Use of Learning Trajectories to Examine Pre-service Teachers' Mathematics Knowledge for Teaching Area and Perimeter: Emerging Issues

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A current concern is student learning outcomes and these are largely a function of teachers' knowledge and their practice. This position paper is premised on the notion that certain knowledge is required for the teaching of mathematics. An exploration of literature demonstrates that such professional knowledge development can be supported by Learning Trajectories (LT). We propose to use LT as theoretical lens to examine pre-service teachers' Content and Pedagogical Content knowledge and advance a research design

The examination of the quality of mathematics teaching continues to be high on the agenda of national and international debates and reform movements. The issue has been given added urgency in Australia in light of the less than satisfactory performance of Year 4 students in the latest international measures of mathematics achievement (Mullis, Martin, Foy & Arora, 2012). While there is a general consensus that mathematics teaching is a complex activity (Empson, 2011) efforts are now afoot in identifying and studying aspects of teaching that can be improved so that learning outcomes of students can be lifted.

Fundamental to a teacher's work in the classroom is his/her knowledge for teaching mathematics (Shulman, 1986). In the past decade, there has been a surge of interest in examining what constitutes teacher knowledge and how this knowledge impacts on the teacher's decisions about content and strategies that are used to engage learners. Two well-known dimensions of teacher knowledge have been the subject of interest for mathematics teachers and researchers: discipline-specific knowledge and pedagogical-content knowledge (Ball, Lubienski & Mewborn, 2001). The model of teacher knowledge proposed by Ball, Thames and Phelps (2008) provides a powerful theoretical lens within which to examine and describe changes in teacher knowledge as they engage in pedagogical practices (Figure 1).

Recent research into children's Learning Trajectories (Baroody, Cibulskis, Lai & Li, 2004) has a focus on how children's learning progresses, in contrast to research into specific content knowledge and skills that they are required to attain within mathematical streams or topics. Thus, Learning Trajectories (Daro, Mosher & Corcoran, 2011) provide windows into expected on-going changes in discipline-specific knowledge and pedagogical-content knowledge. Recent developments suggest that Learning Trajectories frameworks can provide the basis for instructional decisions and models of teaching (Sztajn, Confrey, Wilson & Edgington, 2012).

This position paper examines teacher knowledge development that is necessary for the support of deep mathematical understanding within one stream of primary mathematics, namely, area and perimeter. In so doing, our aim is to highlight emerging issues about the complexity of these concepts and teachers' understandings about teaching that require further study.

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Background and literature review

A key issue in raising international educational standards is teachers' mathematical knowledge. Literature shows that pre-service teachers often have weak discipline knowledge (Zevenbergen, 2005). Furthermore, research indicates a common finding that pre-service teachers often have the same misconceptions in streams of mathematics as Primary School students (Ball, 1993). In the stream of measurement, literature shows that many preservice teachers falsely assume a c onstant relationship between area and perimeter (Baturo & Nason, 1996). These findings indicate insufficient instruction in conceptual knowledge (Ma, 2010). Conceptual knowledge is based on the development of a concept structure of reasoning and representations (Barmby, Harries, Higgins, & Suggate, 2009) and knowing why mathematical processes work . In comparison is procedural knowledge which is limited to following step-by-step processes to obtain the answer (Ball, 1991). Procedural knowledge can thus be applied without conceptual knowledge (Ma, 2010). Procedural knowledge rather than conceptual knowledge of area and perimeter was evidenced in a study of fifty-four preservice teachers (Menon, 1998). Ultimately, preservice teachers' knowledge at course commencement is a product of their experiences as learners of mathematics. Reform, therefore seems to lie with teacher education (Ma, 2010; Mewborn, 2001). Literature indicates a need to find out how to interrupt the cycle which perpetuates lack of conceptual knowledge.

Two specific research efforts predominantly guide contemporary education improvement. The first is the need for identification and articulation of teacher knowledge for teaching. There is wide and ongoing agreement that knowledge of subject specific content is necessary for teaching (Ma, 2010); however, current literature calls for investigation into the complex and multidimensional nature of teacher knowledge (Ball et al., 2008). Since Schulman's (1986) proposal of special knowledge for teaching, termed pedagogical content knowledge, there has been widespread research of this professional knowledge (Chick, Baker, Pham & Cheng, 2006). This description of a particular domain of teacher knowledge has appeal for researchers because it has the seeming potential to bridge subject specific content knowledge and the practice of teaching (Ball et al., 2008). In order to progress their students to more sophisticated understandings, teachers need to surmount individual learner's difficulties (Ball, 2000); a prerequisite is the ability to translate teacher content knowledge into effective teacher practice (Butterfield & Chinnappan, 2010). Ongoing research (Butterfield & Chinnappan, 2011) identifies that preservice teachers have insufficient pedagogical content knowledge when designing problems and providing support for possible misconceptions. An area for further research identified is the development of knowledge of student misconceptions which is closely aligned with knowledge of children's learning progression.

The second research effort is a focus on the creation of learning paths frameworks that represent cognitive progression, for example, linear measurement (Cobb, 2003). Researchers have categorised students' cognitive development in terms of levels of sophistication (Battista & Clements, 1996). To aid student learning progress, the literature suggests the potential of Learning Trajectories construct for the teaching of mathematics (Sztajn et al., 2012). The Learning Trajectories construct incorporates subject specific content knowledge which has pedagogical content knowledge implications. Furthermore, literature indicates that Learning Trajectories facilitate the "important aspect of pedagogical thinking involved in teaching mathematics for understanding" (Simon & Tzur, 2004, p.92). Pedagogical thinking requires teachers to make sense of the frameworks and make sense of an individual child's learning in relation to these frameworks.

Concomitantly, a teacher's own content knowledge is required. Therefore, the dual knowledge components for teaching of mathematics, content knowledge and pedagogical content knowledge, are inherent in Learning Trajectories. This line of research has made a contribution to teacher knowledge of typical progression of learning in streams of mathematics.

Recent calls have been made for research into using Learning Trajectories construct as a basis for instruction (Sztajn et al., 2012) and this could be implemented in the mathematics teaching cycle of preservice teacher education. While there is an emerging body of research into trajectories of learning in various streams of mathematics, for example, area (Outhred & Mitchelmore, 2000; Samara, Clements, Barrett, Van Dine & McDonel, 2011), there is limited literature on how preservice teachers attain necessary content knowledge and pedagogical knowledge to teach measurement concepts. To address the issue of how to develop preservice teachers' knowledge for teaching, a review of the literature has been undertaken into the aforementioned two research efforts. The purpose of this paper is to examine the theoretical perspective of a learning trajectories construct in relation to teacher knowledge for teaching mathematics. Research implications are then outlined.

Learning Trajectories

The construct of Learning Trajectories has recently come to the fore as a means to focus research on a child's learning instruction and assessment. However, various standpoints of learning trajectories are emerging (Baroody et al., 2004) with researchers (Daro et al., 2011) alerting the need for shared meanings and further clarifications. As such, the construct of a learning trajectory is still in an exploratory stage. This paper examines theoretical perspectives and possible practical implementation into research design.

From an historical outlook, Simon's (1995) seminal work presented the term "learning trajectory" and he prefaced this term with "hypothetical". He describes a learning trajectory as being hypothetical because an "actual learning trajectory is not knowable in advance" (Simon, 1995, p. 135) and defines a hypothetical learning trajectory as a construct for teaching. His idea is that a teacher makes conjectures of where individual students "are at" and where the teacher could next take them. In this hypothetical learning trajectory, individual students are prospectively assessed by teachers according to research-based developmental pathways and then the teacher decides what the students could learn next and the way they could learn it.

Subsequent research shows both the controversy and the complexity of conceptualising learning trajectories (Daro et al., 2011). Empson (2011, p.576) argues that "learning sequences" have been the focus of research "over decades", although not metaphorically coined "a learning trajectory". Indeed, different terminology has been used to describe "learning pathways". Ma (2010, p. 122) explains "longitudinal coherence" as teachers' understanding of what precedes and follows crucial concepts in topics. This teacher knowledge consists of what can be reviewed and what students are going to learn; thus proper foundations can be laid for future learning. In the Australian context, "growth points" in specific topics with a series of appropriately sequenced instructional tasks are termed "Learning Frameworks" (Clarke, Cheeseman, McDonough & Clarke, 2003). Outhred, Mitchelmore, McPhail and Gould (2003) have articulated a Learning Framework for measurement based on unit iteration structure. As such, this framework can be used as a tool to guide teaching (Outhred, 2001). However, O'Keefe and Bobis (2008) also investigated practising teachers and determined a lack of explicit knowledge of this Learning Framework for measurement. An implication determined from this later study is how to meaningfully present this information of concept progression in preservice teacher coursework.

A synthesis of research into students' thinking as they develop concepts in mathematics over time forms the basis of the Learning Trajectories construct (Daro et al., 2011). Additionally, despite varying terminology, a commonality in the Learning Trajectories construct for the various streams of mathematics is the focus on a teaching goal, activities and students' mathematical thinking at different levels of cognitive proficiency (Sztajn et al., 2012). Hence, Learning Trajectories may facilitate pedagogical thinking to develop conceptual knowledge (Simon & Tzur, 2004). Congruous with all these perspectives is the hypothetical learning trajectory construct.

Simon (1995) notes that a hypothetical trajectory is made up of three components: the learning goal that determines the desired direction of teaching and learning, the activities to be undertaken by the teacher and students, and a hypothetical cognitive process, "a prediction of how the students' thinking and understanding will evolve in the context of the learning activities" (p. 136). The clarifier, "hypothetical" denotes that the three components enable a flexible and adaptable approach (Mousley, Sullivan & Zevenbergen, 2004). In addition, there is synergism between the last two components (Clements & Samara, 2004). Herein, teacher knowledge of student thinking is paramount. A premise is that teachers plan tasks from their individual student's perspectives (Samara & Clements, 2009). A study of practising teachers shows that task differentiation enables children with diverse learning abilities to succeed in a class' common goal (Mousley et al., 2004). Thus Learning Trajectories provide an important window into teacher knowledge.

Simon (1995) demonstrated how the continually changing knowledge of the teacher creates change in expectations of how students might learn a specific idea. Likewise, Bobis, Clarke, Clarke, Thomas, Wright, Young-Loveridge, and Gould (2005) suggest that teachers' use of Learning Trajectories could provide important insight into their developing knowledge about how children learn mathematics.

The challenge for researchers is how best to capture and describe the changing knowledge of teachers. Literature indicates that Learning Trajectories could provide support structure to help bridge research on learning and research on teaching (Daro et al., 2011). The learning trajectories construct could enable connections with multidimensional teacher knowledge for teaching mathematics (Sztajn et al., 2012). Multidimensional teacher knowledge incorporates two main interdependent and interconnecting components. These are Subject Matter Knowledge and Pedagogical Content Knowledge (Ball et al., 2008). Within each component there are several knowledge facets (Figure 1).

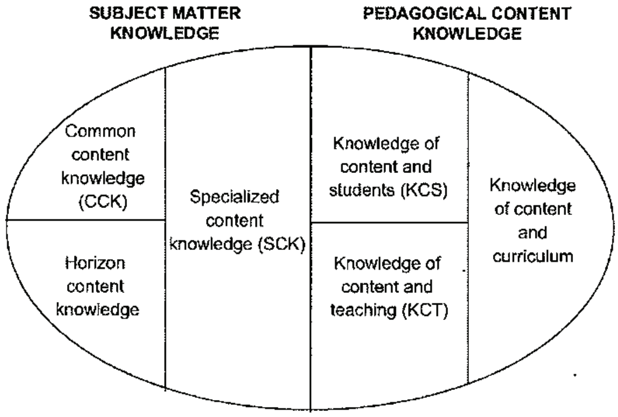

Figure 1. Framework for examining mathematical knowledge for teaching (Ball, Thames & Phelps, 2008).

Many studies find that teachers' content knowledge of mathematics is alarmingly deficient, for example Ball (1993) and Ma (2010). A majority of these studies have focused on division,_fractions and rational numbers (Ball 1993). A smaller base has investigated teachers' content knowledge of measurement (Outhred et al., 2003). While the determination of inadequate knowledge is a useful platform to establish the need for improvement, few studies explicate the development of knowledge that teachers need to support teaching (Ball et al., 2008) that is demanded by current mathematics education reforms.

Mathematical knowledge for teaching is the knowledge required to actually teach (Ball et al., 2008). As such, it is concerned with knowledge of curriculum content, "tasks involved… and mathematical demands of these tasks" (p. 395); "Teaching", therefore is "everything that teachers must do to support the learning of their students". Thus teacher knowledge encompasses subject matter knowledge and pedagogical content knowledge (Hill, Ball & Schilling, 2008). Subject matter knowledge includes conceptual and procedural knowledge. Subject matter knowledge is also a component of a Learning Trajectory. Based on a synthesis of research findings the Learning Trajectories construct incorporates typical developmental pathways to more sophisticated understanding (Samara & Clements, 2009). Therefore, inherent in the Learning Trajectories construct is the premise that teacher knowledge should include understanding of the "big ideas" that underlie a wide range of mathematics concepts (Baroody et al. 2004) or "basic ideas" (Ma, 2010). Also, inherent is pedagogical content knowledge such as anticipation of what students are likely to think and ways to support their understanding.

Emerging research informed by a Learning Trajectories theoretical lens indicates that both pedagogical and content knowledge can be developed. Clarke and his colleagues indicate that teacher knowledge of Learning Frameworks can assist teacher planning of appropriate activities (Clarke et al., 2003). Mojica (2010) determines that a teacher's own content knowledge develops as a result of the practical use of Learning Trajectories in the planning and implementation of activities. Such studies of professional development for in-service teachers, based on Learning Trajectories, show teacher knowledge development for teaching mathematics. There is a n eed to examine teacher development from the perspective of Learning Trajectories and Mathematical Knowledge for Teaching. This strategy holds promise for use in teacher education programs for preservice teachers.

Emerging issues

A common theme in research is that teacher knowledge for teaching mathematics is paramount to high quality student learning and this issue is fundamental to professional development (Australian Institute for Teaching and School Leadership, 2012). Consequently, further research into the character of this knowledge and its development is critical. A Learning Trajectories theoretical lens (Daro et al., 2011) can provide insights into the above issues concerning teacher knowledge. While a number of Learning Trajectories for mathematics have been articulated in recent times, their practical application in teaching contexts is in the early stages of investigation. This is so in the stream of measurement. In particular, there is exiguous examination of the use of Learning Trajectories as a means to develop and support teacher knowledge; including informed instructional decision-making as well as personal growth of subject matter knowledge (Sztajn et al., 2012). Taken together, there is a need to examine new research strategies in the examination of teacher knowledge for effective mathematics practice. Significantly, recent calls for improvement in mathematics engagement and performance of our children (Attard, 2011) have implications for research into what teacher knowledge and learning experiences are essential for the preparation of future teachers of mathematics.

A possible research strategy

We propose a research design that involves a cohort of pre-service teachers (PSTs) whom are enrolled in a Graduate Diploma of Education (Primary) program. In this program, PSTs follow a sequence of lectures/tutorials and Professional Experiences (PEX). This will provide opportunity to examine the interplay between PEX and coursework (Grossman, Hammerness & McDonald, 2009); as well, documentation of knowledge pre-coursework and knowledge during coursework (Charalambos, Hill & Ball, 2011).

Data collection could be conducted in three phases. Phase 1 (Pre-PEX Activities) is aimed at identifying PSTs' content knowledge of concepts and processes in Measurement, particularly in Area and Perimeter. Also, PSTs' perceptions of how children's content knowledge develops and how this determines pedagogical practice (O'Keefe & Bobis, 2008) will be investigated. Phase 2 (PEX Activities) involves PSTs planning and implementing teaching episodes with a particular cohort of Primary School students in mind. Phase 3 (PEX Activities) involves students from these particular cohorts to provide data on their own knowledge development of concepts and processes in area and perimeter after the teaching episodes.

Conclusion and expected outcomes

Research indicates that a special and complex kind of knowledge is required for teaching mathematics. The development of such knowledge requires explication into more visible forms in order to meet the needs of contemporary education standards. This proposed research therefore aims to elucidate the nature of content knowledge for teaching and pedagogical content knowledge. Using a Learning Trajectories approach, professional knowledge for teaching will be examined in the domain of Measurement. The Learning Trajectories for area/perimeter is expected to provide fine-grained data on teachers' cognition about students' learning as well as their own understanding of the concepts, thereby amplifying both content and pedagogical content knowledge. Implications from this research may inform pre-service mathematics education courses. Such information is necessary as current literature clearly indicates a global concern to improve teacher knowledge for mathematics. Furthermore, Ball, Lubienski and Mewborn (2001) ascertain that no a greement has been established on w hat constitutes teachers' knowledge for effective teaching and this lack of consensus has ramifications for policy makers, teacher educators and practising teachers.

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