The Impact of a Professional Learning Intervention Designed to Enhance Year Six Students' Computational Estimation Performance

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This paper reports on the analysis of a study of a professional learning intervention focussing on computational estimation. Using a multiple case study design it was possible to describe the impact of the intervention of students' beliefs and computational estimation performance. The study revealed some noteworthy impacts on computational estimation performance and that beliefs were altered to a lesser extent.

Curriculum documents globally are calling for primary mathematics classrooms to be sense making environments with students able to think flexibly when computing (Australian Curriculum and Assessment Reporting Authority, 2011; Department for Education, 2011; National Council of Mathematics Teachers, 2006). Students who are able to interpret and solve number problems with understanding can be described as possessing 'number sense'. Number sense is an emerging term without an established definition. This study used McIntosh, Reys, Reys Bana and Farrell's (1997) definition of number sense:

> A person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgements and to develop useful and efficient ways of managing numerical situations (p.3)

An essential component of a students' number sense is their ability to estimate in computations – both as a computational choice in its own right and used as a checking tool.

It was of interest to the Researcher, who lecturers in primary mathematics education at a tertiary level, to consider how primary school teachers could be better supported to teach computational estimation to Year 6 students. This paper reports on a professional learning intervention designed to improve the teaching of computational estimation and its impact on Year 6 students' beliefs and performance on estimation tasks. This intervention was part of the Researcher's PhD and took place in the school year of 2009. In this paper it outlines how some noteworthy impacts in students' use of estimation as a computational choice occurred. This study was part of a larger research project which also looked at the teachers' beliefs and teaching approaches that were developed as a result of the professional learning intervention.

Conceptual Framework and Theoretical Perspective

Underpinning the study was sociocultural theory, that claims students learn by a process of gradual individualisation rather than the acquisition of rote knowledge (Sfard, 2008). At the forefront of sociocultural theory is the importance of culture in the construction of new knowledge. It is through conversations that new ideas are constructed. The major semiotic tool in sociocultural theory is language and the teacher plays an important role in mediating the different types of classroom discourse.

Effective Teaching and Learning of Computational Estimation and Mathematics

Traditionally computational estimation is differentiated from numerosity and measurement estimation, but with the latest research findings supporting using contexts, such as authentic measurement tasks, this differentiation may be less useful. In the past

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computational estimation was often taught in an isolated manner (Dowker, 2003). It has recently been asserted, however, that conceptual estimation be taught within a problem-centred contextual approach (Reys & Reys, 2004). Yoshikawa (1994) argues that where computational estimation is set in a meaningful context, students are more likely to appreciate that estimation can save them computational time and energy. Research has shown that students can use computational estimation strategies. For example, Neill (2006) found that Year 8 students were able to implement a variety of estimation strategies. The use of computational estimation has been impeded by some negative connotations that estimation has in schools. Teachers find it ambiguous and students think that mathematics can only have one exact answer (Yoshikawa, 1994). Changing beliefs may be very important in encouraging the use of computational estimation in classrooms.

The Professional Learning Intervention

The professional learning intervention was underpinned by four principles which were derived from the research literature. The first three were concerned with effective mathematics teaching generally and they were that mathematics teaching is effective when it is active (Franke, Kazemi, & Battey, 2007), metacognitive (McKeachie, Pintrich, Lin, & Smith, 1986), and contextual (Gravemeijer & Terwel, 2000). The fourth principle was that computational estimation as a component of number sense is a valuable computational choice (Reys & Reys, 2004). Various estimation strategies have been identified by mathematics educators (Reys & Reys, 2004) and a detailed taxonomy of these was created by the Researcher (Mildenhall, 2009). The strategies in the taxonomy were, rounding, nice/compatible numbers, front-end loading, benchmarking, range and the sample strategy. These strategies were shared with the teachers in the professional learning intervention. The teachers were then asked to consider how to best teach these strategies to Year 6 students.

Features of effective teacher professional learning were considered in the design of the intervention. These included providing support to the teachers throughout the professional learning program(Ingvarson, Meiers, & Beavis, 2005), encouraging an action research and collaborative approach (Bray, 2002), and creating opportunities for the teachers to reflect on how the planned ideas translated into the classroom (Fennema et al., 1996).

The morning sessions of the professional learning workshops focused on the teachers sharing their personal reflections with the group on their teaching of computational estimation. They then focused on developing their personal PCK of estimation and mathematics. In the afternoon sessions the teachers were presented with suggested teaching activities which were designed to include features of the four principles outlined earlier, and in small groups of colleagues they designed a series of estimation activities that would develop computational estimation. It was also suggested that 'estimation as a checking device' be incorporated in to their mathematics lessons.

Purpose and Research Question

The purpose of this study was to investigate the impact of the professional learning intervention on the teaching and learning of computational estimation. The main focus in this part of the study was the impact of the intervention on the students. The following question was therefore explored: How did the teaching approaches impact on students' beliefs about estimation, their mathematical knowledge and their computational estimation performance?

Methodology

This study used a multiple case study design to generate rich descriptions of the impact of the intervention in specific contexts (Merriam, 1998). Initially there were three individual case studies comprising teachers and their Year 6 students and then a cross-case analysis was conducted. Conducting multiple case studies allowed the researcher to gain a more comprehensive understanding of how the professional learning intervention impacted on the Year 6 students' beliefs and computational estimation performance in different school contexts (Miles & Huberman, 1994).

Participants

Experienced Year 6 teachers from five non-government schools in Perth, Western Australia were invited to take part in the study. Six teachers in total, from four schools agreed to take part in the professional learning program. Three teachers were selected for case studies due to their different levels of engagement in the program (Figure 1.). Bob exhibited a high level of engagement in the program, Wendy was engaged in the professional learning but this engagement was affected somewhat by her school context and Peter exhibited the lowest level of engagement. The individual contexts of the three classrooms were important when evaluating the outcomes. It is noted, for instance, in Figure 1 that only Bob's school organisation allowed him the freedom to not use a textbook. As a result of engaging in the professional learning intervention, the three teachers each used different pedagogies to teach computational estimation. Generally Bob used the estimation tasks set in meaningful contexts and integrated estimation into all of his teaching. He also introduced all of the estimation strategies to the students. Wendy taught the estimation work set in meaningful tasks as a separate topic and also introduced all of the estimation strategies to the students. Peter did not introduce the meaningful tasks until towards the end of the study and did not introduce any estimation strategies other than rounding to the students.

Contextual Factors	Wendy	Bob	Peter
School	Low fee independent	Low fee independent	Low fee independent
Organisation of class	Mixed ability	Top stream of 2 classes	Mixed ability
Curriculum	School text book	Broad outline provided	School text book
ISEA value (mean 1000)	1118	1003	1038
Years of teaching	25	16	9
Number of students in the class	32	25	32
Mean Year 5 NAPLAN score (Australian National Testing)	476	478	474

Figure 1. Individual teachers' contexts.

Instruments

The following data collection instruments were used in this part of the study to capture the students' beliefs and computational estimation performance: the Researcher's audit trail journal, teacher interviews, focus group interviews with six students from the class case study, and students' written work and computational estimation timed pre and post-tests. The computational estimation test was adapted from an existing number sense test, which had been used in internationally recognised research (McIntosh et al., 1997). Two of the six questions were drawn from other sources, but they were adapted to fit the same format as the number sense test.

Data Collection

The pre-tests, teacher interviews and student focus-group interviews were administered before the study began in February 2009. The study of the teacher professional learning intervention took place over one year and involved the teachers taking part in three one-day workshops. There were four cycles to the research with data collection involving classroom observation, and collection of classroom artefacts taking place during each of these cycles. At the end of the study in November 2009 the post-test and final student focus-group interviews were administered.

Data Analysis

A content analysis was made of the qualitative data that was gathered from the multiple sources of data. Using multiple sources of data increased the internal validity of the study. Using the computer software NVIVO 9 (QSR, 2008) as a data management system, the Researcher undertook a deductive and inductive coding process so that themes which described students' beliefs emerged. Qualitative data concerning students' computational performance was also collected and analysed using the same process. This triangulation supported the findings that emerged from quantitative data analysis and again increased its validity. The quantitative data was collated and analysed using PASW (SPSS inc, 2010). Using this software it was possible to calculate changes in estimation performance on the timed Computational Estimation Test. The analysis focused on the pre- and post-test mean scores for each question and the estimation strategies used by the students. For this analysis it was decided to focus on specific strategies used and after this initial categorisation the specific strategies where grouped into either a reasoned or unreasoned strategy.

Results

Student Beliefs

At the beginning of the study most of the students believed that mathematics was something to be done quickly, was about four operations, addition, subtraction, multiplication and division and one correct answer. Generally this perception did not change by the end of the professional learning intervention. It appears that the professional learning intervention did not impact on the students' general beliefs about mathematics as they they perceived the estimation work as something outside normal school mathematics.

When asked questions that specifically focussed on estimation most students showed changes from the beginning to the end of the study. Their understanding of computational estimation was somewhat limited at the beginning of the study, viewing it as a checking device and as a type of guess. They did not perceive that computational estimation could be a choice in its own right when initially deciding how to solve a mathematics problem. At the end of the study all of the students in the three focus groups discussed estimation as an extra activity to normal calculations. Their beliefs about estimation appeared broader and two of the three focus groups students' showed an awareness of the variety of estimation strategies. This change was reflected in the drawing of a concept map at the beginning of the study and at the end. Hannah (a pseudonym) explained at the beginning of the study that she believed estimation to be a "guess instead of finding out the real answer" (Hannah, 10/2/2009). But at the end of the study she produced this concept map shown in Figure 2 below, which shows that Hannah had an awareness of a variety of computational estimation strategies and some different contexts in which estimation could be useful . This appreciation of how estimation is relevant in certain contexts reflects the socio-cultural view of effective learning taking place in situated activity. The students in the class that spent the least amount of time on the estimation learning activities had the least awareness of strategies; they were only aware of the term rounding.

Most of the students had enjoyed the estimation work. For example, Peter said " I thought it was fun, especially surf shop 'cos we were selling things to our friends" (Focus group interview, 18/11/2009). Most of the students found the estimation work as something that made mathematics easier. The exception to this was a few Year 6 boys who were very successful in the normal mathematics lessons. Adam explains:

> I am not one for estimating, I normally have a rough idea of what I am after ... I don't like Fermi problems 'cos there is no one particular answer which defines the purpose of mathematics. Mathematics is either black or white.

These boys clung to the notion that maths was about one exact correct answer.

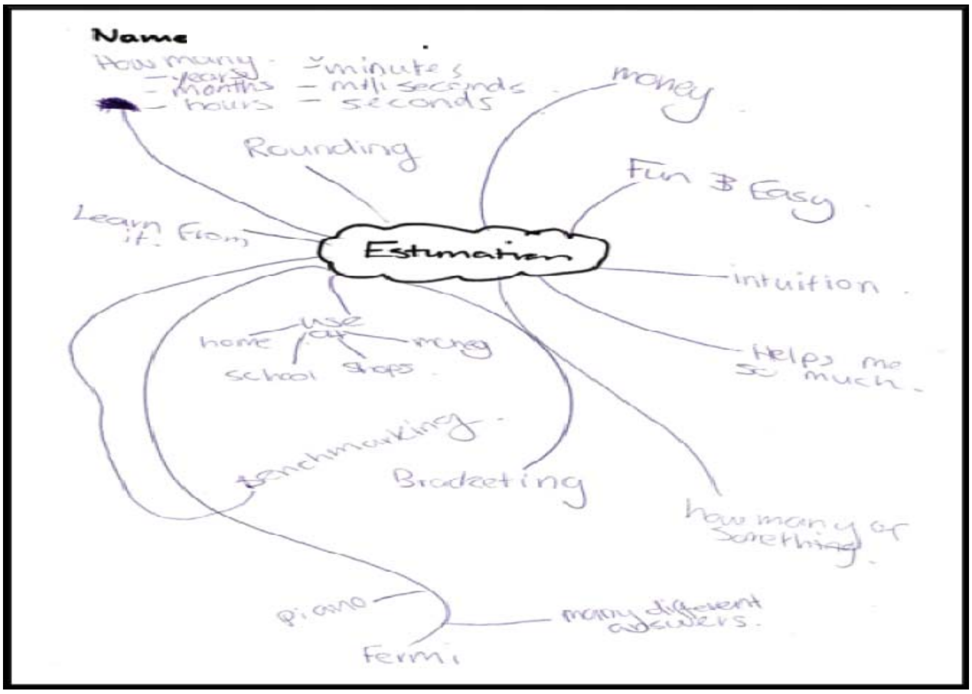

Figure 2. Hannah's post concept map of computational estimation.

Students' Computational Estimation Performance

The professional learning intervention did have an impact on the students' computational estimation performance and their use of reasoned estimation strategies. The qualitative record of the students' work in the classroom showed that students used far more estimation language when solving mathematical problems for which estimation was the main computational choice. There was also a significant improvement in the performance in the computational estimation test. Over the three classes, the students mean pre-test score of 2.94/6 increased to 3.89/6 following the intervention and mean scores increased for all six questions.

In question 4 of the computational estimation test, the students were required to select the best estimate for 45 X 105 ≈. Rounding was an important strategy to answer this question and this strategy was taught explicitly to all three classes and it was observed that this strategy was used extensively when the students solved their problems set in meaningful tasks. This appears to have supported the students' estimation performance as in all three classes more students used a reasoned estimation strategy when comparing the pre and post test responses (Wendy 23% increase, Peter 20% increase and Bob 18 % increase).

At the beginning of the study few students (16%) were able to select the best estimate when adding two fractions. The question from the computational estimation text regarding fractions is shown in Figure 3.

Without calculating the exact answer, circle the best estimate for: $\frac{12}{13} + \frac{7}{8}$

Figure 3. Question 2 concerning the estimation of fractions in the computational estimation test.

At the end of the study 40% of the students were able to select the best estimate in question 2. All three classes used more reasoned estimation strategies in the post-test in comparison with the pre-test to answer this question. The increase by Wendy's students was 36% and Bob's students was 29%. These classes were taught all of the estimation strategies explicitly, including the benchmarking strategy. Peter's students were not taught the benchmarking strategy explicitly and the increase in using reasoned estimation strategies in his class was only 5%. When analysing how many actually used the benchmarking strategy in the post test, 26.7% of Wendy's class used the benchmarking strategy in comparison to 10% in Peter's class.

Wendy also spent time explicitly teaching the front end loading strategy and it was noted that her class results showed a 57% increase in using a reasoned estimation strategy when responding to question 5 (Figure 4) suggesting that explicitly teaching the strategies have a positive impact.

Without calculating the exact answer, circle the best estimate for:

$$\begin{array}{r} 27 \\ 38 \\ 65 \\ +\ 81 \end{array}$$

Figure4. Question 5 in the computational estimation test.

As the study progressed it became apparent that there may be a correlation between students who used a reasoned estimation strategy to answer the question and those who were more proficient at selecting the best estimate in the computational estimation test. A Pearson product correlation coefficient was computed to assess the relationship between students using reasoned estimation strategies and selecting the best estimates. Increases in students' reasoned estimation strategy use were correlated with increases in estimation performance and there was a strong and significant correlation between the two variables $r = 0.499$, $n = 67$, $p = 0.001$. The question which asked the students to estimate when adding fractions had a particularly strong relationship between students using a reasoned estimation strategy and selecting the best estimate , $X^2(1, N = 67) = 34.87$, $p = 0.000$. This suggests that teaching the benchmarking estimation strategy explicitly enhances students' ability to estimate in computational situations.

Discussion and Conclusion

Inspired by recent research suggesting that beliefs are an important factor in effective learning (Carter & Norwood, 1997), the Researcher decided to make student beliefs a focus of this study. It was hypothesised that, for computational estimation to be used by the students, they needed to perceive that it was a worthwhile component of mathematics. The students in the three case studies generally held a view that mathematics was concerned with one correct answer and this view did not change substantially as a result of the intervention. This finding suggests that the students were exhibiting absolutist beliefs and a non-cultural view of mathematics. For student beliefs to change (Gravemeijer & Terwel, 2000), it may be necessary to have a whole school approach to estimation so that students receive a consistent message that computational estimation is a worthwhile computational choice. This perspective is supported by sociocultural theory that emphasises the social setting of the classroom and the creation of a community of learners (Rogoff, 1995).

However, students' beliefs did change in some respects. At the end of the study their awareness of computational estimation had greatly increased and they had developed positive perceptions of this computational choice. The research data suggests teaching computational estimation using fun and engaging meaningful tasks helped students understand the value of computational estimation.

Overall, the students' computational estimation performance and their use of reasoned estimation strategies were enhanced as a result of their teachers being involved in the professional learning intervention. This finding confirms that Year 6 students are able to understand and use a variety of estimation strategies when solving mathematics problems. The study also suggests that when children are explicitly taught about these strategies it enhances their computational estimation performance. The researcher aims to build on these noteworthy findings and implement elements of the intervention in pre-service teacher education programs, so that computational estimation may take its rightful place in the primary mathematics classroom as a valuable computational strategy.

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