Siegler, R. S. (in press). Continuity and Change in the Field of Cognitive Development and in the Perspectives of One Cognitive Developmentalist. *Child Development Perspectives*, early bird on-line publication, 2-29-16, doi: 10.1111/cdep.12173—*anticipated publication date: 2016*

Continuity and Change in the Field of Cognitive Development and in the Perspectives of One Cognitive Developmentalist

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Author's Note

The research reported in this article was supported in part by the Institute of Education Sciences, U.S. Department of Education, through Grants R305A150262 and R324C100004:84.324C, Subaward 23149 to Carnegie Mellon University, in addition to the Teresa Heinz Chair at Carnegie Mellon University and the Siegler Center of Innovative Learning, Beijing Normal University. The opinions expressed are those of the author and do not represent views of the Institute or the U.S. Department of Education.

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Abstract

In this article, I examine changes in the field of cognitive development and in my own thinking over the past 40 years. The review focuses on three periods. In the first, Piaget's theory was dominant, and my research and that of many others was aimed at understanding the many fascinating changes in children's thinking that Piaget documented and at correcting inaccuracies in his theory. The second period involved generating an alternative to Piaget's approach, overlapping waves theory, and specifying through microgenetic methods and computer simulations how development can be produced by variability of strategy use, adaptive choices among strategies, and discovery of new strategies. In the third period, my thinking and research, and that of many others, has increasingly focused on the interface between cognitive development and education. I close by suggesting that generating domain-specific integrated theories of cognitive development may provide a way forward for the field.

Keywords: Cognitive development, numerical education, Piaget, overlapping waves

Continuity and Change in the Field of Cognitive Development and in the Perspectives of One Cognitive Developmentalist

Many fortuitous events contributed to my decision to study cognitive development, but the choice was not totally fortuitous. For as long as I can remember, I've been fascinated by change – changes over eons of evolution, centuries of history, years of a human life, weeks of a football season, and seconds or minutes of a learning event. The basic questions are the same regardless of the time scale and domain: What set the changes in motion, what did and did not change, and what mechanisms underlay the changes and continuities? These were the fundamental questions when I entered the field, and they remain fundamental today.

Once I decided to focus on psychology, it may have been inevitable that my interests would gravitate to child development. Although questions about change can be asked in any area of psychology, they are especially central to the study of development. A researcher who focuses on cognitive, social, or perceptual psychology might or might not be interested in change, but it would be a strange developmentalist indeed who was not interested in it. Moreover, children's energy, candor, playfulness, and originality have always appealed to me. A rare privilege of working in developmental psychology is that thousands of colleagues in the field are similarly fascinated by children and change. That gives us a lot to talk about.

A continuing theme of my research has been how children's mathematical and scientific problem solving changes with age and particular experiences. However, the concepts and problem solving skills that I have studied, and the theories and methods that I apply to them, have changed considerably. These changes reflect trends in the field as much as my own intellectual development.

The Age of Piaget

When I began to study cognitive development in the early 1970's, Piaget's theory was dominant. Some people argued for the theory and some against it, but it was the touchstone for new research in a way that no later theory has been.

My early studies examined Piagetian tasks, including conservation of number, liquid quantity, and solid quantity, balance scales, shadows projection, probability, fullness, time, speed, and distance (1-3). One lesson of these studies was that providing young children with relevant rules, feedback, and instructions led to greater learning and generalization than Piaget thought possible for 4- to 6-year-olds (1, 4). Research conducted by other investigators at about the same time made the same point, either through training studies or through tasks that facilitated more advanced reasoning on these concepts (e.g., 5). However, observing young children also led me to see how tenaciously they clung to their scientific and mathematical misconceptions and how hard it was to dislodge them. These observations led me to an enduring appreciation for Piaget's genius in designing tasks that revealed surprising aspects of children's thinking.

The rule assessment approach that I developed to assess reasoning on tasks like those studied by Piaget (1, 2) indicated that individual children consistently followed a developmental sequence of rules similar but often not identical to those that Piaget posited. The fit of the children's predictions to the rules was especially close for 5- and 6-year-olds. On all of the tasks listed above and many others, 5- and

6-year-olds based their answers on a single dimension of each problem. For example, presented a balance scale like that in Figure 1, 5- and 6-year-olds consistently judge that the side of the arm with more weight will go down, regardless of the distance of the weight from the fulcrum. Similarly, on liquid quantity conservation, a task that typically involves water being poured from a taller, thinner beaker to a shorter, wider one, 5- and 6-year-olds consistently judge that there is more water in the taller, thinner beaker, regardless of the cross sectional areas of the glasses. The question was why.

Common explanations at the time were that immature mental structures or limited processing capacity precluded children from thinking in more advanced ways (6, 7). However, data from training studies and simplified versions of Piaget's tasks indicated that children of these ages could generate more mature reasoning. These findings argued against the cognitive structure and processing capacity explanations, but left unexplained why 5- and 6-year-olds often relied on a single dimension on the same tasks where 7- to 10-year-olds considered multiple dimensions. For example, why would older but not younger children consider distance as well as weight, when neither group of children had experience with balance scales?

It finally struck me that much of the explanation resided in how Piaget chose his tasks. In all of the cases listed above and many others, the tasks 1) were novel for the children, and 2) included a single perceptually or conceptually salient dimension, reliance on which led to the wrong answer (e.g., numbers or sizes of weights on each side of the fulcrum or heights of the liquid columns). This interpretation suggested that 5- and 6-year-olds' failure on these tasks might be due to their not encoding the less salient dimensions on the problem, not because they were incapable of doing so but rather because they did not know that those dimensions were important.

Results of a number of studies proved consistent with this interpretation. When asked to reconstruct balance scale problems they had seen, younger children correctly reconstructed only the more salient dimension, whereas older children correctly constructed both more and less salient dimensions. Telling the younger children that both the less and the more salient dimensions were important led to their correctly reconstructing both dimensions and learning more advanced rules from feedback that had not helped age peers previously. This type of encoding training proved useful for a wide variety of problems, both Piagetian (e.g., 3) and non-Piagetian (e.g., 8).

Overlapping Waves Theory

This analysis of how Piaget chose his tasks led me to study a different type of problem: ones with which children have direct experience. Problems that are unfamiliar and have a dimension that is both salient and misleading are a small minority of the problems that children encounter in the world. A great many problems are familiar, but solving them remains difficult for many children. Therefore, in part to examine development in the context of tasks with which children have direct experience, and in part because I was interested in numerical development, I began to study everyday mathematical problems, such as addition and subtraction (e.g., 9).

Those studies yielded two surprising, and I believe important, findings. One was that children generally used multiple strategies rather than a single consistent approach. For example, when adding numbers with sums of 10 or less, preschoolers sometimes put up their fingers and counted from one, sometimes put up their fingers and answered without counting, sometimes counted without any external referent, and sometimes retrieved an answer from memory (9). When 6- to 8-year-olds encountered problems with sums up to 14, they used some of the same strategies and also counted up from the larger addend (adding 2+5 by counting "6, 7" and used decomposition (e. g., adding 3+9 by thinking "3+10=13, 13-1=12) This strategic variability did not reflect some children using one strategy and other children another. Individuals averaged more than three different strategies apiece (10). Similar variability has been documented in word identification (11), syntactic judgments (12), locomotion (13), scientific reasoning (14), communication (15), moral reasoning (16), search for hidden objects (17), tool use (18), and many other domains. As these examples document, the strategic variability is seen from infancy (e.g., 13) to old age (e.g., 19).

Such findings helped stimulate the overlapping waves theory of cognitive development (20). As opposed to the staircase models suggested by Piagetian and neo-Piagetian theory, in which children abruptly transition from a less advanced to a more advanced approach (Figure 2a), the overlapping waves model posits that on most problems with which children have experience, multiple ways of thinking and acting coexist and compete over prolonged periods of time (Figure 2b). With age and experience, children progress toward greater use of more advanced approaches.

For example, in learning single digit addition, children initially count from one most often, then they count-on from the larger addend most often, and eventually they most often retrieve the answer from memory, though previously dominant strategies and others such as decomposition continue to be used. Overlapping waves theory also proposed that new ways of thinking and acting emerge fairly often, whether through problem solving experience, analogies to related problems, or instruction.

Findings of strategic variability in the domains noted above and many others raised at least two important questions: How do children choose among the varied strategies, and how do they discover new strategies? Children's strategy choices proved to be impressively adaptive from early in development. For example, if a ramp is not too steep for their capabilities, infants tend to descend in their usual locomotor posture (crawling or walking); if the ramp is steeper, they tend to adopt a less risky posture (sliding down feet first, head first, or on their behind); if the ramp is yet steeper, they often refuse to descend altogether (13). Older children's and adults' strategy choices proved to be similarly adaptive in a wide variety of domains (20).

Observing such strategic variability also raised the issue of how children discover new strategies. Such discoveries must be quite frequent, given the variety of approaches that children know and use, but relatively little was known about the discovery process. Microgenetic methods (21) were well suited to examining discoveries of new strategies. Such methods have three main characteristics: observations span the period of rapid change in the competence of interest; the density of observations is high relative to the rate of change in the competence; and observations are subjected to intensive trial-by-trial analysis, with the goal of inferring the processes that give rise to the change. Most often, performance is observed on a trial-by-trial basis, which allows identification of the exact trial where a strategy was first used, as well as analysis of what led up to the discovery and how it was generalized beyond its initial context.

Illustrative of this approach, in one microgenetic study, Siegler and Jenkins (22) presented up to 200 single-digit addition problems to 4- and 5-year-olds who initially knew how to add by using the *sum strategy* (counting from one) but did not know the more efficient *min strategy* (adding by counting from the larger addend). The children were asked immediately after they answered each problem how they had solved it.

Most preschoolers discovered the min strategy during the course of the experiment. Analyses of the trials immediately before the discovery revealed lengthier solution times, more verbal disfluencies, and use of a brief-lived approach, the *shortcut sum strategy* that combined characteristics of the sum and min procedures (e.g., on 2+5, counting "1,2,3,4,5,6,7", unlike the sum strategy, which involves counting "1, 2 –1,2,3,4,5—1,2,3,4,5,6,7" or the min strategy (counting "6, 7"). Analyses of trials following the discovery revealed that most preschoolers used the new strategy only occasionally until they were presented with challenge problems such as 2+19, which were easy to solve with the min strategy but difficult with the sum or shortcut sum strategies. After encountering these challenge problems, children used the min strategy much more often.

These findings are representative of consistent patterns that have emerged from microgenetic studies on other tasks and with older and younger participants (21). One frequent finding is that immediately before a discovery, performance becomes more variable. Shortly before the first observed use of a new strategy, solution times often become much longer than previously, disfluencies become more common, and children often generate short-lived transition strategies (23, 24). Another common finding is that even the most advantageous new strategies are often generalized slowly, with less effective previous approaches persisting for prolonged periods of time, even when children can explain why the new strategy is better (25). A third common finding is that greater initial variability of thinking is often related to superior learning (e.g., 26). Thus, microgenetic studies have proven useful for providing detailed depictions of discovery processes, as well as for providing invaluable data for guiding computer simulations of those processes (27).

Cognitive Development and Education

In the past 10-15 years, much of the most interesting research on cognitive development has examined development of academic skills: reading, writing, mathematics, and scientific reasoning. There are a number of reasons for the increased focus on these areas. One is that the sharp distinction between developments inside and outside of classrooms never made much sense. Cognitive development does not emerge in a vacuum. The focus on unfamiliar tasks of Piaget and many successors shifted attention away from some of the most significant sources of cognitive development. How could cognitive tools as omnipresent and informative as reading, writing, science, and mathematics not influence cognitive development? The field's earlier focus on unfamiliar tasks allowed us to pretend that development just happened, either independent of experience or through unspecified "general experience," but few contemporary researchers would defend that proposition. Focusing solely on either unfamiliar tasks or familiar ones inevitably distorts theories of development, as shown by the very different patterns of development that occur on them.

A further impetus for research bridging between cognitive development and education is the increasingly serious societal challenge of helping students acquire the skills and knowledge required by STEM fields and other cognitively demanding occupations. Reflecting the pressing nature of this challenge, granting agencies have increasingly emphasized application to education and other societal problems as a criterion for funding. There are few societal problems more pressing than how to improve education, and there are few research problems more pressing than how to obtain funding. Yet another motivation is that attempts to cross the traditional divide between cognitive development and education have yielded a great many exciting and surprising empirical findings. For all of these reasons and more, an increasing number of cognitive developmentalists are focusing on development of academic competencies.

Some of my own research illustrates the benefits that can accrue from applying cognitive developmental theories, methods, and empirical findings to education. When children begin school, their numerical knowledge already varies greatly. Children from impoverished families typically start school a year or more behind in numerical knowledge, relative to children from middle-income backgrounds (e.g., 28). These early differences have lasting consequences: 4-year-olds' numerical knowledge predicts 15-year-olds' math achievement test scores, above and beyond relevant factors such as children's IQ and working memory and their parents' income and education (29).

Both theories and empirical findings from cognitive psychology and cognitive development indicate that people organize numerical knowledge in a way that resembles a mental number line (30). In Western and East Asian cultures and many others, smaller numbers are represented on the left and larger numbers on the right. Preschoolers from low-income backgrounds, however, often have not formed this representation.

To help them do so, Geetha Ramani and I devised the numerical board game shown in Figure 3 (31, 32). An adult and a child alternate spinning a spinner and moving their token in accord with the outcome; the first player to reach 10 won. Children needed to say each number as they moved through the corresponding square; the adult playing with them would help them if they did not know the number to say. This game was expected to promote formation of a mental number line, because it provides redundant cues to numerical magnitudes. For example, it takes twice the hand movements, counts, time, and distance traveled to reach "8" as to reach "4". Such redundant cues promote learning in a wide range of tasks and age groups (e.g., 33).

Playing the number board game not only helped children to represent the numbers 1-10 on a mental number line but also to count, identify, and add the numbers (31, 34). The benefits remained two months later. A version of the game involving a 10 X 10 matrix helped kindergartners learn about the numbers 1-100 (35). Numerous other applications of cognitive developmental theories and findings have also proved effective in promoting early mathematics and reading knowledge, especially of low-income children (e.g., 36, 37).

The Future of Cognitive Developmental Theories and Research

During the three periods described in this article, cognitive developmental theories have become more accurate but less satisfying. For all of its flaws, Piaget's theory provided a unified and encompassing depiction of children's cognitive development. The theory was unified in positing a set of stages and transition mechanisms that applied to all developmental acquisitions, and it was encompassing in depicting the development of an exceptionally broad range of important concepts and problem solving skills from infancy through adolescence.

There were good reasons for the theory losing popularity; it underestimated infants' and young children's conceptual understanding, overestimated adolescents' understanding, was unduly dismissive of the role of specific experience and learning, and was vague about how its transition mechanisms operated (38). Nonetheless, none of its successors -- information processing, neo-nativist, sociocultural, and dynamic systems theories -- have matched its applicability to diverse domains and age groups. In moving on to newer theories, we have traded a rough and sometimes inaccurate depiction of the forest for innumerable more accurate depictions of specific trees (and often their branches, twigs, leaves, and chloroplasts).

An alternative that potentially could capture some of the best qualities of Piagetian theory and its successors is formulating domain specific integrated theories. The concepts identified by Kant and Piaget as fundamental to understanding the world -- space, time, number, causality, morality, mind, etc. – seem especially promising areas for such integrations. I have devoted much of my recent research to formulating such an integrated theory for numerical development (39, 40), and other researchers are pursuing similar goals with regard to spatial development (41), moral development (42), and other areas. As with Piaget's theory, these domain specific integrated theories strive to provide a unified depiction of development from infancy through adulthood and to include a wide variety of specific acquisitions and sources of growth within the domain. As with the successors to Piaget's theory, these new approaches recognize the importance of the particulars of development in each domain and of real-time influences on problem solving and reasoning involving them.

Formulating domain-specific integrated theories of development for multiple concepts might also allow a degree of integration across concepts. Consider Lourenco and Longo's (e.g., 43) results examining transfer of learning among space, number, and time concepts. They found that infants transferred learning across all six permutations of initially learned dimension and transfer dimension; for example, after learning that a particular decoration always accompanied a larger shape, infants dishabituated when the decoration accompanied the less numerous set of objects or the objects that were on the screen for less time. This finding suggests that in addition to having specific concepts of space, number, and duration, infants also have an amodal concept of quantity that transcends the particular dimensions. The finding also suggests that Piaget (e.g., 44) was correct in suggesting that infants possess a general quantity concept that transcends specific quantitative dimensions (though he clearly was incorrect in suggesting that they could not represent the specific dimensions.) My hope is that well-grounded domain-specific developmental theories will provide a basis for unified and encompassing general theories of development as well, ideally by the time when I will have been studying cognitive development for 50 years.

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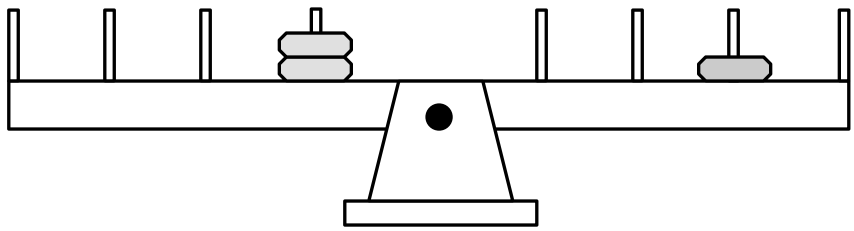
Figure Captions

Figure 1. The balance scale

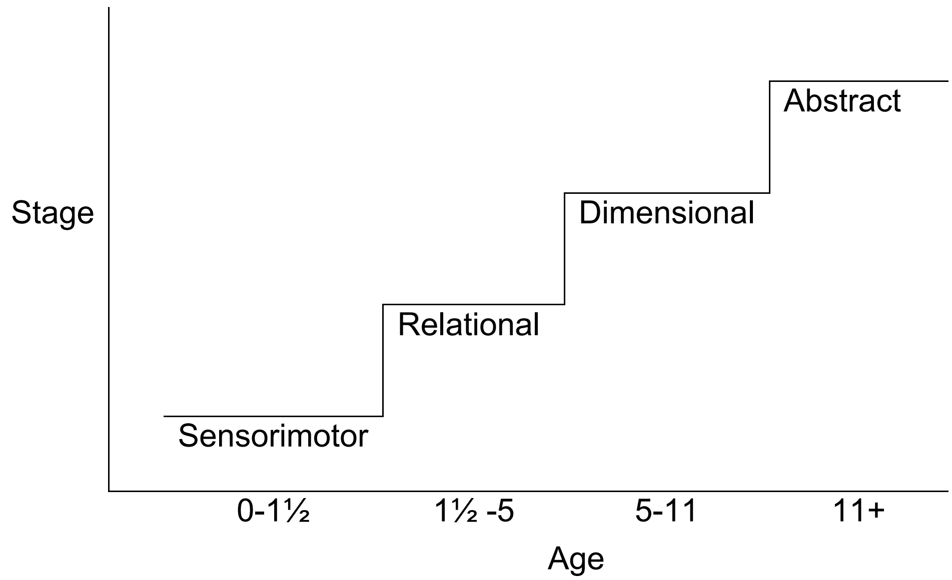
Figure 2. A prominent staircase model (2a) and the overlapping waves model (2b)

Figure 3. The board used in the number game. From "Improving the numerical understanding of children from low-income families," by R. S. Siegler, 2009, p. 121, © 2009, by R. S. Siegler & the Society for Research in Child Development (45). Reprinted with permission.

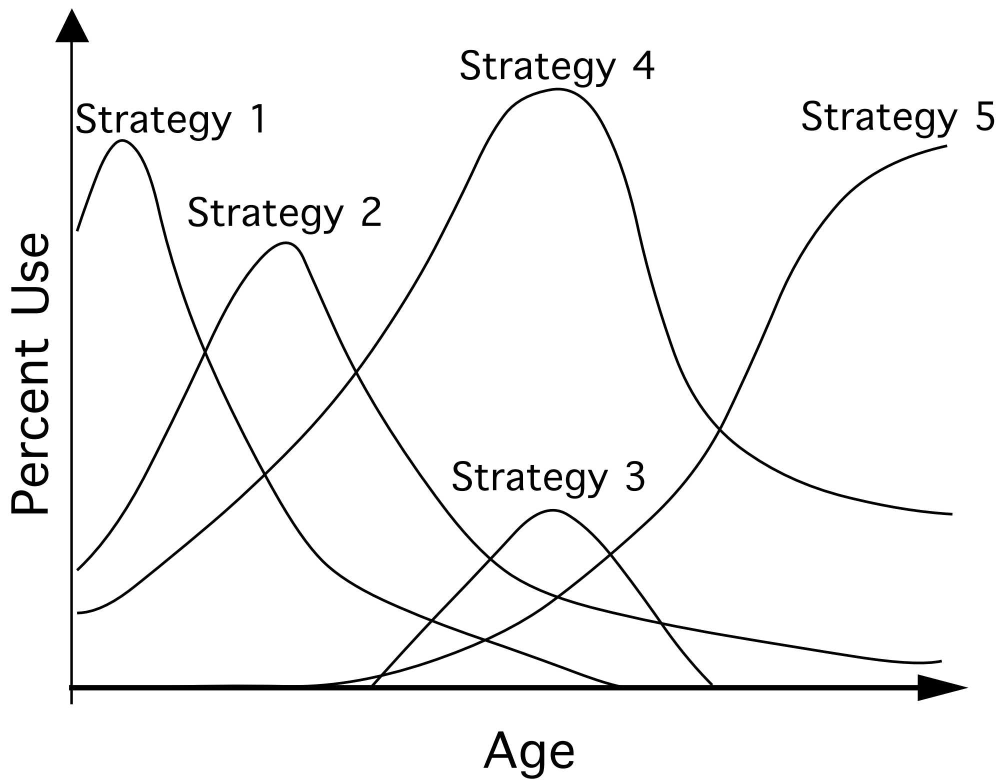
(Figure 1)



(Figure 2A)


Stage
Abstract
Dimensional
Relational
Sensorimotor
0-1½
1½ -5
5-11
11+
Age

(Figure 2B)


Strategy 1
Strategy 2
Strategy 3
Strategy 4
Strategy 5
Percent Use
Age

(Figure 3)