Primary teachers' algebraic thinking: Example from Lesson Study

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Previous studies have reported on primary children's algebraic thinking and generalising in a range of problem settings but there is little evidence of primary teachers' knowledge of algebraic thinking. In this paper the development in algebraic thinking of one primary teacher who taught a research lesson in a Japanese Lesson Study project involving teachers from three primary schools is presented. The findings suggest the need for professional learning in algebra and reasoning and indicate the value of Lesson Study.

Number and Algebra is a content strand in the Australian Curriculum: Mathematics and Patterns and Algebra a content sub-strand for all students from Foundation level. Students are expected to be able to describe, extend and create patterns including patterns resulting from additive (Grade 3) and multiplicative processes (Grade 4) and to generate rules for these patterns (Grade 6) (Australian Curriculum Assessment and Reporting Authority, 2012). While the elements of this sub-strand have been included in previous state curricula for the primary years of schooling, algebra as a content area has not been a title in some previous state based mathematics curricula.

Researchers have previously established that children in the primary years of schooling are capable of discerning spatial and number patterns and relations and generalising relations depicted in growing patterns (Cooper & Warren 2008, Fujii & Stephens 2001, Mulligan & Mitchelmore 2009, Radford 2012). Researchers have identified a number of factors that are important for the development of children's algebraic thinking. In many of these studies a member of the research team has taught the lesson or planned the task and lesson for the teacher to implement. So it is not clear what teachers know and understand about pattern and algebraic thinking and the pedagogical practices that scaffold children's reasoning and algebraic thinking. In this paper I will present and analyse one teacher's algebraic thinking and understanding as they worked with colleagues in a Japanese Lesson Study (JLS) planning team to plan and teach a research lesson using an algebra pattern problem. It is one element of a larger study that investigated teachers' implementation of a problem-solving approach through engagement in JLS[1].

Comparing and contrasting, generalising and justifying are three proficiencies that define reasoning in the Australian Curriculum. Each is strongly aligned with algebraic thinking (Mason 1996). The capacity to see what is the same and different spatially and numerically is key to analytical reasoning even for children in the early years (Mulligan & Mitchelmore 2009, Radford 2012). Radford (2000) argued that there are two components to generalising: grasping generality by noticing and expressing a generality. There are three stages of generalising: (1) recursive or arithmetic generalising, when students see and express repeated addition; (2) quasi-generalisation, when students instantiate the general relation with particular number cases; and (3) explicit or algebraic generalisation when students express a general rule using language and symbols (Cooper & Warren 2008, Fujii

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& Stephens 2001, Radford 2012). These frameworks are used to analyse primary teachers' algebraic thinking as they prepared to teach a research lesson as part of JLS project designed to implement a problem-solving approach.

The Study

The study which is the context for this paper adopted Design Based Research (DBR) methodology with two cycles, one in Term 3 and the second in Term 4 of 2012. Design-based research sees researchers and teacher-practitioners working together to produce meaningful change in classroom instruction. Three primary schools from a network of schools in Victoria took part in this study. Six teachers and four numeracy coaches from the schools and network worked collaboratively in two across-school planning teams. Two researchers joined each planning team to observe and facilitate the planning process. Four planning meetings, each meeting lasting two hours, were conducted in each cycle to prepare a public research lesson. Various data were collected for the larger study. The data collected and used in preparation of this paper included observation and video recording and field notes of one whole day professional learning day involving all participants, four planning meetings and the research lesson. Teachers' work on problems during the professional learning day and the first planning meeting in the cycle were also collected.

At the beginning of the project, teachers and numeracy coaches in the project were introduced to lesson study and the structured problem-solving approach during a whole-day professional learning workshop. During one session one of the researchers modelled a problem solving lesson using the Marble Stack Problem (see Figure 1). The teachers completed the problem designed to elicit algebraic thinking and, following the practice of *neriage* (Takahashi, 2008), participated in a whole-class discussion about the solutions designed to compare and contrast solutions to elicit thinking related to the lesson goals.

Early algebra was the theme for the first lesson study cycle. It was chosen because the research team did not expect that the teachers at these schools had the topic of pattern and algebra in their curriculum plan and so teachers could teach it as a "once-off" lesson. This turned out to be the case. So while the problem and potential learning objectives were closely connected to other topics that teachers had already taught or were planning to teach, this topic enabled these teachers to ease into JLS by planning the research lesson without the constraint of planning it as part of a unit of work. Of course this also meant that these teachers may have had little knowledge of early algebra or experience of using these types of problems in their classroom. The Matchstick Problem (see Figure 2) intended to be used in the research lesson was introduced at the first planning meeting by one of the researchers following the lesson structure typical of JLS. For the remainder of this planning meeting and the other three planning meetings in Term 3 the teachers and coaches in the planning teams were in charge of planning the research lessons for their team.

Two research lessons were taught, observed by teachers drawn from the three schools and other invited observers and discussed during a post-lesson discussion as is the practice of JLS. One was taught to a Grade 3 class and the other to a Grade 4 class. While each lesson used the same problem, The Matchstick Problem, the two planning teams developed different plans for the implementation of the lesson. Trevor, the teacher discussed in this paper taught the research lesson in the Grade 3 class.

Trevor's responses to the Marble Stack problem are recorded in Figure 1. The teachers were asked to find the number of marbles in the 8^{th} stack, 100^{th} stack and the n^{th} stack.

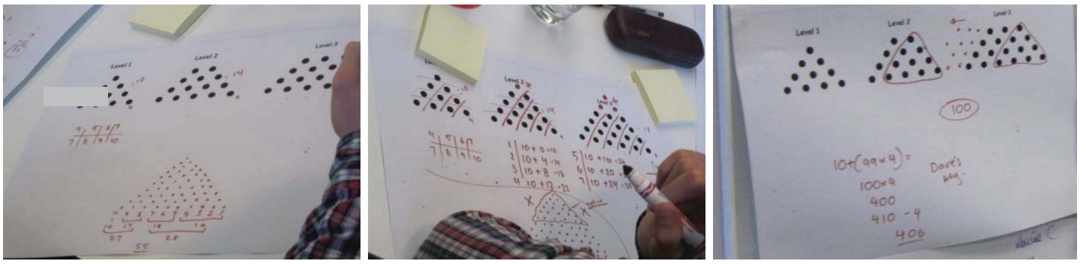

Figure 1. Trevor's responses to the Marble Stack Problem.

Trevor begins by counting all. He sees a triangle structure and counts and records the number of marbles on the bottom row and records these in a table showing the stack number and number of marbles in the bottom row. To find the number of marbles in the 10th stack, Trevor draws a triangular stack rather than a truncated triangle and counts in chunks to find the total number. He realises his thinking is incorrect as indicated by his recording of the truncated nature of the triangle in the next diagram. In his second attempt he sees that the 10 marbles in the first stack are present in each other stack and records the total in each stack as an addition number sentence: 10+0=10, 10+4=14, 10+8=18, 10+12=22 etc. He's not yet seeing the repeated addition of diagonal rows of 4 marbles (that is 10+4+4+4). Following the presentation, explanation and discussion of the multiple solutions generated by the teachers, the researcher invites the teachers to consider which method and expression is most useful for finding the 100th stack. Trevor chooses one of the methods presented by a colleague that instantiates the relation for this particular case: 10+(99×4). The discussion enabled Trevor to grasp and express a quasi-generality.

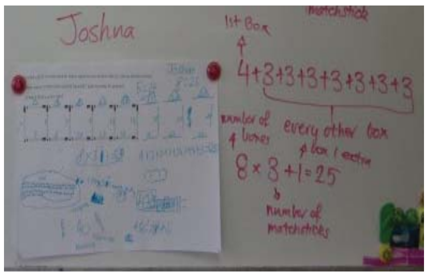

Figure 2 Trevor's Matchstick Problem solutions. *Figure 3* Trevor's recording of children's responses.

In the matchstick problem teachers were invited to use the diagram to generate an expression to determine the number of matchsticks for 5 squares and then to consider the number needed for 8 squares and 100 squares. Trevor (see Figure 2) sees that there is the same number of the matches on the top and the bottom and records that there are six vertical matches: 5×2+6. He writes 5×3+1 and crosses it out as he doesn't yet see this relationship in the diagram. On the next worksheet he marks the three matches on the diagram and instantiates the generalization by calculating the number of matches for 8 and 100 squares. During the orchestrated discussion it becomes clear that a number of teachers have grasped this generalization and expressed it in various ways visually and recorded an equivalent number sentence in different ways: 1+3×5=16; 3×5+1=16; 2×5+5+1=16. One of the teachers generalised this relation and recorded it as a=no. of squares, 3a+1=no. of matchsticks needed, and 2a+(a+1)=no. of matchsticks needed.

Over the course of the next three planning meetings the teachers discussed objectives for the lesson, anticipated student responses and planned the lesson in detail including questions to elicit children's thinking and reasoning during the discussion (*neriage*). After Trevor was randomly selected by the team to teach the research lesson he trialled the lesson and problem with another Grade 3 class at the school. His trial of the lesson was video-recorded and discussed with his numeracy coach, a member of the other Lesson Study planning team, and with his own team. As a result Trevor was comfortable and familiar with the range of possible solutions and the thinking that was likely to emerge in his classroom. Figure 3 shows a section of his board at the end of his research lesson that he produced during *neriage*. It shows his recording of one child's thinking, elicited through questioning that sought their explanation of the numbers and operations recorded in the number sentences and visual displays of their thinking. His recording communicates to the other children in the grade this child's expression of recursive and quasi-generality.

Conclusion

Trevor's learning journey regarding algebraic thinking is typical of the other teachers in the JLS project and follows the stages of algebraic thinking recorded in the literature though this teacher did not demonstrate algebraic generalization only quasi-generalisation (Cooper & Warren, 2008, Radford 2012). The problem-solving structure typical of JLS research lessons, where the whole-class discussion of student generated responses is the most important part of the lesson, demands flexible and adaptable knowledge of teachers to orchestrate this discussion well. The deep and detailed planning including trialing of this lesson enabled this teacher to be well prepared and demonstrates the value of Japanese Lesson Study for developing teachers' mathematical and pedagogical knowledge.

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