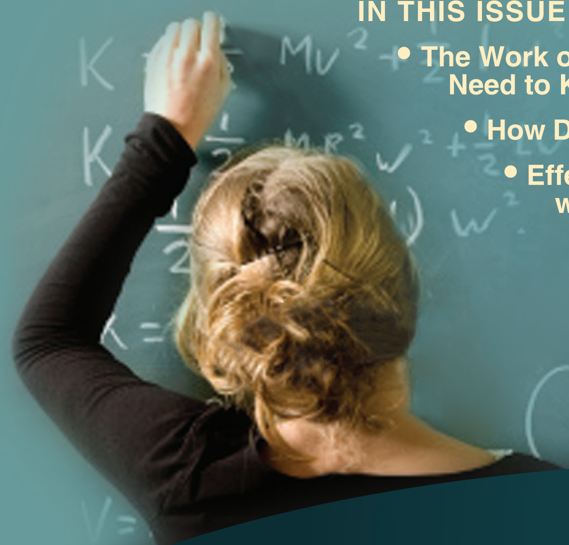

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Effective Mathematics Instruction

By Kathlyn Steedly, Ph.D., Kyrie Dragoo, M.Ed., Sousan Arafeh, Ph.D., & Stephen D. Luke, Ed.D.

> **“Can you do addition?”** the White Queen asked. “What’s one and one and one and one and one and one and one and one and one and one?”
>
> “I don’t know,” said Alice. “I lost count.”
>
> —Lewis Carroll
> *Through the Looking Glass*

We start this *Evidence for Education* with an odd, little quote that illustrates several things quickly about math. It’s easy to get lost, especially if the question comes at you too fast, and once you get lost, well…

Well, we don’t want students to get lost in math. This *Evidence for Education* is about helping students stay on track in math, building concept upon concept in a steady progression of skills. This is as much a national priority as it is a practical necessity for the students themselves, because daily life *involves* math—from the check-out counter at the school store to the express line in the grocery, from our most routine jobs to the high-paying, high-profile ones in engineering, technology, and science (Lee, Grigg, & Dion, 2007; U.S. Government Accountability Office, 2005).

So—two questions naturally arise. What do students need to know how to do, mathematically? And what instructional approaches are effective in teaching those skills?

This *Evidence for Education* addresses these questions, and one more: What do we do when disability affects a student’s ability to learn math skills? That’s the reality for literally millions of students in our schools; certain disabilities *do* add to the challenge of learning an already challenging subject. Therefore, what the research has to say about effective math instruction for students with disabilities is a vital tool in the hands of school personnel responsible for designing and delivering math programming. This publication offers just such research-based tools and guidance to teachers, administrators, and families.

We’ve divided the discussion into four sections, as follows:

- The work of four expert panels, which have recommended what students need to learn in math and what we need to teach;
- How disability can affect math learning;
- Four instructional approaches emerging from the research as effective math interventions for students with disabilities; and
- Suggestions for moving research into practice.

PART 1:

The Work of Expert Panels: What Students Need to Know and We Need to Teach

Four advisory panels have been appointed since 1999 alone to advise the nation on how best to teach mathematics: The National Commission on Mathematics and Science Teaching for the 21st Century, National Research Council, the RAND Mathematics Study Panel, and the National Mathematics Advisory Panel. The reports emerging from each are detailed, often technical, but well worth reading, especially for those involved in math education, because they capture what each expert panel concludes schools must teach and students learn in math.

What these reports make clear is that mathematics teaching and learning are complex undertakings. The National Research Council, for example, refers to "mathematical proficiency" as five intertwined strands, described in the box on page 3 (Kilpatrick, Swafford, & Findell, 2001). Learning each of these strands is an ongoing process that builds on itself. As new concepts and skills are learned, new terms and symbols must also be learned and older skills remembered and applied. The final report of the National Mathematics Advisory Panel (2008) speaks clearly to the need for math curricula that fosters student success in algebra (and beyond) and experienced math teachers who use researched-based instructional strategies. The report also stresses the "mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic recall of facts" (National Mathematics Advisory Panel, 2008, p. xiv). Math teachers know this already—and recognize the very real consequences of students not achieving a level of mastery with foundational math concepts. Disability can further compromise student learning (Spear-Swerling, 2005), especially if the disability affects recall of information and the generalization of skills from one learning situation to another.

Which brings us to the next two parts in this *Evidence for Education*: How disabilities can affect math learning and how to effectively address these special learning needs.

Mathematics Advisory Panels and Their Reports

- **National Commission on Mathematics and Science Teaching for the 21st Century—*Before It's Too Late***
  http://www.ed.gov/inits/Math/glenn/index.html
- **National Research Council—*Adding It Up: Helping Children Learn Mathematics***
  http://www.nap.edu/catalog/9822.html
- **RAND Mathematics Study Panel—*Mathematical Proficiency for All Students***
  http://www.rand.org/pubs/monograph_reports/MR1643/index.html
- ***Foundations for Success: The Final Report of the National Mathematics Advisory Panel***
  http://www.ed.gov/about/bdscomm/list/mathpanel/report/final-report.pdf

PART 2:

How Disabilities Can Affect Math Achievement

Many different disabilities can affect children's math learning and performance, but none more than disabilities that affect cognition—mental retardation, traumatic brain injury, attention-deficit/hyperactivity disorder, and learning disabilities, to name a few. Several specific areas of disability are clearly connected to math learning difficulties. Visual processing, visual memory, and visual-spatial relationships all impact math proficiency in that they are threads in the fabric of conceptual understanding and procedural fluency (Kilpatrick et al., 2001). Specific math learning difficulties also can affect a student's ability to formulate, represent, and solve math problems (known as strategic competence).

The term learning disabilities (LD) certainly appears throughout the literature on math difficulties. This is not especially surprising: LD is the most frequently referenced disability affecting math learning and performance, with a well-documented impact on the learning of 5% to 10% of children in grades K-12 (Fuchs & Fuchs, 2002; Garnett, 1998; Geary, 2001, 2004; Mazzocco & Thompson, 2005).

That's more than 2.8 million children (U.S. Department of Education, 2007). While some of these children are primarily affected in their ability to read or write, many others struggle predominantly in the math arena, a manifestation of LD known as *dyscalculia*. This can be seen in the Federal definition of LD, which captures well the variable impact of the disability:

> Specific learning disability means a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken or written, **that may manifest itself in the imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations....**
>
> 34 C.F.R.§ 300.8(c)(10)(i)

The "imperfect ability to…do mathematical calculations" accurately describes how LD affects many students. However, not all children with LD have math troubles, and not all children with math troubles have a learning disability. The commonality of interest here, then, is *trouble with math*, not what disability a child may have. That's one very good reason to look beyond labels and focus on what teachers can do, instructionally speaking, to support students who are struggling in math. Which we're going to do right now.

PART 3:
Effective Mathematics Instruction for Students with Learning Difficulties in Math: Four Approaches That Improve Results

WE KNOW A GREAT DEAL about effective math instruction for students with disabilities, especially students who have LD. There have been five meta-analyses on the subject, reviewing a total of 183 research studies (Adams & Carnine, 2003; Baker, Gersten, & Lee, 2002; Browder, Spooner, Ahlgrim-Delzell, Harris, & Wakeman, 2008; Kroesbergen & Van Luit, 2003; Xin & Jitendra, 1999). The studies combined in these meta-analyses involved students with a variety of disabilities—most notably, LD, but other disabilities as well, including mild mental retardation, AD/HD, behavioral disorders, and students with significant cognitive disabilities. The meta-analyses found strong evidence of instructional approaches that appear to help students with disabilities improve their math achievement. We now also have the National Mathematics Advisory Panel Report (2008) that further investigates successful mathematical teaching strategies and provides additional support for the research results.

According to these studies, four methods of instruction show the most promise. These are:

- **Systematic and explicit instruction**, a detailed instructional approach in which teachers guide students through a defined instructional sequence. Within systematic and explicit instruction students learn to regularly apply strategies that effective learners use as a fundamental part of mastering concepts.
- **Self-instruction**, through which students learn to manage their own learning with specific prompting or solution-oriented questions.
- **Peer tutoring**, an approach that involves pairing students together to learn or practice an academic task.
- **Visual representation,** which uses manipulatives, pictures, number lines, and graphs of functions and relationships to teach mathematical concepts.

Of course, to make use of this information, an educator would need to know much more about each approach. So let's take a closer look.

The National Research Council's Concept of "Mathematical Proficiency"

"The integrated and balanced development of all five strands of mathematical proficiency [shown below] …should guide the teaching and learning of school mathematics."

- **Conceptual understanding—** comprehension of mathematical concepts, operations, and relations
- **Procedural fluency—**skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- **Strategic competence—**ability to formulate, represent, and solve mathematical problems
- **Adaptive reasoning—**a capacity for logical thought, reflection, explanation, and justification
- **Productive disposition—**habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick, et al., 2001, p. 11).

Explicit and Systematic Instruction

Explicit instruction, often called direct instruction, refers to an instructional practice that carefully constructs interactions between students and their teacher. Teachers clearly state a teaching objective and follow a defined instructional sequence. They assess how much students already know on the subject and tailor subsequent instruction, based upon that initial evaluation of student skills. Students move through the curriculum, both individually and in groups, repeatedly practicing skills at a pace determined by the teacher's understanding of student needs and progress (Swanson, 2001). Explicit instruction has been found to be especially successful when a child has problems with a specific or isolated skill (Kroesbergen & Van Luit, 2003).

The Center for Applied Special Technology (CAST) offers a helpful snapshot of an explicit instructional episode (Hall, 2002), shown in Figure 1 below. Consistent communication between teacher and student creates the foundation for the instructional process. Instructional episodes involve *pacing* a lesson appropriately, allowing adequate *processing* and *feedback* time, encouraging *frequent student reponses*, and *listening* and *monitoring* throughout a lesson.

Systematic instruction focuses on teaching students *how to learn* by giving them the tools and techniques that efficient learners use to understand and learn new material or skills. Systematic instruction, sometimes called "strategy instruction," refers to the strategies students learn that help them integrate new information with what is already known in a way that makes sense and be able to recall the information or skill later, even in a different situation or place. Typically, teachers model strategy use for students, including thinking aloud through the problem-solving process, so students can see when and how to use a particular strategy and what they can gain by doing so. Systematic instruction is particularly helpful in strengthening essential skills such as organization and attention, and often includes:

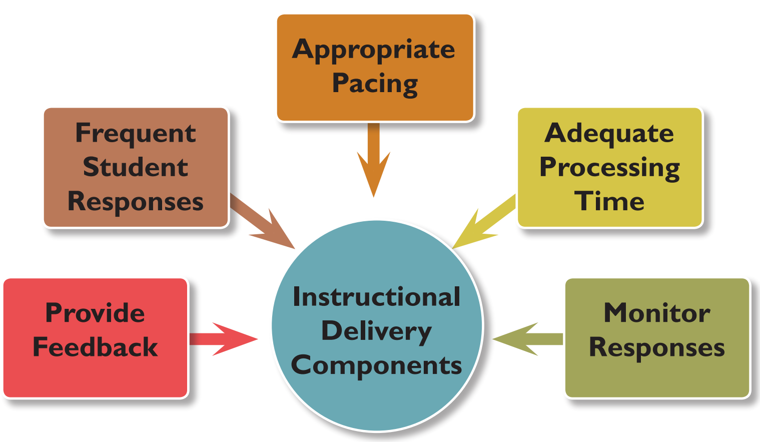

Figure 1. Standard instructional delivery components essential to all explicit instructional episodes (Hall, 2002).

- Memory devices, to help students remember the strategy (e.g., a first-letter mnemonic created by forming a word from the beginning letters of other words);
- Strategy steps stated in everyday language and beginning with action verbs (e.g., read the problem carefully);
- Strategy steps stated in the order in which they are to be used (e.g., students are cued to read the word problem carefully *before* trying to solving the problem);
- Strategy steps that prompt students to use cognitive abilities (e.g., the critical steps needed in solving a problem) (Lenz, Ellis, & Scanlon, 1996, as cited in Maccini & Gagnon, n.d.).

All students can benefit from a systematic approach to instruction, not just those with disabilities. That's why many of the textbooks being published today include overt systematic approaches to instruction in their explanations and learning activities. It's also why NICHCY's first *Evidence for Education* was devoted to the power of strategy instruction. The research into systematic and explicit instruction is clear—the approaches taken together positively impact student learning (Swanson, in press). The National Mathematics Advisory Panel Report (2008) found that explicit instruction was primarily effective for computation (i.e., basic math operations), but not as effective for higher order problem solving. That being understood, meta-analyses and research reviews by Swanson (1999, 2001) and Swanson and Hoskyn (1998) assert that breaking down instruction into steps, working in small groups, questioning students directly, and promoting ongoing practice and feedback seem to have greater impact when combined with systematic "strategies."

What does a combined systematic and explicit instructional approach look like in practice? Tammy Cihylik, a learning support teacher at Harry S. Truman Elementary School in Allentown, Pennsylvania, describes a first-grade lesson that uses money to explore mathematical concepts:

"[Students] use *manipulatives*," she explains, "looking at the penny, identifying the penny." Cihylik prompts the students with explicit questions: "What does the penny look like? How much is it worth?" Then she provides the answers herself, with statements like, "The penny is brown, and is worth one cent." Cihylik encourages students to repeat the descriptive phrases after her, and then leads them in applying that basic understanding in a systematic fashion. After counting out five pennies and demonstrating their worth of five cents, she instructs the students to count out six pennies and report their worth. She repeats this activity each day, and incorporates other coins and questions as students master the idea of *value*.

Within this example, the relationship between explicit and systematic instruction becomes clear. The teacher is leading the instructional process through continually checking in, demonstration, and scaffolding/extending ideas as students build understanding. She uses specific strategies involving prompts that remind students the value of the coins, simply stated action verbs, and metacognitive cues that ask students to monitor their money. Montague (2007) suggests, "The instructional method underlying cognitive strategy instruction is explicit instruction."

Self-Instruction

Self-instruction refers to a variety of self-regulation strategies that students can use to manage themselves as learners and direct their own behavior, including their attention (Graham, Harris, & Reid, 1992). Learning is essentially broken down into elements that contribute to success:

- setting goals
- keeping on task
- checking your work as you go
- remembering to use a specific strategy
- monitoring your own progress
- being alert to confusion or distraction and taking corrective action
- checking your answer to make sure it makes sense and that the math calculations were correctly done.

When students discuss the nature of learning in this way, they develop both a detailed picture of themselves as learners (known as metacognitive awareness) and the self-regulation skills that good learners use to manage and take charge of the learning process. Some examples of self-instruction statements are shown on the next page.

To teach students to "talk to themselves" while learning new information, solving a math problem, or completing a task, teachers first model self-instruction aloud. They take

For More Information on Explicit and Systematic Instruction

- **National Institute for Direct Instruction (NIFDI)**
  http://www.nifdi.org/
- **The Access Center's *Direct or Explicit Instruction and Mathematics***
  http://www.k8accesscenter.org/training_resources/DirectExplicitInstruction_Mathematics.asp
- **Special Connections' *Direct Instruction: Math***
  http://www.specialconnections.ku.edu/cgi-bin/cgiwrap/specconn/main.php?cat=instruction&section=main&subsection=di/math
- **NICHCY's *The Power of Strategy Instruction***
  http://research.nichcy.org/strategy_instruction1.asp
- **The Access Center's *Strategy/Implicit Instruction and Mathematics***
  http://www.k8accesscenter.org/training_resources/Strategy_ImplicitInstructionandMath.asp
- **University of Nebraska-Lincoln's *Cognitive Strategy Instruction: Math***
  http://www.unl.edu/csi/math.shtml

Examples of Seven Statements Associated with Self-Instruction

Problem Definition *(defining the nature and demands of a task)*	"OK. I need to read the problem carefully and decide whether we're adding or subtracting here." "What's my next step?"
Focusing Attention and Planning Ahead *(attending to task and generating plans)*	"I need to stop horsing around and concentrate." "What's the best way to do this problem?"
Strategy-Related *(engaging and using a strategy)*	"I need to remember to use my strategy." "OK, what I need to do is remember my 4 Bs strategy."
Self-Monitoring *(being alert to one's own progress and to confusions and distractions)*	"So far, so good." "This isn't making sense. Let me go back and read the problem again."
Self-Evaluation *(detecting errors and correcting them)*	"Does this answer make sense?" "Oops, this can't be right. I need to check my math."
Coping *(dealing with difficulties/failures)*	"I can do this if I keep at it." "This isn't rocket science. I know I can do it." "Take a deep breath and relax."
Self-Reinforcement *(rewarding oneself)*	"I did it! Great job!" "I worked hard and I got it right!"

Adapted from Reid, R. (2006). *Strategy instruction for students with learning disabilities.* New York: Guilford Press.

a task and think aloud while working through it, crafting a monologue that overtly includes the mental behaviors associated with effective learning: goal-setting, self-monitoring, self-questioning, and self-checking. Montague (2004) suggests that both correct and incorrect problem-solving behaviors be modeled.

> Modeling of correct behaviors helps students understand how good problem solvers use the processes and strategies appropriately. Modeling of incorrect behaviors allows students to learn how to use self-regulation strategies to monitor their performance and locate and correct errors. Self-regulation strategies are learned and practiced in the actual context of problem solving. When students learn the modeling routine, they then can exchange places with the teacher and become models for their peers. (p. 5)

The self-statements that students use to talk themselves through the problem-solving process are actually *prompting* students to use a range of strategies and to recognize that certain strategies need to be deployed at certain times (e.g., self-evaluation when you're done, to check your work). Because learning is a very personal experience, it's important that teachers and students work together to generate self-statements that are not only appropriate to the math tasks at hand but also to individual students. Instruction

For More Information and Guidance on Self-Instruction

- **Penn State's *Self-Regulation Abilities, Beyond Intelligence, Play Major Role in Early Achievement***
  http://www.pop.psu.edu/searchable/press/apr0407.htm
- **The National Research Center on the Gifted and Talented (NRC/GT)'s module, *Self-Regulation***
  http://www.gifted.uconn.edu/Siegle/SelfRegulation/section0.html
- **The Access Center's *Math Problem Solving for Primary Elementary Students with Disabilities***
  http://www.k8accesscenter.org/training_resources/mathprimaryproblemsolving.asp
- **The Access Center's *Math Problem Solving for Upper Elementary Students with Disabilities***
  http://www.k8accesscenter.org/training_resources/MathPrblSlving_upperelem.asp
- **Federal Way Public Schools' *Adaptations Are Essential: Early Years Mathematics***
  http://www.fwps.org/cur/adaptations/math.pdf
- **Marjorie Montague's 2007 article, "Self-Regulation and Mathematics Instruction" in *Learning Disabilities Research & Practice*, 22(1), 75–83. (This article appears in a special issue of the journal devoted to math instruction.)**
  http://www3.interscience.wiley.com/journal/118480756/abstract

also needs to include frequent opportunities to practice their use, with feedback (Graham et al., 1992) until students have internalized the process.

Peer Tutoring

Peer tutoring is a term that's been used to describe a wide array of tutoring arrangements, but most of the research on its success refers to students working in pairs to help one another learn material or practice an academic task. Peer tutoring works best when students of different ability levels work together (Kunsch, Jitendra, & Sood, 2007). During a peer tutoring assignment, it is common for the teacher to have students switch roles partway through, so the tutor becomes the tutee. Since explaining a concept to another person helps extend one's own learning, this practice gives both students the opportunity to better understand the material being studied.

Research has also shown that a variety of peer-tutoring programs are effective in teaching mathematics, including Classwide Peer Tutoring (CWPT), Peer-Assisted Learning Strategies (PALS), and Reciprocal Peer Tutoring (RPT) (Barley et al., 2002). Successful peer-tutoring approaches may involve the use of different materials, reward systems, and reinforcement procedures, but at their core they share the following characteristics (Barley et al., 2002):

- The teacher trains the students to act both as tutors and tutees, so they are prepared to tutor, and receive tutoring from, their peers. Before engaging in a peer-tutoring program, students need to understand how the peer-tutoring process works and what is expected of them in each role.
- Peer-tutoring programs benefit from using highly structured activities. Structured activities may include teacher-prepared materials and lessons (as in Classwide Peer Tutoring) or structured teaching routines that students follow when it is their turn to be the teacher (as in Reciprocal Peer Tutoring).
- Materials used for the lesson (e.g., flashcards, worksheets, manipulatives, and assessment materials) should be provided to the students. Students engaging in peer tutoring require the same materials to teach each other as a teacher would use for the lesson.

For More Information and Guidance on Peer Tutoring

- **Vanderbilt's *Peer-Assisted Learning Strategies (PALS)***
  http://kc.vanderbilt.edu/pals/
- **The Institute of Education Sciences' *Intervention: Peer-Assisted Learning Strategies (PALS)***
  http://ies.ed.gov/ncee/wwc/reports/english_lang/pals/index.asp
- **The Institute of Education Sciences' *Intervention: ClassWide Peer Tutoring (CWPT)***
  http://ies.ed.gov/ncee/wwc/reports/beginning_reading/cwpt/index.asp
- **The Access Center's *Using Peer Tutoring for Math***
  http://www.k8accesscenter.org/training_resources/mathpeertutoring.asp
- **The Access Center's *Using Peer Tutoring to Facilitate Access***
  (Reviews the use of CWPT, RPT, and PALS in teaching mathematics and other subjects)
  www.k8accesscenter.org/training_resources/documents/PeerTutoringFinal.doc
- **Fulk and King's *Classwide Peer Tutoring at Work***
  http://www.dldcec.org/pdf/teaching_how-tos/classwide_peer_tutoring.pdf
- **Special Connection's *An Introduction to ClassWide Peer Tutoring***
  http://www.specialconnections.ku.edu/cgi-bin/cgiwrap/specconn/main.php?cat=instruction&section=main&subsection=cwpt/main
- **The Center for Effective Collaboration and Practice's *Classwide Peer Tutoring: Information for Families***
  http://cecp.air.org/Peer_Tutoring.pdf

- Continual monitoring and feedback from the teacher help students engaged in peer tutoring stay focused on the lesson and improve their tutoring and learning skills.

Finally, there is mounting research evidence to suggest that, while low-achieving students may receive moderate benefits from peer tutoring, effects for students specifically identified with LD may be less noticeable unless care is taken to pair these students with a more proficient peer who can model and guide learning objectives (Kunsch, Jitendra, & Sood, 2007).

Visual Representations

Mathematics instruction is a complex process that attempts to make abstract concepts tangible, difficult ideas understandable, and multifaceted problems solvable. Visual representations bring research-based options, tools, and alternatives to bear in meeting the instructional challenge of mathematics education (Gersten et al., 2008). Visual representations, broadly defined, can include manipulatives, pictures, number lines, and graphs of functions and relationships. “Representation approaches to solving mathematical problems include pictorial (e.g., diagramming); concrete (e.g., manipulatives); verbal (linguistic training); and mapping instruction (schema-based)” (Xin & Jitendra, 1999, p. 211). Research has explored the ways in which visual representations can be used in solving story problems (Walker & Poteet, 1989); learning basic math skills such as addition, subtraction, multiplication, and division (Manalo, Bunnell, & Stillman, 2000); and mastering fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003) and algebra (Witzel, Mercer, & Miller, 2003).

Concrete-Representational-Abstract (CRA) techniques are probably the most common example of mathematics instruction incorporating visual representations. The CRA technique actually refers to a simple concept that has proven to be a very effective method of teaching math to students with disabilities (Butler et al., 2003; Morin & Miller, 1998). CRA is a three-part instructional strategy in which the teacher first uses *concrete* materials (such as colored chips, base-ten blocks, geometric figures, pattern blocks, or unifix cubes) to model the mathematical concept to be learned, then demonstrates the concept in *representational* terms (such as drawing pictures), and finally in *abstract* or symbolic terms (such as numbers, notation, or mathematical symbols).

During a fraction lesson using CRA techniques, for example, the teacher might first show the students plastic pie pieces, and explain that, when the circle is split into 4 pieces, each of those pieces is ¼ of the whole, and when a circle is split into 8 pieces, each piece is ⅛ of the whole. After seeing the teacher demonstrate fraction concepts using concrete manipulatives, students would then be given plastic circles split into equal pieces and asked what portion of the whole one section of that circle would be. By holding the objects in their hands and working with them concretely, students are actually building a *mental* image of the reality being explored physically.

For More Information and Guidance on Visual Representations

- **The Access Center’s *Concrete-Representational-Abstract Instructional Approach***
  http://www.k8accesscenter.org/training_resources/CRA_Instructional_Approach.asp
- **TeachingLD’s *Teaching Students Math Problem-Solving Through Graphic Representations***
  http://www.teachingld.org/pdf/teaching_how-tos/journal_articles/Article_5.pdf
- **Special Connection’s *Concrete-Representational-Abstract (C-R-A) Instruction***
  http://www.specialconnections.ku.edu/cgi-bin/cgiwrap/specconn/main.php?cat=instruction&section=main&subsection=math/cra
- **MathVIDS’ *Concrete-Representational-Abstract Sequence of Instruction***
  http://coe.jmu.edu/Mathvids2/strategies/cra.html
- ***Using Research-based Methods to Teach Fraction Concepts: What REALLY Works***
  http://app.outreach.psu.edu/math/fromdatabase/3770-ButlerFraction_Presentation1.pdf

After introducing the concept of fractions with concrete manipulatives, the teacher would model the concept in *representational* terms, either by drawing pictures or by giving students a worksheet of unfilled-in circles split into different fractions and asking students to shade in segments to show the fraction of the circle the teacher names.

In the final stage of the CRA technique, the teacher demonstrates how fractions are written using abstract terms such as numbers and symbols (e.g., ¼ or ½). The teacher would explain what the numerator and denominator are and allow students to practice writing different fractions on their own.

As the Access Center (2004) points out, CRA works well with individual students, in small groups, and with an entire class. It’s also appropriate at both the elementary and secondary levels. The National Council of Teachers of Mathematics (NCTM) recommends that, when using CRA, teachers make sure that students understand what has been taught at each step before moving instruction to the next stage (Berkas & Pattison, 2007). In some cases, students may need to continue using manipulatives in the representational and abstract stages, as a way of demonstrating understanding.

PART 4:

Putting the Research to Work: Choosing and Using Effective Math Intervention Strategies

WE'VE BRIEFLY EXAMINED four approaches to teaching mathematics to students with disabilities that research has shown to be effective (Adams & Carnine, 2003; Baker, Gersten, & Lee, 2002; Kroesbergen & Van Luit, 2003; Xin & Jitendra, 1999). Each is worthy of study in its own right, so we hope that the sources of additional information we've provided will help teachers, administrators, and families bring these research-based practices into the math classroom.

When it comes time to determine how you can best teach math to your students, select an instructional intervention that supports the educational goals of those students based on age, needs, and abilities. Research findings can and do help identify effective and promising practices, but it's essential to consider how well-matched any research actually is to your local situation and whether or not a specific practice will be useful or appropriate for a particular classroom or child. Interventions are likely to be most effective when they are applied to similar content, in similar settings, and with the age groups intended for them. That's why it's important to look closely at the components of any research study to determine whether the overall findings provide appropriate guidance for *your* specific students, subjects, and grades—apples to apples, so to speak.

Of great value to those seeking to better understand the evidence base for math (and other) educational interventions are these three sources of information:

- **What Works Clearinghouse**
  http://ies.ed.gov/ncee/wwc/
- **Best Evidence Encyclopedia (BEE)**
  http://www.bestevidence.org
- **Center on Instruction**
  http://www.centeroninstruction.org/index.cfm

Both of these sources look closely at existing research on educational interventions and report on their effectiveness. Both have evaluated a wide range of commercial math series and materials in use around the country and have categorized them by how much *evidence of effectiveness* they show. What Works uses six categories (strong positive, potentially positive, mixed, potentially negative, strong negative, and no discernible evidence), while the Best Evidence Encyclopedia uses five (strong, moderate, limited, insufficient to make a judgment, and no qualifying studies), either of which offers valuable information to decision makers. To help teachers and administrators investigate those interventions most relevant to their local situation and need, interventions are also broken out by level (elementary and middle and/or high school).

The instructional approaches on which we have focused take place in classrooms where they often coexist, support, strengthen, and work together to effectively teach students. These approaches can be seen as threads in the fabric of the classroom. In the complex world of the classroom, students benefit from one-on-one guidance from a teacher who can model problem-solving techniques and control the difficulty of tasks through feedback and cues.

And speaking of feedback…Consistent and ongoing feedback has been shown to be quite effective in improving student performance. That's why it's an aspect that should be incorporated into all classrooms, regardless of the intervention. In particular, the value of *immediate* feedback stands out. Regular feedback helps students guide and improve their own practice, even as *giving* feedback helps teachers guide and tailor their own instruction.

When you consider the interventions described here, it's exciting to realize that they are ready tools in our hands and in our classrooms, and can serve us well as a means of improving students' math proficiency and outcomes. We hope they do just that.

Glossary

Concrete-Representational-Abstract Techniques – a three-part instructional strategy where the teacher uses concrete materials (manipulatives) to model a concept to be learned, then uses representational terms (pictures), and finally uses abstract, symbolic terms (numbers, math symbols).

Explicit and Systematic Instruction – teacher-led demonstrations of various strategies that students, in turn, use to guide their way through the problem-solving process.

Dyscalculia – a form of learning disability that causes an individual to have difficulty in understanding concepts of quantity, time, place, value, and sequencing, and in successfully manipulating numbers or their representations in mathematical operations.

Intervention – specific services, activities, or products developed and implemented to change or improve student knowledge, attitudes, behavior, or awareness.

Mathematical Proficiency – a term developed by the National Research Council that describes five interrelated strands of knowledge, skills, abilities, and beliefs that allow for mathematics manipulation and achievement across all mathematical domains (e.g., conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition) (Kilpatrick et al., 2001).

Operations – mathematical procedures such as addition, subtraction, multiplication, and division.

Peer Tutoring – an instructional arrangement where students work in pairs to help one another learn material or practice an academic task.

Representation Techniques – visually or schematically presenting the ideas or information contained in word problems in language or visual media.

Self-Instruction – an instructional approach that teaches students to use a variety of self-statements or verbal prompts to guide themselves through the problem-solving process.

Strategy Instruction/Strategy Training – an instructional approach that teaches students how to use the same tools and techniques that efficient learners use to understand and learn new material or skills.

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