regular textbook had very little hands-on activities at all. Her students who “really struggled” enjoyed the hands-on activities, especially the puppet-making in the *Geometry & Measurement for All Shapes & Sizes* Unit activity. She found that many students chose the more difficult shapes, such as the pentagon; they looked for challenge rather than avoiding it, as many did before the What Works Mathematics Curricula.

Real-world Activities Fostered Student Engagement and Understanding

Treatment classroom teachers also frequently mentioned the benefits of the real-world nature of the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. Some teachers discussed how the real-world connection facilitated student acquisition of specific skills. Ms. Arnold noted, “Using real data collected over time was helpful in teaching why/how we use line graphs” (TL, George Washington, NE). In addition, Ms. Maxwell commented, “Students developed a decent understanding of median, mode, maximum and minimum. It also helped them to see a connection between mathematics in school and the real-world” (TL, Morrowind, NE). Teachers in the focus groups also noted that students connected math to the real-world. One teacher in Focus Group 21 talked about how she saw students being more aware of the math around them. She mentioned the real-world connections that the What Works Mathematics Curricula makes for the students and how those helped her students to look around and become more aware of things like fractions, for instance (SW).

In some classrooms, treatment classroom teachers mentioned that students felt purposeful and proud of their work when participating in a real-world activity. Ms. Flores explained:

> My students really enjoyed collecting clothing for the nurse's office. They felt it was important to take care of others and do their part. They researched what was really needed and then collected these items. They felt that they needed underwear for the younger kids, because they might have more accidents and they had other ideas based on ages of students. (*Greening Up With Graphing: Recycle, Reduce, & Reuse*, TL, Grand Arch, NE)

Ms. Davidson noted, "The students loved the recycling project. They loved collecting the data over time. They loved reporting results to classmates. They really took ownership of this project and were proud to show how they used the computer to graph results" (*Greening Up With Graphing: Recycle, Reduce, & Reuse*, TL, Shady River, MW).

Overall, the treatment teachers commented that the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit was real and meaningful to students. The recycling interventions seemed to affect the entire school in some instances. Teachers commented that students felt a sense of purpose completing the interventions in the unit. In addition, the administrators' interviews reflected a positive response to the method of having students act as mathematicians as they work through mathematical content to achieve authentic learning. One method the administrators felt that the What Works Mathematics Curricula used to promote students thinking as mathematicians was the use of real-world content and topics that related to the students' lives.

Conclusion

Overall, Cohort III treatment teachers and administrators appreciated the challenging nature of the What Works Mathematics Curricula and recognized the benefits of challenging their students. They also appreciated the real-world applications within the units as well as using a hands-on approach to instruction. These approaches appeared to support some struggling students, which was an initial concern for some of the treatment teachers and administrators.

Questioning and Discourse

Analysis of the treatment and control classroom observations revealed several categories of questioning strategies and mathematical discourse. The ways teachers used divergent questioning and discourse are presented in two sections.

Divergent questioning was defined as instances in which open-ended questions were asked by teachers to elicit multiple or a range of acceptable responses from students. Discourse was defined as students explaining, writing, challenging, relating, defending a point of view, elaborating, predicting, conjecturing, generalizing, questioning, answering, or asking for clarification about mathematical problems. During the analysis, the items coded were collapsed into two major categories: (a) discourse that focused on students' prior knowledge, and (b) discourse that focused on students' ability to explain their thinking.

Divergent Questioning

In the treatment teacher observations, 74% showed evidence of the use of divergent questions. In the control teacher observations, 64% showed evidence of the use of divergent questions as well. Two major categories will be discussed: (a) multiple response questions and (b) comprehension questions.

Multiple Response Questions

The most common classification of divergent questions used by Cohort III treatment teachers and control teachers was that of questions eliciting "multiple responses." Teachers asked students to find multiple strategies or multiple solutions while solving or discussing mathematical problems.

Cohort III Treatment Teacher Observations

Divergent questions eliciting multiple responses included questions in which students were asked to find multiple strategies or solutions to mathematics problems. They also included questions in which the teacher asked students to brainstorm multiple solutions to real-world problems.

Treatment teachers asked students to find multiple strategies to arrive at a solution by asking questions such as "Is there another way?" or ". . . now come up with a different way to solve this problem." During a classification lesson from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, Ms. Howard directed students to empty the contents of a bag and sort the contents based on the shapes of the objects:

> After this sorting activity, students shared any strategies used to work collaboratively together to sort the objects by shapes. For example, one student said, "We worked together and sorted the shapes and then organized them."

> Another student shared, "We all identified one shape and organized it and then moved on." Then Ms. Howard challenged students by stating, "Everyone has eight different shapes, and every group has a different amount of each shape. Come up with a plan for sorting another way." (TTO, Historic Cove, NE)

The students then sorted the shapes by color, if the object had a hole in it or not, rounded edges, sharp edges, symmetrical versus non-symmetrical, or if objects moved or did not move. The objective of the lesson was for students to understand the purpose for sorting data by similarities and differences. The observer also noted that students enjoyed working on the activities.

Students had the opportunity to share their mathematical strategies during observations from the other What Works Mathematics Curricula. In a lesson observation from the *Geometry & Measurement for All Shapes & Sizes* Unit, Mr. Armstrong initiated the lesson by asking students, "What might you need to make a perfect circle?" (TTO, Forge Hill, NE). Students brainstormed multiple strategies such as using "a protractor, a water bottle, or even the bottom of your pencil." He gave students an envelope with paper clips, a rubber band, a pencil, and some string for students to use to try to discover a method to create their own perfect circle. He then asked, "Was anyone able to make a perfect circle?" Multiple students responded. One stated, "Yes, I used a paper clip to make one half, then flipped it to make the other half," and another shared, "I had my partner hold the paper while I used a paper clip and pencil without stopping to make the perfect circle." After exploring three more methods, students had the opportunity to reflect through writing which method they felt worked the best.

Treatment teachers also prompted students to elicit multiple solutions by asking divergent questions that required students to predict, determine open-ended solutions, share opinions and observations, make real-world connections, respond to "why?" and "how?" questions, or brainstorm multiple solutions to real-world problems. For instance, at the end of a *Geometry & Measurement for All Shapes & Sizes* lesson, the teacher asked students to look around the room to identify what was and what was not symmetrical. The students were able to point out multiple symmetrical and asymmetrical objects (TTO, Sunnyside, SW). This is an example in which the students elicited multiple solutions to a divergent question by making real-world connections between the content of the math lesson and their classroom environment.

In another example, Ms. Davidson had students brainstorm multiple solutions to what might be recycled as a means of initiating the investigation:

> Ms. Davidson asks them, "What does it mean to recycle?" After students respond, she asks them for some things that they recycle at home and at school. Many students build upon others' ideas. Ms. Davidson continues and asks about what students might reuse at home and at school. She then transitions to things that students *wish* they could recycle at school or at home that are not already being recycled. The students come up with many different and creative ideas. They pay

attention to each other and comment on each other's responses in a positive, encouraging way. (TTO, Apple Tree, MW)

Cohort III Control Teacher Observations

Similarly, some teachers in the control classrooms elicited multiple responses from students by asking them to find multiple strategies or solutions to mathematical problems. For instance, Ms. Long reviewed how to solve money word problems by asking "What if $1 is shared with 5 people?" (CTO, Skinner, NE). After reminding students of the strategies that they might use to solve similar problems, Ms. Long allowed students to solve the problems on their own. Then she asked students to share how they arrived at the answer. She prompted students by asking questions such as: "How many of you drew coins?" "Did anyone else do it with pictures?" "Did anyone do it mentally?" "Any other methods?" Students demonstrated or explained their strategies in response to Ms. Long's questions.

Control teachers also asked students to consider multiple solutions to a mathematics problem. For example, Ms. Reid asked students to "try to complete eight ways to make 10" using only the numbers 1, 2, and 4 (CTO, First Sun, NE). In addition, there were some instances when control teachers asked for multiple responses in the form of real-world connections. For example, during a lesson on geometry, Ms. Price asked open-ended questions about where in the students' world they would find parallel or intersecting lines (CTO, George Washington, NE). Students gave multiple responses such as on the street, train lines, on the floor, or on the ceiling.

Comprehension Questions

The second type of divergent question asked by both the Cohort III treatment and control teachers were comprehension questions. Observations were coded in this classification for analysis if teachers asked students open-ended comprehension, review, or reflection questions. Also included in this classification were questions asking students to explain their thinking. Finally, instances in which teachers utilized Talk Moves (Chapin et al., 2009) such as *revoicing, repeating, reasoning*, and *adding-on* were also included in this category. How teachers used these Talk Moves will also be addressed in the next section on discourse. Examples of the questions subsumed in the "comprehension" category are presented for both the Cohort III treatment and control teacher observations.

Cohort III Treatment Teacher Observations

Divergent comprehension questions that were not necessarily review, reflection, Talk Moves, or "explain your thinking" questions were also coded simply as comprehension questions within the larger category of "comprehension." An example of a comprehension question was exemplified when Ms. Wagner asked students to look at a line graph of temperatures across different time points in one day. Students were asked to explain what happened to the temperature after 4 pm (TTO, Governor's Park, NE). The observer noted that three different responses were elicited from the students as Ms. Wagner monitored the students' comprehension of the lesson objectives.

Reflection questions were those in which the teacher asked students to reflect back on their learning process from the lesson. For instance, after completing review work in stations from each of the three units, Ms. Higgins asked the following reflection questions during a whole class discussion: "What did you learn? What did you find challenging? What did you find easy?" (TTO, Staten Ridge, MA).

Divergent review questions were those in which the teacher asked students to share specifically either content or skills that they learned from the unit or lesson. A review question used by Mr. Goldberg included, "What else did you learn about patterns?" (TTO, First Sun, NE).

Treatment teachers also asked students to explain their thinking in response to a mathematics question. An observer made note of an instance in which Ms. Wilson probed the student by stating, "That doesn't really tell me much. Explain a little more" (TTO, Solsbury Valley, SE).

Talk Moves were used frequently by the treatment teachers. Primarily teachers used the *reasoning* Talk Move by asking students if they agreed or disagreed with classmates when discussing mathematics or they used a combination of the Talk Moves. For instance, an observer noted that Ms. Nelson asked if students agreed or disagreed with each other after each response was read during a discussion about an activity they completed in their journals from a *Greening Up With Graphing: Recycle, Reduce, & Reuse* lesson (TTO, Stone Mill, NE). Some, but not all, students explained why they agreed or disagreed. In another example, Ms. Horton asked "Do you agree?" A student responded "Yes. The reason why I agree is because you have to get 2 fish out of the way." She then used another Talk Move question as a follow-up when she asked the students, "Can you say it in a different way?" (TTO, Morrowind, NE). Some teachers had students respond physically rather than engaging in discourse. For example, Ms. Jennings asked students, "Thumbs up if you agree with what [male student] did" (TTO, Shale Rock, SW). Again, these are examples of divergent questions as the teacher was not probing for one finite answer.

Treatment teachers also used *adding on* and *repeating* Talk Moves, but to a lesser extent than the *reasoning* Talk Move. For example, Ms. Zimmerman asked, "Who can explain that a little more?" to encourage students to add on to the discussion about how students solved a repeating pattern question from the *Awesome Algebra* Unit (TTO, Terracotta, NE). An example of using the *repeating* Talk Move to help facilitate discourse, as well as comprehension, can be seen in the following observation notes: "Ms. Austin visited the back table. 'Can you tell me, [male student] what [female student] just said?' She knew that several of the students understood but some students were struggling" (TTO, Deer Creek, NE).

Cohort III Control Teacher Observations

Control teachers also utilized divergent comprehension, review, explain your thinking type questions, and Talk Moves that were all subsumed into the larger theme of "comprehension." However, there were no control observations coded as reflection questions.

As an example of a divergent comprehension question from the control observations, Ms. Lake asked students to "write everything you know about the fraction 5/3" (CTO, Solsbury Valley, SW). A review question used by Mr. Black included, "What are the different kinds of graphs?" (CTO, Deer Creek, NE). The observer noted that the students responded with different answers such as bar, circle, line, and pictographs. An illustrative observation that was coded as an "explain your thinking" question asked by Ms. Long was: "Who wants to volunteer to explain their thinking?" The observer noted that all the students' hands went up, and one group was chosen to go up to the front of the room to explain the thinking (CTO, Skinner, NE).

Reasoning, repeating, revoicing, and *adding on* Talk Move questions were used by control teachers who were not trained to use Talk Moves specifically by the NRC/GT research team. However, an administrator from a district with multiple participating schools indicated that the state adopted Talk Moves as part of their curriculum initiatives, which potentially influenced the control classroom observation data. As was the case with the treatment teacher observations, the *reasoning* Talk Move was used most often by the control teachers. In almost all instances in which teachers used the *reasoning* Talk Move questions, teachers only asked students if they agreed or disagreed with their classmates' thinking instead of also asking students why. For example, one observer noted: "This time Ms. Kim makes them write their own explanations down. She then has students share their explanations. She asks students to do thumbs-up or thumbs-down if they agree or disagree" (CTO, First Sun, NE).

Conclusion

Treatment teachers asked more divergent questions than the control teachers based on the analysis of the classroom observations. "Explain your thinking" questions were more prevalent in the treatment teacher observations. It should be noted as well that "explain your thinking" questions were discussed previously in the challenge section of this research monograph. Divergent questions used by the teachers were those that did not require a singular response; rather teachers accepted more than one response as being appropriate. Treatment and control teachers utilized two major types of divergent questions: multiple responses and comprehension questions. Multiple response questions included questions that elicited multiple solutions and strategies to mathematics problems. The classification of comprehension questions subsumed review, reflection, "explain your thinking" questions, and Talk Move type questions. This classification also included divergent comprehension questions in general that were not necessarily review, reflection, "explain your thinking," or Talk Move type questions. Also, treatment teachers asked more questions within each of the subthemes as well. Treatment teachers most commonly asked multiple solution type questions. Control teachers most commonly asked questions in which students were asked to explain their thinking.

Discourse

Seventy-three percent (*n*=58) of Cohort III treatment teachers used discourse in their classrooms during the observation session. Fifty-two percent (*n*=26) of control teacher observations were coded as containing discourse. During the analysis two major categories were developed: (a) discourse that focused on students' prior knowledge, and (b) discourse that focused on students' ability to explain their thinking. Discourse included interactions among multiple students or discourse that emerged from teacher-led discussions. It should be noted that the What Works Mathematics Curricula had many embedded opportunities for discourse.

Discourse—Prior Knowledge

Teachers often initiated discourse by asking what students noticed, what they already knew, or any connections they could make. Sometimes, it was used to check to see if students had a basic understanding of a particular topic of study before moving on to other topics.

Cohort III Treatment Teacher Observations

In 24% (*n*=19) of the Cohort III treatment teacher observations, teachers asked students to identify what they knew about a particular mathematics topic. Brainstorming, clarifying, and asking for multiple responses were ways in which teachers sought to find out what students knew about a topic.

In one class, Ms. Goodwin (TTO, Calder, MA) asked the students to identify properties of various shapes. Students listed what they knew about several shapes including triangles, rectangles, and squares. Students were able to list multiple attributes of each shape. The teacher further probed, "What else can you tell me about this shape?" One student responded, "[It has] Four lines; line segments; 4 sides." A second student stated that it has "Four angles." Finally, a third student mentioned that the shape has "Four vertices." This conversation continued with a discussion of the attributes of several other shapes.

In another class, there was a discussion about arrays, and students agreed about the definition of an array. Ms. Reynolds (TTO, Shelbyfield, SE) began with a variety of questions: "Which color, which pattern, is an array? Blue? How many think blue is an array? Do you agree or disagree with that, [female student]? Which one is a non-example?" The observer noted that the discussion continued in both whole and small groups:

> One student offers that he/she thinks the black one is an array because it has rows and columns. Finally, the students were asked, "What is an array?" Students were asked to turn and talk to their neighbor about the answer to the question. Students were then asked as an entire group what conclusions they had come to. "Objects arranged into equal columns and rows," one student offered. "What can you tell me about patterns in rows and columns? Think of your hundreds chart (up on the board behind them). Yes, the ones column numbers stay the same, but the tens column numbers increase by one each time down." (TTO, Shelbyfield, SE)

Teachers used discourse to clarify what students knew about topics. The depth of these discussions varied. Often, students' responses were brief, and teachers attempted to help clarify or solidify student understanding about mathematical content and concepts.

Cohort III Control Teacher Observations

In 20% (*n*=10) of the control teacher observations, students were asked to relate information that they already knew. Often, the teachers asked a question, and students gave brief responses followed by the teacher confirming or correcting the student answer. In one lesson Ms. Lily (CTO, Pegasus, MA) reviewed what they had learned about intersecting and parallel lines:

> T: What is the name for #1?
> S: Line segment.
> T: Right. Why is this not a line?
> S: It doesn't have arrows.
> T: What do the arrows tell us?
> S: It keeps going.

When asking students to identify what they already knew, control teachers also used brainstorming techniques. For instance, students in Ms. Fox's (CTO, Franklin Bridge, MW) classroom responded to a series of questions that asked them to list things they are sure will happen, things they are not sure will happen, and things that may happen. The students were sharing responses when the observer entered the classroom:

> T: Can you be sure?
> S1: No, we may go to Blueberry Café.
> S2: We may have a test.
> S3: [I am] sure the time will change on Sunday.
> S4: [I am] sure that North America will sink in 10 years.
> S5: [I am] not sure night will become day.

Discourse—Explain Your Thinking

In both treatment and control Cohort III classroom observations, students were asked to explain their thinking more frequently than just to identify prior knowledge. Sometimes students were called up to the board to write their explanations.

Cohort III Treatment Teacher Observations

In 49% (*n*=39) of the Cohort III treatment teacher observations, students were asked to explain how they solved a problem. Teachers attempted to help students self-correct errors in thinking. Several teachers let students work through problems using incorrect methods, and then used questioning techniques guiding students to see where they made an error.

In a lesson that focused on discovering patterns, the observer noted the following conversation:

> Ms. Sanders asks students for a mathematical sentence that would represent the pattern. One student answers, "6+2." The teacher asks, "Is it always a 6?" trying to get them to think about variables. However, one student says that it is 6x2. She does not correct him, but carries on with the analysis, making him go through it step-by-step until he realizes it is wrong. (TTO, Grand Arch, NE)

Ms. Sanders used discourse to understand how the student is thinking about the mathematics and helped him understand the logic in his thinking. Because this conversation happened as a whole group, the class was allowed to participate in the deliberation of the problem.

Other teachers used discourse to challenge students to see that they could find the answer in multiple ways. In a lesson involving discovering how knowing a pattern can help solve a problem, students are trying to figure out which letter would be 18th if the last name DAVIS were repeated continually. Ms. Bloomstein asked, "How did you solve this problem?" One student responded by stating "count the letters by 1s." Another student stated that you could "count the letters by fives." Ms. Bloomstein probed, "Who can explain that a little more?" Another student responded, "Count by 5–5, 10, 15–18 is 3 more than that—so 18 is 3 more so it is 'V'." Another student responded, "Count by 5s two times, then add 8 (5x2)+8=18 V because it has 3, 8, 13, 18. I did it in my head first, then tried it with the table" (TTO, Terracotta, NE). The students are encouraged to share their answers and explanations. In addition, multiple methods to solving the problem are discussed.

Cohort III Control Teacher Observations

As with the treatment classrooms, observed discourse generally focused on students explaining how they solved a problem. This occurred in 32% (*n*=16) of the control teacher observations. The observed teachers appeared to be trying to find out what students are thinking and to ensure that all students in class are exposed to a variety of methods.

Many control teachers asked multiple students to explain their thinking. In Ms. Fordam's (CTO, Skinner, NE) classroom, the students were asked to answer the following question: "A boy has a bag of chips. 1/4 are blue chips and 1/5 are yellow chips. Does he have more blue or yellow chips?" The observer noted, "One student answers and explains why. Different students are asked to answer what they think and why. A variety of methods were used."

In another classroom, Ms. Peach (CTO, Solsbury Valley, SW) asks: "What would the problem look like using parentheses? Amy scored 12 points and Josh scored 6 points. If their team scored 41 points, how many points did the rest of the team score?" Ms. Peach is not concerned with the answer but wants the students to write the number model using parentheses. One student comes up to the Smart Board and writes (12+6)+______=41. The student gets the answer 23 and writes it on the Smart Board. The teacher asks, "Anyone thinking of another strategy?" One student explains and writes her thinking 41–(12+6)=23. Ms. Peach asks the class to do a second word problem, and tells the students, "Class, talk to your teammates and help each other." Students discuss strategies. One student goes up and writes on the Smart Board. She asks, "Thumbs up if sharing helps," and over half the class has thumbs up.

In addition to asking multiple students to explain their thinking in a whole group setting, students sometimes engaged in discourse within smaller groups or with a partner before entering into a whole class discussion. The teachers sometimes walked through the classroom and talked to individual groups. At other times, they waited to hear student responses in the whole group discussion. In Mr. Boyd's class, the students worked in groups on developing good estimates for the area of various shapes, and the observer noted:

> Students are working and discussing with their partners. Mr. Boyd stops the class for a moment to point out a type of thinking about if their estimate is good or not . . . students continue to work and discuss with each other. Students are loudly explaining things with each other. (CTO, Centurion, NE)

A whole group discussion followed in which the teacher had multiple students share their responses and worked "students through their thinking." Teachers provided students opportunities to work through their thinking before stepping in, allowing for the students to support each other in the problem solving process.

Discourse—Real-world Connection

In observations of the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, discourse was often connected to the real world. Teachers asked students to think about where and how mathematics is used in everyday situations. The focus on real-world connections was not seen as often in observations of lessons for other units in the treatment classrooms or in the control classrooms.

Cohort III Treatment Teacher Observations

In 54% (*n*=18) of the 33 Cohort III treatment teacher observations of the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, discourse in the classroom was connected to real-world examples. The *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit was designed with a real-world investigation. The majority of observations focusing on real-world connections discourse occurred during lessons from this unit. Throughout the unit, the students were supposed to be working on a project to increase recycling in their school.

Within the real-world discussions, conversations often focused on brainstorming. In the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, students were asked to complete an intervention to increase recycling at their school. This involved brainstorming possible interventions to increase recycling in their school, which often led to dynamic and in-depth whole-group discourse.

> As a class, Ms. Simmons (TTO, Smithton, SE) has them discuss the second question—how they can encourage people to recycle. They talk about the morning news and how they can create a slogan to remind people to recycle. Ms. Simmons brings up billboards and how people use them to advertise; she asks them about smaller ways to advertise like they do on billboards. Students respond with ideas about posters and flyers.

At other points, the real-world discussions centered on the methodology of students' unit projects. Teachers initiated discourse by asking students how they were going to get this assignment done and why they should do things this way. Ms. Arnold's students (TTO, George Washington, NE) discussed different ways to ensure that survey questions were answered in an accurate way that truly reflects the question. In this class, students were discussing how they can find out the five favorite foods of students in their school. At first, they were discussing if you could just watch what students in one class eat in the cafeteria:

> T: There are no right or wrong answers. You can have different opinions. Sometimes surveys are to get specific information.
> S1: I said, "no" because one class is not the whole school.
> S2: They might be eating something, but it is not their favorite.
> S3: I have something to add. It says watch—not ask.
> S4: Everyone might have a different food so you would not know what the favorite was.
> S5: Yes, it would be their favorite food because they packed their own food.
> T: They made their lunch? Could we ask all students to write their favorite foods? Ask all students to write down his/her 5 favorite foods.
> S6: Yes, because if you have them write it down you would know.

S7: No, if you have 5 favorite foods.
T: It might be difficult to narrow it down to 5 favorite foods. Maybe everyone does not have 5 favorite foods.

The students were engaged in a "deep" discussion that reflects the big idea of how to know if something is really true.

The real-world discussions were typically associated with the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. This unit offered multiple opportunities for students to engage in discussions that are connected to the outside world.

Cohort III Control Teacher Observations

In less than 6% (*n*=3) of the control teacher classrooms, real-world examples were incorporated into mathematical discourse. Two of the three discussions were focused on finding fractions in the real world. Teachers helped students understand the utility of fractions, and that they use fractions all the time. Examples of students' responses to the ways that fractions are used in daily life included how pizza and Subway sandwiches are divided into sections, and that while cooking people often need to use portions (e.g., ¼ cup of flour). Teachers and students generally just listed many items, and the focus was on awareness.

Conclusion

Teachers in both Cohort III treatment and control classrooms used discourse in similar ways. They often appear to use discourse to gain a better understanding of what students know, help them understand and express their mathematical thinking, and explore connections between mathematics and the real world. Although treatment and control teachers used discourse in similar ways, treatment teachers were observed using discourse in a higher percentage of classrooms, and there was more depth to the discourse in treatment teacher classrooms. In addition, in the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, discourse was connected to real-world applications of mathematics.

Preassessments and Grouping

In the qualitative data analysis of the treatment and control teacher observations, teachers' logs, teachers' focus groups, and administrators' interviews, the category of grouping emerged. Grouping was defined as instances in which teachers grouped students by ability to work on tiered or differentiated assignments either individually, with partners, or in smaller groups. This section describes how the treatment teachers used preassessments to guide the formation of groups and how often this occurred in comparison to the control groups. In addition, administrators' and treatment teachers' perceptions related to these practices will be shared.

Treatment Teachers Use of Preassessments

As a result of the study, many treatment teachers began to effectively use preassessments to gauge the needs of their students. Analysis of the treatment teachers' logs, teachers' focus groups, and administrators' interviews revealed that both administrators and teachers recognized the growth in the teachers' understanding and use of preassessments to guide grouping practices. Several treatment teachers prior to the study either did not use preassessments, or if they did, it was a district mandate and they did not use the information to create meaningful learning experiences (FG 26, MW; FG 14, MA). Ms. Bridges, an administrator at First Sun, shared, "Any good curriculum takes into account that students learn in different ways and at a different pace. Some teachers find the process easy and do it almost instinctively, while others need more direct instruction" (AI, First Sun, NE). Through their participation in the study, teachers began to see the value in the use of preassessments to gauge students' initial knowledge, to guide grouping practices, and to track student growth.

Gauge Existing Knowledge

Comments from the teachers' logs and administrators' interviews revealed additional evidence that treatment teachers recognized the importance and the logic of using preassessments to determine their students' current levels of knowledge and understanding. For example, Ms. Hardin explained, "Pre-testing and post-testing practices allowed teachers to see the importance of this practice to ensure students are working on skills that they do not already have and make time for learning more optimal" (AI, Grand Arch, NE). Ms. Austin confirmed this view, stating, "I feel that I am able to reach each student's needs with this unit. Grouping them by ability has been beneficial both to me and the students" (TL, Deer Creek, NE). Another teacher agreed in a focus group session, as she mentioned that it "makes so much sense" to pre-assess (FG 14, MA). She questioned why she did not do it before participating in the What Works math study.

Guide Grouping Practices

It is not enough, however, to only know their students' initial knowledge base; teachers must use that information to guide their grouping practices and to design appropriate learning experiences (Kulik, 1992; Rogers 1991). A focus group participant realized the importance of this step and shared, "I learned how beneficial it is to pre-test before we begin everything. It is like reading the question before you read the next passage. It helps guide you" (FG 26, MW). With other curricula, teachers must independently determine how to use the preassessment information, but the What Works study's math curriculum explicitly connected tiered assignments with the unit preassessments. Ms. Jenkins enthusiastically described this:

> I really loved the tiered activities based on the pretest questions in your study. . . . I felt I was making the greatest progress and achievement when using the tiered lessons based on the pretest. Loved it. Loved the scaffolding instruction part and the chapter check-ups that are in the book. . . . (TL, Smithton, SE)

Mr. McDowell agreed, “The tie in between the preassessment and then selecting the appropriate instructional group was effortless” (AI, Smithton, SE).

Track Student Growth

As a result of the assessment data, teachers were also able to discuss the growth students made throughout the unit. Ms. Caldwell reported, “I was surprised by the growth in the students’ learning as demonstrated by the pre- and post-tests” (TL, Skinner, NE). She also reflected about one particular student’s experience and growth during implementation of the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit:

> I had a student, a girl, who scored poorly in the pretest, [it was] obvious that she had limited exposure to graphing. On the posttest, she scored very high. She was engaged in the activity and the data was real to her. The data set was manageable, she understood the concept of intervention and she became confident in her ability to be able to determine the appropriate tool to display data. (TL)

There were a few teachers who shared concerns about using the preassessments. These teachers commented about the time required to score preassessments and utilize the data. Ms. Zimmerman wrote that the “scoring is incredibly tedious” in her Teacher’s Log (Terracotta, NE). Another teacher explained, “I found when it came to splitting up the three groups, the pre-test didn’t help. I really couldn’t tell until I taught a lesson who should really be in a certain ability group” (FG 15, NE). This particular teacher’s comment exemplifies the importance of teachers ultimately being responsible for knowing their students and thus needing to continuously and flexibly use data to differentiate appropriately for students.

Conclusion

While there were a few negative comments about preassessments, the majority of the specific teacher comments about using the preassessments was positive, as evidenced by the quantitative component on the teachers’ logs. When asked about the preassessments, 86% of the teachers agreed or strongly agreed that the pretests helped place students in different readiness groups for the *Awesome Algebra* Unit, and 74% responded similarly for the *Geometry & Measurement for All Shapes & Sizes* Unit. Unfortunately, these data were not collected for the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit. Overall, positive comments from both administrators and teachers, in addition to positive findings from quantitative measures, indicated that treatment teachers effectively used the preassessments provided for them in the What Works Mathematics Curricula units.

Grouping Students for Instruction in Mathematics

The classroom observations, teachers’ logs, teachers’ focus groups, and administrators’ interviews were analyzed to determine how teachers in both the treatment and control classrooms grouped students during math instruction. The findings on how students were grouped is presented in two sections: (a) grouping students by ability, and (b) grouping students for other reasons. Further evidence from other data sources is incorporated into these results.

Grouping Students by Ability

Both the Cohort III treatment and control teachers' classroom observations revealed that students were grouped to work on differentiated assignments. The results of the analysis for the treatment teacher observations are presented first, followed by the results of the control classroom observations.

Cohort III Treatment Teacher Observations

Of the Cohort III treatment teacher observations, 41% (*n*=33) of the observations indicated that students were grouped for the purpose of working on differentiated mathematics work based on students' ability levels. Based on the qualitative analysis of the classroom observations, it was evident that teachers assigned students to work on tiered activities as determined by students' preassessment data. In many instances, teachers used all the separate student groupings as recommended in the unit lessons. For example Ms. Baker divided her students into three separate groups to work on a *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit lesson. Some students worked collaboratively on the carpet as a group while others worked in groups at different tables. An observer in Ms. Baker's classroom noticed, "Students were enthusiastic about working in their mathematicians' groups and started working immediately" (TTO, Seabreeze, SW). Other teachers also expressed students' positive reactions to being grouped. Ms. Freeman wrote, "The students especially liked working in the flexible groups" (TL, Haverbrook, SE).

In the treatment classrooms that used ability grouping, each group had appropriate and honorable tasks to complete (Tomlinson, 2001). For example, Ms. Caldwell taught Lesson 6 of the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit (TTO, Skinner, NE). She formed two separate ability groups with her students. The objective for both groups was to interpret bar graphs, but the complexity was slightly greater for the second group. The observer noted that Ms. Caldwell spent time working with students and asking questions to each group. In Ms. Robertson classroom, students examined repeating patterns during a lesson from the *Awesome Algebra* Unit (TTO, Newton, MW). One group was given a hundreds number chart to provide a numerical visual scaffold of the pattern, another group was prompted through written questions, and a third group was given the problem with no scaffolding. When the class came back together, students shared their groups' unique strategies. The observer also recorded that all students were deeply engaged with the same problem but at various levels.

Even in treatment classrooms with seemingly "homogeneous" students, teachers used the What Works Mathematics Units flexibly to purposefully group students. Ms. Lambert taught a clustered group of higher-level third grade students who did not test out of third grade mathematics curriculum. When asked if she was only using part of the differentiation, she said that, in geometry, she still used the various levels because the students' knowledge was still vastly varied (TTO, Franklin Bridge, MW). Rather than making assumptions about her higher-level students, Ms. Lambert grouped students purposefully based on their math readiness specifically for the *Geometry & Measurement for All Shapes & Sizes* Unit.

Many treatment teachers and administrators cited the study's provision of tiered activities as the impetus behind the treatment teachers' ability to provide challenging learning opportunities that matched students' needs. One teacher described her regular school curriculum: "The textbook does do that a little, but it does not have as much opportunity for differentiation" (FG 26, MW). Further, Ms. Strickland reported that the study's curriculum "allowed for differentiation which kept the students engaged at all levels" (TL, Staten Ridge, MA). Mr. Lowery shared, "The curriculum was appropriately challenging for the gifted math students. The units also provided tiering so that students could work on the material in a way that was appropriate for them" (AI, Franklin Bridge, MW). While differentiation is often thought of as a time-consuming process, this study's curricula seemed to alleviate some of the pressure on the teachers to create higher- and lower-level modifications to their existing curriculum.

Cohort III Control Teacher Observations

There was some evidence that control classroom teachers used ability groups, but this was not observed to the same extent as in the treatment teacher observations. In fact, only six of the control teacher observations (12%) showed evidence of differentiating for students through the use of grouping. In the following example, Ms. Pinkman grouped students by ability to work out different mathematics word problems.

> She utilizes differentiation and tells different students to work on different problems in their groups. They must read the directions out loud within their groups, and they are allowed to work with their "shoulder partner" if they need to have a discussion regarding solving the problem. The yellow group has the more difficult problem. They must use numbers with decimals to solve their problem, whereas the other groups use whole numbers. (CTO, Southeastern, SE)

In some instances, control teachers grouped students by ability to play an educational mathematics game. For example, in Ms. Pierce's classroom, the students were paired up by similar ability to play a card game (CTO, Lucasville, MA). The students were assigned to a particular mathematics game based on their ability levels. The games included a fraction line-up game in which students placed fractions in order from least to greatest, a whole number game in which students combined fractions to get a value of one, and a decimal game where students had to add decimals together.

However, even when the control teachers used ability grouping strategies, they did not always provide high-ability students with challenging work. In Ms. Lily's room, one student was placed in his own group because he passed the preassessment, but instead of going beyond the textbook content that he already mastered, the teacher required the student to complete all the workbook pages (CTO, Pegasus, MA). While the observer was there, the student was on his fourth day of working through the pages.

Grouping Students for Other Reasons

To a lesser extent, teachers in both the treatment and control classrooms had students working in small groups or with partners to discuss their work or to complete an assignment together. When students were grouped in these ways, they did not necessarily work on differentiated mathematics assignments as determined by their ability levels.

Cohort III Treatment Teacher Observations

Analysis of the Cohort III treatment teacher observations revealed that 28% (n=22) of the observations showed evidence of grouping students for reasons other than ability. When treatment teachers grouped students for other reasons, it appeared that they primarily did so to have students work collaboratively or discuss their thinking with either a partner or in a small group. For example, after placing students with a partner for an assignment from the *Geometry & Measurement for All Shapes & Sizes* Unit, Ms. Carpenter explained:

> Sitting at your desks, together with your partners, I want you to make a triangle that is 6 units long and another that is 8 units long. You are going to make a right angle using 6 and 8 and then connect them together. Check each other's work. (TTO, Forge Hill, NE)

In another case, students in Ms. Walker's classroom had the option to select their own partner to complete an assignment from the *Awesome Algebra* Unit.

> The teacher then directs students to partner up and work on the student page in their Student Mathematicians Journals. They appear to only be working on the Diophantus Lab dealing with increasing patterns with terms and numbers. Students are actively engaged and on-task, working quietly together. Some pairs of students are consulting other pairs of students, and the teacher circulates and checks on the pairs of students as they work. (TTO, Seabreeze, SW)

Other treatment teachers grouped students to work collaboratively by counting off by threes or assigning students to a partner they had not worked with before.

Cohort III Control Teacher Observations

As was the case with some of the treatment teacher observations, control teachers also grouped students together to work collaboratively on an assignment or to discuss their mathematical thinking. Thirty-two percent (n=16) of the Cohort III control classroom observations indicated that teachers grouped students for reasons other than ability. In these instances, students were not grouped by ability but for other reasons. In one case, Ms. Bates grouped students together based on who worked well in a group. The students had to work with a partner to find the factors of a given number (CTO, Bald Eagle, MW).

In another lesson involving long division, students were intentionally grouped heterogeneously. Ms. Long explained, "We just did 4 problems that we shared money with different people, now we are going to do it with much bigger amounts of money." The observer noted that the teacher divided students into heterogeneous groups of three to work together on dividing 54 by three (CTO, Skinner, NE).

Conclusion

After an analysis of the observations that were coded under the theme of grouping, it was found that 41% of the treatment teachers (n=33) were observed providing differentiated mathematics assignments to students while working in homogeneous groups. Treatment teachers used ability grouping more often and more purposefully than control teachers, as only 12% of the control teachers (n=6) were observed grouping students homogeneously. Treatment teachers used the What Works Mathematics Curricula that incorporated lesson objectives that were differentiated by depth, complexity, and abstractness. Observations also showed that both treatment and control teachers grouped students heterogeneously to work collaboratively on assignments or to discuss their mathematical thinking rather than working on differentiated mathematics assignments.

Administrator and Teacher Perceptions of Grouping

The data from the treatment teachers' focus groups, administrators' interviews, and teachers' logs were analyzed to determine how the participants reacted to grouping students by ability for mathematical instruction and activities. The results of the analysis are presented below.

Positive Perceptions

Participation in the study influenced the treatment teachers' and administrators' perceptions promoting an excitement for a differentiated classroom. Ms. Walker reflected, "It has had a 'fantastical' effect on me and my classroom in that now I enjoy having my students work in small groups with manipulatives. The class was soooooo into this unit!" (TL, Seabreeze, SW). A teacher in Focus Group 18 shared a similar sentiment:

> I have never seen kids so engaged in their math lessons. They loved the cooperative work, and I loved the differentiation of every lesson. The lessons challenged the highest learners as well as reached our lowest kids. It is very enjoyable to teach. (MW)

Teachers also reflected on their own learning as well as the effect of grouping on student learning. “It definitely helped me see these students who are basic needs have strengths. I liked how the groups could change. I could work on strengths and see areas that they needed help” (FG 28, MA). Teachers discussed specific components of the math curriculum that facilitated the implementation of differentiation practices such as the flexible and responsive nature of grouping by readiness. In the teachers’ log, Ms. Reynolds reflected, “The differentiated groups were great and I love how the students moved in and out of ability groups. My kids loved this!” (Shelbyfield, Southeastern, SE). Another teacher corroborated this idea: “[I] liked it because when I gave the test, I was able to divide up who was in what group. That made it easier.” She also appreciated the flexibility of being able to “bump students into different groups” (FG 16, NE).

In addition, teachers in the focus groups appreciated that the curriculum came with pre-differentiated lessons and flexible guidelines for grouping students. A teacher mentioned how the math curriculum helped her accommodate all learners in her classroom. This teacher explained that students who were previously bored during math became interested and engaged and also added that the groups were an easy way to manage the accommodation of all students (FG 14, MA). Ms. Marshall mentioned, “The differentiated lessons allowed all students to be successful” (TL, Shelbyfield, SE).

Administrators also saw their teachers’ engagement with differentiation increase. Ms. Hardin recognized, “Differentiation is a practice we have been working on and this project allowed them to see the power of this practice in meeting the needs more effectively” (AI, Grand Arch, NE).

Concerns Related to Grouping

Certainly, not all participating teachers shared such enthusiasm about grouping students for differentiated lessons. Time, effort, and lack of coordination with their typical curriculum were concerns for some. Implementing new curriculum can be challenging and time consuming, although as one administrator commented, “This project has really stretched our teachers and pushed them out of their comfort zones. We appreciate the opportunity for our teachers to realize what they’re capable of” (FG 15, NE).

Conclusion

As a result of the study, the majority of the administrators and teachers held positive perceptions of grouping students to work on challenging pre-differentiated and enriched curricula during the course of the math study. Participants shared their enthusiasm for how the lessons accommodated and engaged students at all levels. They also appreciated the flexibility of guidelines for grouping students.

Treatment Teacher and Administrator Reflections

Treatment teachers and administrators engaged in reflection of instructional practices, student learning, and goals. How teachers changed and what they learned from participating in the math study will be presented first, followed by administrators' hopes for future teacher change and learning.

Teacher Change and Learning

The questions "What did you learn about how you teach?" and "How has your knowledge of student learning changed?" were posed to the treatment Teachers' focus groups. The theme of teacher "change" emerged throughout the majority of the focus group discussions of these two questions. Teachers shared instances of how they changed their instructional practices in the classroom, perspectives on how students learn, and expectations of students as a result of participating in this study. Setting new goals and critically reflecting on their attitudes and teaching styles were also categorized under this theme of change. Other teachers felt validated in their established pedagogical practices while implementing the What Works Mathematics Curricula, while a few teachers described the challenges of teaching the units. These findings were also corroborated by comments from the administrators' interviews.

Changes in Instructional Practices

Treatment teachers who participated in the focus group discussions shared how their instructional practices changed. For example, a teacher shared that after teaching gifted students and then returning to the general education classroom she "changed as a teacher with strict guidelines; put limits on what students could do; lowered expectations" (FG 19, SE). This same teacher, however, found joy again in teaching math with the What Works Mathematics Curricula and learned to "break out of the rigidity" of her instructional style. Ms. Rojas, an administrator from Halcyon made note of the benefits of the What Works Mathematics Units, "The math curriculum gave the teachers ideas and activities they could supplement into their instruction. It offered a nice look at covering a topic in depth and broadly across the cognitive range" (AI, NE). Another teacher in Focus Group 19 explained that she found herself "moving away from the books" and the algorithms and shifted to more discovery type learning. Ms. Gentry also noted the learning that occurred from participating in the study, "There are many benefits such as exposure to new curriculum, learning new methods and instructional strategies . . ." (AI, Morrowind, NE).

The integration of elements of discourse and Chapin et al.'s (2009) math Talk Moves—such as *reasoning* and *repeating*—in the What Works Mathematics Curricula also served as a catalyst for changing teachers' instructional practices. One teacher noted that she "had to become an active listener as a teacher" (FG 20, SE) when using these Talk Moves. Another teacher noted that there was "more turning and talking" emphasized with the use of Talk Moves (FG 19, SE). Ms. Jordan stated in her Teacher's

Log that "how to encourage real discourse with the students was the toughest of all—I have definitely grown during this unit" (Shelbyfield, SE).

Other instructional changes included using scaffolding for students and integrating concepts and skills from the What Works Mathematics Curricula into other subject areas. A teacher in Focus Group 13 (MW) explained that she was using scaffolding with students and that she had not done this before. In addition, one teacher said that she used the concepts behind "The Name Game" *Awesome Algebra* lesson in spelling. For instance, she asked students how many e's would be in "persevere" if they wrote it 30 times. She transferred many of the concepts from the What Works Mathematics Curricula to other areas of study (FG 14, MA).

Goal Setting

A very interesting finding was that the treatment teachers not only reflected critically on their current teaching styles, but they also created a range of different goals for themselves to improve their practice as a result of participating in the study. A participant in Focus Group 28 (MA) explained, "I don't give them enough free reign. Not that I stand up and lecture the whole time. We do projects, but I need to let them think more for themselves." A teacher in Focus Group 23 (SW) shared, "I need to make math more fun; math is not my bag; I need to think out of the box." Another teacher in this group stated that she needs to be more organized and that usually she flies "by the seat of my pants" and needs to be reigned in better. Another participant expressed, "I think that we are missing the boat because we need to scaffold more" and work together as facilitators (FG 12, NE). Ms. Decker also corroborated this theme of setting goals in an administrators' interview, "Our participation was extremely worthwhile. It gave us insight into things we could do better, and that's for both students and teachers" (Forge Hill, NE).

Critical Reflections on Prior Instructional Practices

Some focus group participants were critically reflective of their prior teaching styles. One teacher said that the What Works Mathematics Units made her question her use of differentiation in other areas. The observer commented that the teacher said that it was so hard to differentiate curricula on her own, so she appreciated how the What Works Mathematics Curricula have done all of the differentiation for her (FG 14, MA). Another shared, "I tell too much; it's hard to let kids discover" (FG 20, SE). Echoing these sentiments, one teacher realized she "talked too much as a teacher before these units." The observer commented further that the teacher believed that the What Works Mathematics Curricula allowed her to explain the lessons, but then allowed students to investigate on their own (FG 16, NE). A participant in Focus Group 15 (NE) shared that she liked that the What Works Mathematics Curricula "made me encourage my students to explain their thinking more" because "I'm not good at normally doing that or remembering to ask them to do that." Ms. Stafford, an administrator at Southeastern, corroborated this finding, "I believe it [the study] allowed teachers to become more reflective about their practice. It gave teachers an effective model for delving deeper into mathematics concepts while supporting student independence and thinking" (SE).

Changes in Attitudes and Perspectives

Teachers also shared changes about their attitudes towards teaching students math. A teacher in Focus Group 20 (SE) stated that working with the What Works Mathematics Curricula ". . . made me less afraid with whatever answer students give." Another teacher felt "more relaxed and enjoying" teaching math (FG 23, SW).

Not only did teachers' personal attitudes change, but their perspectives about how students understand mathematics changed as well. For example, one teacher mentioned how she realized that "talking math" and explaining thinking helps students understand the material (FG 21, SW). A teacher explained that she learned how students think and "how different kids solve problems" and that there was "a lot of thinking in the program" (FG 18, MW). Another teacher thoughtfully reflected, "Kids say things in different ways that I didn't think of" (FG 20, SE). Many teachers also commented on the benefits of hands-on activities to student learning. For instance a teacher reflected on how she now appreciates how students learn by doing (FG 14, MA).

Changes in Student Expectations

Another theme that emerged from the focus group discussions and administrators' interviews was a change in expectations of students. One teacher thought that the high ability students would do well with the new curriculum, but they didn't do as well as she had expected. She noticed that many of her "lower kids rose to the challenge" (FG 17, NE). A different teacher said that her expectations of her students had changed regarding what they can learn and do. She said, "I have more confidence in them that they can do more" (FG 25, SE). Another teacher said that he was surprised as he shared his initial expectations of his students, "they're never going to get this." He thoughtfully reflected that he "should have given them more credit" (FG 16, NE). Ms. Hobbes noted that the treatment teachers in her district "now have higher expectations for students in math" (AI, Cortana, NE).

Interestingly, one teacher shared that although her students could not "recall from day to day" they could explain that two groups of four "are 2, 4, 6, 8" (FG 22, NE), while in a different group, a teacher explained that memorization of multiplication facts was emphasized in the past, but her "students did not understand multiplication conceptually" (FG 19, SE). A surprised participant in Focus Group 22 (NE) questioned, "Who would have thought someone who can't do basic math facts can see a pattern and come up with a formula?" These particular teachers' experiences reinforce the importance of challenging all students regardless of their ability level to develop conceptual understandings of mathematics.

Validation

Some teachers in the focus groups (*n*=5, 38%) felt a sense of validation for their own teaching styles and instructional strategies after teaching the What Works Mathematics Curricula. One teacher said the way the lessons were set up is the way she typically organizes her class stating, “It’s what I normally do anyways” (FG 25, SE). She said it was nice because the units didn’t require too much change. Similarly, a teacher in Focus Group 20 corroborated stating that the “math units aligned with [my] teaching.” Another teacher shared, “It reminded me that there are some things that I am pretty good at” (FG 22, NE). Feeling validated, a teacher in Focus Group 14 (MA) affirmed, “I know kids love hands-on stuff and that they do better with manipulatives,” and that she enjoyed teaching that way even before the What Works Mathematics Study, but her textbook did not have “those things” so the What Works Mathematics Curricula validated what she already knew about “good teaching.”

Conclusion

Overall, the teachers in the focus groups were thoughtful and reflective in their responses to the questions “What did you learn about how you teach?” and “How has your knowledge of student learning changed?” Treatment teachers shared how their instructional practices and attitudes towards teaching mathematics changed. Teachers also learned that students could live up to higher expectations. Administrators’ interview comments also indicated that teachers’ instructional practices changed, and teachers learned as a result of participating in the study. As Ms. Conway shared, “Being involved in the research study is exciting and we certainly learned a lot” (AI, Savannah, MW).

Administrators’ Hopes for Teacher Change and Learning

Participating administrators in Cohort III were asked to consider the following question: “What strategies or skills do you hope teachers will transfer to other subject areas?” The administrators’ responses to this question revealed that the strategies and skills that they hoped to see teachers transfer to other areas included differentiation, the use of challenging content, and techniques that promote authentic student learning.

Differentiation

The administrators’ interviews revealed that the most common strategy they felt the teachers should transfer to other subject areas was differentiation. Many administrators agreed that the What Works Mathematics Curricula modeled differentiation in a way that the teachers found helpful as a guide for future differentiation. Ms. Hardin stated, “Differentiation is a practice we have been working on and this project allowed them to see the power of this practice in meeting the needs more effectively” (AI, Grand Arch, NE). More specifically, many of the teachers and administrators had such positive experiences with preassessments they wanted to continue the practice in the future. Ms. Atkinson shared thoughts about the future of preassessments in her district when she explained,

> It is hoped that teachers will continue to work collaboratively to develop means to assess students' prior knowledge and develop differentiated lessons based on the results of the pretest. . . . This study demonstrated to our teachers the importance of learning students' prior knowledge and building upon that. (AI, Skinner, NE)

Challenge

The second most prevalent strategy that the administrators believed the teachers should transfer to other subject areas was using more challenging lessons to promote higher levels of thinking. Ms. Shannon explained, "Raising the rigor of the curriculum across the board would be a benefit to our students" (AI, East Point, SE). Ms. Conway simply stated, "I hope my teachers transfer higher-level thinking to other areas" (AI, Vermilion, MW). The What Works Mathematics Curricula facilitated the use of challenge and higher-level thinking by teaching students to act as mathematicians to promote authentic student learning.

Authentic Student Learning

Students' ability to explain their reasoning verbally and in writing was emphasized in the What Works Mathematics Curricula. This was the most commonly mentioned skill that administrators wished to see transferred to other content areas. Administrators saw a connection between students' written and verbal explanations and an increased understanding of the mathematical content. This connection lead the administrators to respond that they would like to see the teachers transfer writing and discussing students' thoughts, processes, and explanations into other content areas. Ms. Strong responded, "I hope that the teachers will get the children to talk and write and use problem solving skills in other subjects the way they did in the math units" (AI, Calder, MA). The increased focus on explanations and communication facilitated the use of teamwork and collaboration between the students. This was well received by the administrators. "Again, I really liked the teamwork aspect of the units" (Ms. Booker, AI, George Washington, NE). Ms. Carey appreciated the hand-on nature of the units and hoped teachers would transfer these methods to other subjects.

> I have found hands on learning and discovery to be very effective in helping students to learn. These activities and ideas would be easily transferable to other areas of the curriculum and my hope is it will inspire them to do so. Involvement in the learning is so much more effective. (AI, Northwest, SE)

Conclusion

The overall findings of the administrators' interviews indicate that the principals, assistant principals, gifted and talented coordinators, and curriculum coordinators believed that a strategy that they would like to see transferred to other subject areas would be differentiation. It was also evident that administrators thought the increased challenge provided by the What Works Mathematics Curricula was beneficial to students, and they would like to see the increase in higher level thinking transferred to other content areas. The final set of skills that the administrators felt would be beneficial to students, if transferred to other content areas, were those associated with the students acting as professionals within the field being studied.

State Standards Influenced Teachers' Implementation of the Mathematics Curricula

Occasionally, treatment classroom teachers and administrators mentioned in their teachers' logs and administrators' interviews the need to supplement the What Works Mathematics Curricula or to teach their regular curriculum along with the What Works Mathematics Curricula because of concerns connected to state standardized tests. Some teachers expressed concern that the units did not cover specific skills that would be tested on their state standardized assessments. In addition, teachers in over half of the focus groups also voiced some concerns about implementing the What Works Mathematics Curricula due to the pressure of state tests and completing the units in enough time. It should be noted that the units were aligned to NCTM Standards and Focal Points; however, the units were not intended to "teach to the test" in each state. Ms. Copeland commented about state standards,

> I had to add/delete from the curriculum due to meeting the [state] standards. For example, the curriculum did not include pictographs and ordered pairs. Also, we only have the timeframe of 2 weeks for graphing so we had to pick & choose what to do with the curriculum. (*Greening Up With Graphing: Recycle, Reduce, & Reuse*, TL, Oyster Harbor, SE)

Ms. Higgins stated, "I did have to do 2 math lessons a day. I had to cover fractions unit during this time also" (*Awesome Algebra*, TL, Staten Ridge, MA). Ms. Berry also expressed the concern that she needed to spend time on other "tested" topics, explaining, "I could not teach this unit each day because we have too many other required topics. Also, we do not use line plots very often in 3rd grade, but do use pictographs which this unit didn't include" (*Greening Up With Graphing: Recycle, Reduce, & Reuse*, TL, Governor's Park. NE). One teacher said that teaching these units was "difficult" because it was a "different type of math," which was "hard to cram it in" with all of the other requirements for district assessments (FG 21, SW). A teacher in FG 26 expressed that there was "so much emphasis on the test, [that it] stresses us out" (MW).

Many of the teachers who expressed these concerns also noted the positive influence of the mathematics curricula. Ms. Fletcher noted:

> I really wish we had less topics to cover in the 3rd grade math curriculum so we could spend more time on units such as this. We are under a lot of pressure in [the state] to have the math curriculum taught by the end of March, so the students are

> prepared to take the (state test) test in April. This unit was fun, especially when you would see "the light go on" with some of the students. (*Awesome Algebra*, TL, Solsbury Valley, SE)

A participant in Focus Group 14 explained that she had a chance to look at the grade 5 math state test and knew that her grade 3 students could complete its graphing section because of their preparation from the What Works Mathematics Curricula (MA).

The few comments from the administrators who mentioned the state standards and testing in the interviews were mixed. Ms. Stafford expressed that "They felt the curriculum was well written and covered math standards adequately" (AI, Southeastern, SE). Ms. Hobbes shared, "They like the curriculum overall and feel that there are many valuable components although the curriculum is not comprehensive enough to cover all of our [state] learning standards" (AI, Cortana, NE).

Summary of Cohort III Qualitative Findings

For Cohort III participants in the What Works in Gifted Education Mathematics study, the research team's qualitative analyses addressed the following broad research question:

> How do teachers and administrators respond to their access to pre-differentiated and enriched curricula in algebra, geometry and measurement, and graphing and data analysis?

Team members reviewed qualitative data from a variety of sources: observation notes from treatment and control classrooms; focus group notes from treatment teachers; interview data from administrators; and treatment teachers' responses to questions from teachers' logs completed after implementing each of the curricular units. Five main themes were identified through inductive coding using QSR's NVivo 9 Qualitative Analysis Software. The major themes that emerged from the qualitative analyses include: (a) challenge, (b) questioning and discourse, (c) preassessments and grouping, (d) teacher and administrator reflections, and (e) concerns over state standards.

These responses from Cohort III teachers and administrators indicate that, despite the curricular units' high level of challenge, teachers effectively attended to the units' emphasis on mathematical discourse and higher-level questioning strategies. Teacher and administrators appreciated the help with differentiating instruction the tiered activities offered and hoped to build more differentiated instruction into their practice in the future. Concerns of the units' alignment with tested state standards or having sufficient time to teach unit objectives in addition to state standards surfaced as well.

CHAPTER 10: Summary and Recommendations

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Summary of Data From Cohorts I, II, and III

Despite the potential of differentiated curricular materials to enhance learning for all students, relatively little research exists to document the impact of high-level pre-differentiated and enriched curricula on students of all ability levels. Perhaps confronting average or struggling learners with abstract and difficult tasks may only result in frustration without learning gains. Perhaps advanced learners, accustomed to a lack of challenge, would fail to respond positively in the face of substantial challenge. Perhaps elementary school teachers would fail to embrace conceptual, hands-on instruction in mathematics, preferring more traditional direct instruction methods to assure success on high-stakes accountability tests. Although systems-level changes in teacher education programs and K-12 schools are necessary to implement pre-differentiated and enriched curricula in its fullest sense, empirical evidence can suggest preliminary answers to what is mainly speculative at present. The What Works in Gifted Education Mathematics Research Study attempted to provide quantitative and qualitative evidence of the impact of using challenging, pre-differentiated and enriched curricula with grade 3 students in general education classrooms.

Cohort I Quantitative Results

In Cohort I, the NRC/GT research team examined the impact of pre-differentiated and enriched curricula on student achievement (ITBS), employing a cluster randomized design. Twenty-one schools were randomly assigned to treatment or control, but one of these schools could not provide posttest data. A total of 1,366 students participated; the analytic sample for the multilevel analysis contained 970 students with complete ability, pre-intervention achievement and post-intervention achievement data. Treatment and control students took standardized ability and mathematics achievement tests prior to the intervention and a standardized mathematics achievement test after the intervention. Treatment students additionally completed researcher-developed unit tests before and after the instruction of each unit; they also completed a brief assessment constructed from grade 4 NAEP items. Each outcome measure presented a different perspective on the effectiveness of the curriculum implementation.

The results on the standardized mathematics test—the ITBS Math Problem Solving and Data Interpretation subtest or the Stanford Achievement Test Mathematics subtest—indicated that the treatment had no effect and that a student's starting score and his or her school's mean achievement were more predictive of post-intervention mathematics achievement than treatment. However, there were two problems with the outcome measure. First, 8.4% of the students received the highest score possible on the posttest. Thus, the true growth of these students was likely not measured accurately and introduced measurement error into the model. The true post achievement of these students was not determined. Second, the skills that were taught in the treatment were not reflected on the post-ITBS or Stanford Mathematics subtest. Many of these skills were above grade level and not captured by the grade level assessments.

The NAEP has provided a national benchmark of students' academic achievement in the United States for over 40 years. As part of participation in the NRC/GT mathematics study, grade 3 students who received the pre-differentiated and enriched curricula were administered 15 mathematics items from the NAEP. The items were chosen for their similarity to the conceptual knowledge contained in the mathematics curricula.

Although students involved in the study were in grade 3 during the curriculum implementation, the items on which they were tested were drawn from the grade 4 NAEP item bank. "Above-grade level" testing targets student achievement above the levels that could be measured by "at-grade level" items. Over 50% of the grade 3 students completed the grade 4 items correctly, with the exception of items 2 and 15. These results represent the 572 treatment students who participated in the first year of the study.

Treatment group students completed pre and post unit tests. Each unit had challenging content and a unit test with a high ceiling so all students could demonstrate growth. Despite the challenging content, across the sites all students showed significant growth from pretest to posttest in all three units. These results represent the following number of treatment students who participated in the first year of the study, Algebra (n=480); Geometry & Measurement (n=462); and Graphing & Data Analysis (n=399). These results demonstrate that all students can learn when presented with challenging and differentiated curricula.

Cohort II Quantitative Results

For Cohort II, the research team examined the impact of the pre-differentiated and enriched curricula on student achievement. Using a cluster randomized design, 17 schools were randomly assigned to treatment or control, and a total of 45 teachers and 846 students participated.

The ITBS results indicated that the treatment had no effect. However, as with Cohort I, test ceiling effects and lack of content alignment precluded accurate measurement. In Cohort II, 10% of the students received the highest score possible on the post-ITBS. Both of the problems could be solved by administering above grade level post assessments.

Treatment students in Cohort II completed 14 mathematics items from the NAEP. Although students involved in the study were in grade 3 during the curriculum implementation, the items on which they were tested were drawn from the grade 4 NAEP item bank. With the exception of items 10 and 13, more than 50% of the grade 3 students mastered the grade 4 items. These results represent the treatment 393 students who participated in the second year of the study who were assigned to treatment status by school.

Cohort III Quantitative Results

For Cohort III, the research team implemented a multisite cluster randomized control trial of grade 3 classrooms across the country that assigned 141 general education classroom teachers from 12 states and 43 schools to treatment or control conditions. The key purpose of the study was to determine if there was a difference between the mathematical achievement of students involved in 16 weeks of differentiated mathematics curricula in algebra, geometry and measurement, and graphing and data analysis and the achievement of students involved in the district's general education mathematics curriculum or "business as usual."

Treatment and control group students completed the ITBS—pretest Math Problems prior to the intervention and the posttest Data Interpretation the 16 week time period. To more fully examine the academic outcomes of students involved in the intervention, the treatment group students completed criterion-referenced pretests and posttests for the algebra, geometry and measurement, and graphing and data analysis units. In addition, treatment group students completed a subset of items related to these same topics from the NAEP. Items from the NAEP were designed for grade 4 students. Thus mastery of these items would provide evidence that students had developed mathematics content, concepts, and skills that extended beyond the typical grade 3 mathematics curricula.

A series of 3-level models constituted the quantitative analyses showing the effects of the differentiated curricula on Cohort III students. Students were nested within classrooms, which were nested within schools: The level-1 variables were student-level factors, level-2 variables were classroom-level factors, and level-3 variables were based on data aggregated to the school level. The final model failed to show a main effect for treatment, but did uncover interesting cross-level interaction effects. Using students' performance on the ITBS as a measure of achievement, Cohort III results showed that high performing students in low performing schools benefited from their involvement with the mathematics intervention. In addition, treatment group students from high performing schools and who were not as strong academically also benefited from their participation in the intervention. Students in the highest category of pretest achievement benefitted from the treatment in schools of all socioeconomic contexts. Additional descriptive analyses indicated the highest achieving students appeared to benefit the most from the treatment, and that the treatment effect was greatest for high achieving students in higher poverty schools. The results underscore the importance of using nuanced comparisons to capture treatment effects. Further evidence about the impact of the differentiated mathematics curriculum was documented by treatment group students' performance on the criterion-referenced assessments.

Pretest and posttest criterion-referenced unit test data indicated that treatment group students successfully learned and applied challenging content and concepts presented in the algebra, geometry and measurement, and graphing and data analysis units: Cohen's *d* effect sizes for each unit ranged from 1.33 to 1.73.

At the conclusion of the intervention, items from the National Assessment of Educational Progress assessed treatment group students' performance in algebra, geometry and measurement, and graphing and data analysis. At least 50% or more of the students mastered the content, concepts, and skills of all but one item typically used to assess grade 4 students.

Thus, this randomized controlled trial to determine the efficacy of using differentiated mathematics curricula with all grade 3 students yielded mixed evidence. Using the ITBS as a criterion, the intervention appeared to do no harm to the treatment students of low and average ability and did appear to benefit the students of high ability. Performance on the criterion-referenced unit tests and NAEP items provided some preliminary evidence of the efficacy of using pre-differentiated and enriched curricula with all grade 3 students; however, the limitations of this statement are fully acknowledged as those assessments were only completed by students involved in the intervention. To truly test the pre-differentiated and enriched curricula, a randomized control trial designed for grade 3 students would need to ensure that appropriate assessments reflecting the units' content were available to measure the growth of students' content, concepts, and skills involved in the intervention and compare it to the growth of students' content, concepts, and skills who continue with the general education mathematics curricula or "business as usual."

Cohort III Qualitative Findings

Multiple sources of qualitative data were collected to obtain a complete picture of the mathematics intervention onsite and at-distance. Treatment teachers completed logs at the end of each unit. The logs consisted of questions that would provide the researchers with documented information about the intervention and the impact on the teachers and their students. The research teams observed treatment and control classrooms, interviewed administrators, and conducted focus groups of treatment teachers. These forms of data were the basis for the qualitative findings described below.

Challenge

The higher percentage of statements coded under the "challenge" category for the treatment teacher observations could be explained by the nature of the challenging curricular content. Treatment teachers in this study received professional development in which they learned the importance of providing appropriate levels of challenge as well as the benefits of students “staying in the struggle.” According to Vygotsky (1978), students learn at an optimal level when they are instructed within a *zone of proximal development*. Within this zone, students are exposed to material that is slightly above their comfort level, but they are still capable of working independently with some support from the teacher (Tomlinson, 2001). Vygotsky stated “the area of immature, but maturing processes makes up the child’s zone of proximal development” (cited in Gredler & Shields, 2008, p. 85).

Larger and More Complex Numbers

As students deepen their mathematical understanding, they can often move from conceptualizing limited quantities or patterns to increasingly large ones. Eventually, students can extend these understandings into abstract algebraic concepts of quantity and patterns that do not require countable objects.

Complex Concepts and Skills

Of the three mathematics units, the *Awesome Algebra* Unit appeared to present the greatest degree of challenge for students in regards to learning complex concepts and skills. In this unit, students learned about the concepts of algebraic functions, growing and repeating patterns, and variables.

Overall, it appears as if the control teachers who did challenge their students with complex concepts and skills did so primarily by asking them to make predictions about formulas, patterns, or equations.

Explaining One’s Thinking

In the Treatment Classroom Observations, it was evident that the purposes for having students explain their thinking was to encourage students to think like mathematicians and to learn how to support and communicate their solutions.

The control teachers appeared to challenge students in this way as a means of monitoring students’ understanding of mathematics concepts.

Struggle

It is important to note in this discussion of “struggle” in response to the mathematics curriculum that participating treatment teachers in the study received professional development in which teachers learned about the importance of students struggling and “staying with the struggle.” This may also explain why there were more instances of students struggling in the treatment observations as compared to the control teacher observations. Furthermore, the nature of the mathematics units was designed to provide students with an appropriate degree of challenge.

Treatment classroom teachers' comments about hands-on activities indicate that they thought these activities helped students develop conceptual understandings of the mathematical topics being studied.

Questioning

The majority of observations coded as "multiple solutions" came from the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit lessons. This may be due in part to the nature of the real-world problems incorporated into the Unit's investigations. In this mathematics unit, the lessons lead students through the process of learning how to use data to determine if a given intervention will increase the number of items recycled at the school.

Grouping

Overall, grouping students by ability levels appeared to be an effective means for addressing academic variability within the classrooms. However, it is important to note that if students are to be grouped by ability, the assignments given to students should also be differentiated based upon the students' ability levels.

Beyond changing teachers' preassessment and grouping practices, this study affected many teachers' attitudes towards differentiation. Swan and Swain (2010) described this effect in their study of specific professional development techniques:

> It might be assumed that in order to change a teacher's practice, one has to first change through persuasion his or her beliefs about teaching. Indeed, this forms the model of many pre-service and in-service professional development courses, where ideas and theories are propounded and illustrated. However, we would suggest that changes in beliefs are more likely to follow changes in practice, after the implementation of well-engineered, innovative methods, as processes and outcomes are discussed and reflected upon. (p. 175)

This same pattern of change in practice leading to change in beliefs was confirmed through this mathematics research study.

Divergent Questioning

Initially, Cohort III treatment and control teacher observations were inductively analyzed to determine what types of questions teachers were using during mathematics instruction. Two broad classifications of classroom questions emerged during the inductive analysis of the treatment and control teacher observations, which were classified as either divergent or convergent. Divergent questions are defined as open-ended questions that elicit multiple responses or a range of acceptable responses from students. Convergent questions are questions that yield a singular or finite range of acceptable responses. As the focus of the pre-differentiated and enriched curricula was not necessarily to increase teachers' use of convergent questions with singular responses, the findings from the analyses of the divergent questioning classification will be the focus of this summary.

Discourse

In the Cohort III observations, it appears that discourse was utilized as an instructional strategy in a few ways. First, teachers often asked students to identify what they knew about a topic or mathematical concept by prompting students with "tell me what you noticed" after giving them a pattern or mathematics-related scenario. In addition, teachers used discourse to engage students in explaining their thinking—usually connected to a problem they had to solve in class. Finally, in the *Greening Up With Graphing: Recycle, Reduce, & Reuse* Unit, they used discourse to make connections between mathematics and the real world.

Discourse generally occurred in whole group settings. There were some instances of teachers asking students to discuss with a partner or in a group. When students discussed in groups or with a partner, there was usually a whole group follow-up. This allowed students to build on other students' responses, and it exposed students to a variety of ideas.

Further investigation on what happens in subsequent lessons based on information gained through discourse would help expand understanding of the role that discourse plays in the classroom.

Teacher Change and Learning

Responses were thoughtful and reflective as well as varied in the way that these questions were addressed during the discussions.

Recommendations

The What Works in Gifted Education Mathematics Study involved schools throughout the country. The theoretical framework was based on Renzulli and Reis's (1997) act of learning, which is central to the change process if educators want to ensure that all students are exposed to challenging and differentiated curricula. Too many times, struggling students are faced with repetitive curricula that is not tailored to what they know and what they need to know to fully understand the content, concepts, and skills, while average achieving and high achieving students are not presented with appropriate levels of challenge that will foster continued growth in learning. These students may be waiting to learn new concepts or just re-learning prior concepts. The act of learning emphasizes the interaction between student, teachers, and curricula. Educators must consider:

1. present achievement levels . . . [in mathematics],
2. the learner's interest in particular topics and the ways in which they can enhance present interests or develop new interests, and
3. the preferred styles of learning that will improve the learner's motivation to pursue the material that is being studied. (Renzulli & Reis, 1997, p. 35)

The critical importance of the interactions between and among the teacher, student, and curricula was constantly reviewed as the research team developed the algebra, geometry and measurement, and graphing and data analysis units using models developed by Tomlinson, Kaplan, and Reis and Renzulli. The curricula were purposely designed to be challenging; therefore, lessons were scaffolded to provide more or less details to help students master the concepts and to offer curricular extensions when students demonstrated their knowledge and understanding of the mathematics.

In the curriculum development process, the pre-differentiated and enriched curricula were developed to provide in-depth lessons in the three areas of mathematics. The modified, pre-differentiated units focused on conceptual thinking, replacing 16 weeks of the general education grade 3 mathematics curriculum. We hypothesized that treatment group students would outperform control group students on the ITBS Math Problem Solving and Data Interpretation subtest. Instead, the post-ITBS scores of students in the treatment group were equal to those in the control group. Several conclusions can be posited:

1. The ceiling on the norm-referenced test was not high enough to record students' true level of content, concepts, and skills mastered in problem solving and data interpretation.
2. The norm-referenced ITBS was not a good match to content in the algebra and geometry and measurement units.
3. The lack of a main effect illustrated that eliminating 16 weeks of the "business as usual" curricula for the treatment group students did not have a negative impact on students involved in the intervention.
4. The curricula benefited students differentially depending on the achievement status of their schools and their designation as treatment group or control group students.

A defensible interpretation of the results from this study is to consider that treatment teachers were able to replace grade level curriculum with more challenging, pre-differentiated and enriched curricula without negatively impacting standardized test scores. In the current age of increased accountability, teachers are often afraid to stray from the mainstream curriculum for fear of jeopardizing their state test scores. Assuming the ITBS Level 9 Math Problem Solving and Data Interpretation subtest and the Stanford Achievement Test mathematics section measure the typical grade 3 mathematics curricula, the current study provides some evidence that teachers can replace typical at-grade level curriculum with more challenging, pre-differentiated and enriched curricula without suffering adverse consequences on standardized assessments. Viewed through this lens, the results of this study should encourage teachers to consider stepping out of the lock-step curriculum to differentiate their math curriculum.

The results of data analyses focusing on the unit tests and NAEP test provided a more positive picture of the efficacy of using pre-differentiated and enriched curricula with students of all abilities. This statement is made with full recognition that the more stringent research design of comparing the results of the treatment and control groups was not operative. Further study of the impact of the differentiated curricula is warranted.

The study produced several recommendations for future researchers. First, it is essential that the outcome measure adequately assesses the constructs that are taught as part of the curriculum. Standardized assessments may have very narrow content coverage. Second, seriously consider using out-of-level assessments, even when assessing the entire grade level, as ceiling effects are clearly evident for on-grade assessments. Third, consider administering researcher-developed measures to both the treatment and control conditions. Tremendous learning gains occurred between the unit tests administered to the treatment group students at pretest and posttest. Unfortunately, because those tests were not administered to the control group students, there is no way of comparing the growth of the treatment group students to the growth of the control group students. Future research will incorporate researcher-developed measures into the assessment plan for both treatment and control group students. As with all research, finding answers to initial hypotheses often results in myriad questions. Some answers provide guidance for future studies, while other questions remain.

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Appendix A: What Works in Gifted Education Study Instruments

The National Research Center on the Gifted and Talented
What Works in Gifted Education Study

University of Connecticut
Mathematics Study

Focus Group Questions
Teachers

1. **Share an example of a student's reaction to the math curriculum.**

2. **What was your comfort level with mathematics before you started the unit(s)?**

3. **Describe your comfort level with one of the units.**

4. **What did you learn about how you teach?**

5. **To what extent has the math curriculum helped you accommodate diverse learners?**

6. **How has your knowledge of student learning changed?**

7. **How would you compare the approach to mathematics in these units to your school's textbooks?**

8. **What strategies or techniques do you think you will transfer to other units in math? Other content areas?**

9. **To what extent have the differentiated lessons assisted you with teaching the mathematics content?**

The National Research Center on the Gifted and Talented
What Works in Gifted Education Study

University of Connecticut
Mathematics Study

Administrator Interviews

1. **What are teachers' reactions to the curriculum?**

2. **How does the math curriculum benefit students?**

3. **What strategies or skills do you hope teachers will transfer to other subject areas?**

4. **How this study supported any other initiatives in your school?**

5. **What would you tell other administrators about being involved in a research study?**

What Works Classroom Observation Scale *(Treatment Classrooms)*

Observer: ______________________________ Teacher: ______________________________ School: ______________________________

Date: ___________________ Unit/Lesson: ____ TIME: Start: __________ Finish: ___________

Rate each item on a 1-to-4 response scale in which 1 indicates that the item occurs to a lesser extent and 4 indicates that the item occurs to a greater extent. If the item is not applicable to the lesson observed, check N/A (Not Applicable).

	N/A	**1** Occurs to a lesser extent	**2**	**3**	**4** Occurs to a greater extent
1. Students understand the "Big Idea" (i.e., understand concept in different context) of the lesson.					
2. The materials provided for lesson implementation are used appropriately.					
3. Appropriate mathematical language is used.					
4. Students have a clear understanding of directions for lesson activities.					
5. The students and/or teacher make(s) connections to prior concepts.					
6. Student learning is assessed through observation, listening, and/or information gathering.					
7. Students are grouped according to the suggested levels of differentiation.					
8. Students are presented with challenging content.					
9. Discourse (whole group, small group, peer) about mathematical problems occurs.					
10. Students are invited to find multiple strategies or solutions to the mathematical problem.					
11. Students are encouraged to explain or justify their responses.					
12. Students are engaged in the lesson.					

What Works Classroom Observation Scale *(Treatment Classrooms)*

Field Notes	1 Low fidelity	2	3	4 High fidelity
Fidelity to curriculum:				

What Works Classroom Observation Scale *(Control Classrooms)*

Observer: ______________________________ Teacher: ______________________________ School: ______________________________

Date: ___________________ Unit/Lesson: ____ TIME: Start: __________ Finish: __________

Rate each item as Not/Observed (N/O) or Observed (O).

	Not Observed (N/O)	**Observed (O)**
1. Students understand the "Big Idea" (i.e., understand concept in different context) of the lesson.		
2. The materials provided for lesson implementation are used appropriately.		
3. Appropriate mathematical language is used.		
4. Students have a clear understanding of directions for lesson activities.		
5. The students and/or teacher make(s) connections to prior concepts.		
6. Student learning is assessed through observation, listening, and/or information gathering.		
7. Students are grouped according to the suggested levels of differentiation.		
8. Students are presented with challenging content.		
9. Discourse (whole group, small group, peer) about mathematical problems occurs.		
10. Students are invited to find multiple strategies or solutions to the mathematical problem.		
11. Students are encouraged to explain or justify their responses.		
12. Students are engaged in the lesson.		

What Works Classroom Observation Scale *(Control Classrooms)*

Field Notes	1	2	3	4
Overall Rating				

The National Research Center on the Gifted and Talented
What Works in Gifted Education Research Study
Awesome Algebra 08-09
Teacher's Log

Name: ______________________________ Date: ___________________________

School: ______________________________ District/State: _____________________

Instructions: Please complete this log when your class finishes the *Awesome Algebra* unit.

Question 1

Please circle your level of agreement with the following statement: The teacher's manual provided me with enough information to teach the lessons successfully.				
1 Strongly Disagree	2 Somewhat Disagree	3 Neutral	4 Somewhat Agree	5 Strongly Agree
What would have helped you to be more successful?				

Question 2

Please circle your answer to the following question: How well did the pacing chart (p. 14) estimate the time that each lesson required?					
Many lessons took a **shorter** amount of time than estimated.	**Some** of the lessons took a **shorter** amount of time than estimated.	**Some** of the lessons took a **shorter** amount of time while others took **longer**.	**Most** lessons were estimated **accurately**.	**Some** of the lessons took a **longer** amount of time than estimated	**Many** lessons took a **longer** amount of time than estimated.
In what ways could the pacing chart for the unit be improved? In your response please identify specific lessons for which the pacing chart was inaccurate.					

Question 3

Please circle your response to the following question:
How would you rate the resources provided to support this unit?

	Not Helpful	Somewhat Unhelpful	Neutral	Somewhat Helpful	Very Helpful	Not Utilized
Videos	1	2	3	4	5	NU
Other Instructional Resources on CD	1	2	3	4	5	NU
Website	1	2	3	4	5	NU
UCONN Contact	1	2	3	4	5	NU

How could the existing resources be more helpful?

What additional resources would have been beneficial?

Question 4

Please circle your response to the following question:
How would you rate the preassessments tools including the rubrics found within this unit in terms of their helpfulness in placing the students within appropriate readiness groups?

1 Not Helpful	2 Somewhat Unhelpful	3 Neutral	4 Somewhat Helpful	5 Very Helpful

How could the preassessments tools including the rubrics be made more helpful?

Question 5	
	Did you use pretests prior to teaching this unit? If not, have you started using pretests in any other content area?
	Will you continue to pretest students in math after this study?

Question 6					
	Please circle your level of agreement with the following statement: The lesson activities within the different sections seemed to challenge the students.				
	1 The lesson activities challenged none of the students.	2 The lesson activities challenged a few of the students.	3 The lesson activities challenged half of the students.	4 The lesson activities challenged most of the students.	5 The lesson activities challenged all of the students.
	Which lesson seemed to challenge the students the most? Which lesson seemed to challenge the students the least?				
	How did the different ability levels within your class respond to the challenge level of the lessons?				

Question 7

Please circle your level of agreement with the following statement:
The lesson activities within the different sections seemed to engage the students.

1	2	3	4	5
The lesson activities engaged none of the students.	The lesson activities engaged a few of the students.	The lesson activities engaged half of the students.	The lesson activities engaged most of the students.	The lesson activities engaged all of the students.

Which lesson seemed to engage the students the most? Which lesson seemed to engage the students the least?

How did the different ability levels within your class respond to the engagement level of the lessons?

Question 8

Please describe any activity within the unit that your students had difficulty completing. How did you modify the lesson to accommodate their needs?

Question 9	
	How did you utilize the Talk Moves within the lessons?
	To what extent did the Talk Moves help you develop relevant questions and/or spur student discussions?
	How have you used them in other content areas?

Question 10	
	How did your students react to the Think Beyond cards?

Question 11	
	What kind of changes would you make in your instruction if you were teaching this unit again?

Question 12	What lessons/topics would you like to see added to *Awesome Algebra*?

Question 13	Additional Comments/Suggestions:

After you have completed the form, please return it to your University of Connecticut liaison through one of the following methods:

- Send by email as an attachment
- Send by fax to 860.486.2900
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  Storrs, CT 06269-3007

The National Research Center for Gifted and Talented
What Works in Gifted Education Research Study
Geometry & Measurement for All Shapes & Sizes 08-09
Teacher's Log

Name: ______________________________ Date: ____________________________

School: ______________________________ District/State: _____________________

Instructions: Please complete this log when your class finishes the *Geometry & Measurement for All Shapes & Sizes* unit.

Question 1

Please circle your level of agreement with the following statement: The planning format within the manual (Big Mathematical Ideas, Lesson Objectives, Materials, Mathematical Language, Lesson Preview) was useful.				
1 Strongly Disagree	2 Somewhat Disagree	3 Neutral	4 Somewhat Agree	5 Strongly Agree
What would you change about the format to further support teachers?				

Question 2

Please circle your answer to the following question: How well did the pacing chart (p. 8) estimate the time that each lesson required?					
Many lessons took a **shorter** amount of time than estimated.	**Some** of the lessons took a **shorter** amount of time than estimated.	**Some** of the lessons took a **shorter** amount of time while others took **longer**.	**Most** lessons were estimated **accurately**.	**Some** of the lessons took a **longer** amount of time than estimated	**Many** lessons took a **longer** amount of time than estimated.
In what ways could the pacing chart for the unit be improved? In your response please identify specific lessons for which the pacing chart was inaccurate.					

Question 3

Please circle your response to the following question:
How helpful was the culminating project, *A Shapely Living Room*, in determining what the students learned and understood from the unit?

1 Not Helpful	2 Somewhat Unhelpful	3 Neutral	4 Somewhat Helpful	5 Very Helpful

How could the post-assessment or culminating project be made more helpful?

Question 4

To what extent have your students' math communication skills changed throughout this unit? Please provide specific examples.

Question 5

How did this unit affect your views about differentiation?

Have you used some of the differentiation pieces—such as tiered assignments—within other subjects or lessons? Please explain.

Question 6	Did this unit appropriately challenge all students within your classroom? Please explain.

Question 7	*Please circle your response to the following question:* In general, what effect did this unit have on student motivation in math?				
	1 Very Negative	2 Somewhat Negative	3 No Effect	4 Somewhat Positive	5 Very Positive
	Please give a specific example.				

Question 8	What aspects of the unit did you find to be most successful for you and/or your students?

Question 9	What lessons/topics would you like to see added to *Geometry & Measurement for All Shapes & Sizes*?

Question 10	What advice would you give to a teacher who is beginning this unit?

Question 11	Additional Comments/Suggestions:

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  Storrs, CT 06269-3007

The National Research Center for Gifted and Talented
What Works in Gifted Education Research Study
Greening Up With Graphing: Recycle, Reduce, and Reuse 08-09
Teacher's Log

Name: ______________________________ Date: ____________________________

School: ______________________________ District/State: _____________________

Instructions: Please complete this log when your class finishes the *Greening Up With Graphing: Recycle, Reduce, and Reuse* unit.

Question 1

Please circle your level of agreement with the following statement: My students have exhibited a greater command and use of mathematical language in their small group and whole class discussions as a result of this unit.				
1 Strongly Disagree	2 Somewhat Disagree	3 Neutral	4 Somewhat Agree	5 Strongly Agree
Please give a specific example to explain your choice.				

Question 2

Please circle your level of agreement with the following statement: My students' ability to communicate mathematical concepts in their written work has improved as a result of this unit.				
1 Strongly Disagree	2 Somewhat Disagree	3 Neutral	4 Somewhat Agree	5 Strongly Agree
Please give a specific example to explain your choice.				

Question 3

Please circle your level of agreement with the following statement: My students have demonstrated a greater capacity to tackle challenging problems that require analysis and problem solving skills as a result of this unit.				
1 Strongly Disagree	2 Somewhat Disagree	3 Neutral	4 Somewhat Agree	5 Strongly Agree
Please give a specific example to explain your choice.				

Question 4

Please circle your level of agreement with the following statement: My students' are better able to draw conclusions based on data they have collected as a result of this unit.				
1 Strongly Disagree	2 Somewhat Disagree	3 Neutral	4 Somewhat Agree	5 Strongly Agree
Please give a specific example to explain your choice.				

Question 5

To what extent did this unit address students' varied learning styles?

Question 6

Please circle your answer to the following question:
How well did the pacing chart (p. 14) estimate the time that each lesson required?

Many lessons took a **shorter** amount of time than estimated.	**Some** of the lessons took a **shorter** amount of time than estimated.	**Some** of the lessons took a **shorter** amount of time while others took **longer**.	**Most** lessons were estimated **accurately**.	**Some** of the lessons took a **longer** amount of time than estimated	**Many** lessons took a **longer** amount of time than estimated.

In what ways could the pacing chart for the unit be improved? In your response please identify specific lessons for which the pacing chart was inaccurate.

Question 7

What lessons/topics would like to see added to *Greening Up With Graphing*?

Question 8

How did you utilize the concept check-ups?

Question 9	Did the students have any misconceptions? If so, please describe what the misconceptions were and how they were corrected.

Question 10	What was the most challenging aspect of implementing this unit?

Question 11	Describe any changes in your professional knowledge and skills throughout this study.
	What knowledge, skills, or strategies will you apply from these units to other units of study?

Question 12	
	Please compare these units to your regular curriculum with regard to student interest and growth.
	Please compare these units to your regular curriculum with regard to depth and complexity of content.
	Please compare these units to your regular curriculum with regard to ease of implementation.

Question 13	
	Additional Comments/Suggestions:

After you have completed the form, please return it to your University of Connecticut liaison through one of the following methods:

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  The National Research Center for Gifted and Talented
  c/o E. Jean Gubbins
  2131 Hillside Road, Unit 3007
  Storrs, CT 06269-3007

The National Research Center on the Gifted and Talented
What Works in Gifted Education Research Study
Awesome Algebra 09-10
Teacher's Log

Name: Date:

School: City/State:

Instructions: Please complete this log when your class finishes the *Awesome Algebra* unit. For items 1-11, rate how strongly you agree or disagree with the statements by marking the checkbox. Thank you for your thoughts and reflections.

Item	**Strongly Disagree**	**Disagree**	**Neither Agree nor Disagree**	**Agree**	**Strongly Agree**
1. The preassessment helped me place students in readiness groups.	☐	☐	☐	☐	☐
2. I have noticed a positive difference in my students' writing abilities in math and other subjects because of this curriculum.	☐	☐	☐	☐	☐
3. The lessons in *Awesome Algebra* challenged all of my students.	☐	☐	☐	☐	☐
4. I found the additional study resources (CDs, DVDs, and website) very helpful.	☐	☐	☐	☐	☐
5. My students seem more excited about math with this curriculum.	☐	☐	☐	☐	☐
6. The ability level of my students was higher than I had expected.	☐	☐	☐	☐	☐
7. My students are now better at discussing mathematical concepts with their peers and adults.	☐	☐	☐	☐	☐
8. Implementing this curriculum has improved my abilities to differentiate.	☐	☐	☐	☐	☐
9. The culminating project was helpful to gauge what my students had learned in *Awesome Algebra*.	☐	☐	☐	☐	☐
10. The teacher's manual was easy to comprehend and implement.	☐	☐	☐	☐	☐
11. I enjoyed teaching this unit.	☐	☐	☐	☐	☐

12. Describe a specific student's reaction to *Awesome Algebra*.

13. Describe your overall experience with the *Awesome Algebra* curriculum.

14. You may use this space to provide us with any additional comments, suggestions, or concerns.

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3. Mail in the business reply envelope.
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THANK YOU!

The National Research Center on the Gifted and Talented
What Works in Gifted Education Research Study
Geometry & Measurement for All Shapes & Sizes 09-10
Teacher's Log

Name: Date:

School: City/State:

Instructions: Please complete this log when your class finishes the *Geometry & Measurement for All Shapes & Sizes* unit. For items 1-10, rate how strongly you agree or disagree with the statements by checking the appropriate box. Thank you for your thoughts and reflections.

Item	**Strongly Disagree**	**Disagree**	**Neither Agree nor Disagree**	**Agree**	**Strongly Agree**
1. The preassessment helped me place students in readiness groups.	☐	☐	☐	☐	☐
2. My students looked forward to math class when we were working on this unit.	☐	☐	☐	☐	☐
3. This unit challenged all of my students.	☐	☐	☐	☐	☐
4. My students were engaged with the lessons in this unit.	☐	☐	☐	☐	☐
5. This unit helped me think about some geometry and measurement concepts in a new or unique way.	☐	☐	☐	☐	☐
6. I witnessed my students making considerable conceptual growth throughout this unit.	☐	☐	☐	☐	☐
7. My students benefited from working with other students in their assigned groups.	☐	☐	☐	☐	☐
8. The teacher's manual was easy to comprehend and implement.	☐	☐	☐	☐	☐
9. My students were able to demonstrate their learning through the culminating project.	☐	☐	☐	☐	☐
10. I enjoyed teaching this unit.	☐	☐	☐	☐	☐

11. Describe a specific student's reaction to the geometry curriculum.

12. Describe a surprising experience you have had with this math curriculum.

13. You may use this space to provide us with any additional comments, suggestions, or concerns.

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2. Fax to 860.486.2900, attn: Lisa Rubenstein.
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THANK YOU!

The National Research Center on the Gifted and Talented
What Works in Gifted Education Research Study
Greening Up With Graphing: Recycle, Reduce, and Reuse 09-10
Teacher's Log

Name: Date:

School: City/State:

Instructions: Please complete this log when your class finishes the *Greening Up With Graphing: Recycle, Reduce, and Reuse* unit. For items 1-9, rate how strongly you agree or disagree with the statements by checking the appropriate box. Thank you for your thoughts and reflections.

Item	Strongly Disagree	Disagree	Neither Agree nor Disagree	Agree	Strongly Agree
1. My students' ability to communicate mathematical concepts in their written work has improved as a result of this unit.	☐	☐	☐	☐	☐
2. My students have demonstrated a greater capacity to approach and tackle challenging problems using analysis and problem solving skills as a result of this unit.	☐	☐	☐	☐	☐
3. This unit addressed my students' varied learning styles.	☐	☐	☐	☐	☐
4. My students are better able to draw conclusions from data as a result of this unit.	☐	☐	☐	☐	☐
5. My students were able to understand and answer the questions in the Student Journal.	☐	☐	☐	☐	☐
6. This unit added depth and complexity to the way graphing is usually taught in our third grade curriculum.	☐	☐	☐	☐	☐
7. My students exhibit a greater command and use of mathematical language in small group and whole class discussions as a result of this unit.	☐	☐	☐	☐	☐
8. I enjoyed teaching this unit.	☐	☐	☐	☐	☐
9. The teacher's manual was easy to comprehend and implement.	☐	☐	☐	☐	☐

10. Give a specific student's reaction to the graphing and data analysis curriculum.

11. Describe how this unit has affected you and/or your classroom.

12. You may use this space to provide us with any additional comments, suggestions, or concerns.

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THANK YOU!

Appendix B: Data Source Key

Data Source Key

Source	**Code**
Administrators' Interviews	AI
Control Teacher Observation	CTO
Teachers' Focus Groups	FG
Teachers' Logs	TL
Treatment Teacher Observation	TTO

Appendix C: Cohorts I & II Treatment Teacher Focus Group Pseudonyms

Cohorts I & II Treatment Teacher Focus Group Pseudonyms

Cohort I Treatment Schools

Focus Group #	Pseudonym School	Pseudonym Location†
1	Vista; Heritage	SE
2	Stone Mill	NE
3	Crowder Point; Historic Cove	NE
4	Rosewood Park	SE
5	West Valley; Pleasant View	MW
6	Lakeshore	MW
7	New Horizon	SE

† NE: Northeast; MA: Mid-Atlantic; SE: Southeast: MW: Mid-west; SW: Southwest

Cohort II Treatment Schools

Focus Group #	Pseudonym School	Pseudonym Location†
8	Lakeshore	MW
9	Pleasant View	MW
10	Stone Mill	NE
11	West Valley	MW

† NE: Northeast; MA: Mid-Atlantic; SE: Southeast: MW: Mid-west; SW: Southwest

Appendix D: Sample Grouping Guide for Differentiated Instructional Groups

Sample Grouping Guide for Differentiated Instructional Groups

Greening Up With Graphing: Recycle, Reduce, & Reuse
Lesson 6: Bar Graphs—Displaying Shape Data

	Babbage Group	**Galileo & Falconer Group**
How student scored on #3 on the pretest	Scored 0-2 points on item	Scored 2-5 points on item
Student Pages for Lesson	*Reaching our Goal*—Babbage	*Reaching our Goal*—Galileo & Falconer

Appendix E: Cohort III Treatment Teacher Focus Group Pseudonyms

Cohort III Treatment Teacher Focus Group Pseudonyms

Focus Group #	Pseudonym School	Pseudonym Location†
12	Morrowind; First Sun; Terracotta	NE
13	Franklin Bridge	MW
14	Lucasville; Staten Ridge; Calder	MA
15	George Washington; Skinner; Cortana	NE
16	Cedar Brook; Halcyon; Deer Creek	NE
17	Forge Hill; Centurion	NE
18	Apple Tree; Shady River	MW
19	Springville; Shelbyfield; Southeastern	SE
20	Smithton; Old Toll Road	SE
21	Shade Rock; Sunnyside	SW
22	Grand Arch	NE
23	Sycamore; E. Halsey; Seabreeze	SW
24	Evergreen Street; Vermillion; Savannah	MW
25	Oyster Harbor	SE
26	Newton	MW
27	Bald Eagle; Mustang Ranch	MW
28	Pegasus	MA

† NE: Northeast; MA: Mid-Atlantic; SE: Southeast: MW: Mid-west; SW: Southwest

Research Monograph

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University of Connecticut
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