Developing Pre-Service Teacher Capacity to Make Appropriate Choices of Tasks and Resources through Diagnostic Assessment of Children's Work

Chris Hurst
Curtin University
<c.hurst@curtin.edu.au>

This paper reports on one phase of a long-term project investigating mathematical content knowledge of pre-service teachers. A cohort of second year PSTs conducted a diagnostic assessment and a series of associated tutoring sessions with a primary aged child. The focus here is on the PSTs' ability to make appropriate task choices following the diagnostic process. Results of the study suggest that PSTs are capable of making sound choices of tasks and associated resources based on their mathematical and pedagogical content knowledge following a targeted diagnostic assessment process.

Since Schulman's (1986) seminal paper, there has been much said about content knowledge and associated pedagogical knowledge needed by teachers to effectively teach mathematics and make appropriate task choices. The three domains identified by Schulman have been interpreted in different ways by various education researchers such as Ball, Thames and Phelps (2008) who incorporated his 'curricular knowledge' as part of 'pedagogical content knowledge' in a broader model. Their model also included in Schulman's 'subject matter knowledge', other ideas such as 'common content knowledge', 'horizon content knowledge' and 'specialised content knowledge' (Ball, Thames & Phelps, 2008). Others have continued to interpret teacher knowledge in different ways. Beswick, Callingham and Watson (2012) considered it as a single entity with a hierarchical structure that positioned particular mathematical knowledge for teaching at the higher end of a scale.

Tatto et al. (2008) noted that knowledge for teaching requires both mathematical content knowledge (MCK) and mathematical pedagogical content knowledge (PCK or MPCK) and offered a framework for MPCK which included curricular knowledge, knowledge of planning for teaching mathematics and enacting mathematics for teaching and learning. In their report on the Teacher Education and Development Study in Mathematics (TEDS-M), Tatto et al. (2008, p. 39) noted that "In order to interpret and evaluate students' mathematical solutions ... it is necessary to possess the abilities of analysing and diagnosing which are assigned to the sub-domain of enacting mathematics for teaching and learning". However, in considering the results of the TEDS-M study, they also reported that "much of the instructional time in teacher education is spent in the domain of general pedagogy" (Tatto et al. p. 185). This poses the question: How does this situation position pre-service teachers (PSTs) regarding their capacity to diagnose, analyse and make effective task choices? The answer may be 'not very well' based on findings of Callingham et al. (2011) who reported that PSTs made some puzzling choices of tasks and resources. They also noted that both MCK and PCK are essential but that "mathematics understanding alone is not sufficient" (Callingham et al., 2011, p. 906).

The solution may lie in the suggestion by Ball et al. (2008, p. 398) that MPCK may be a way to "build bridges between the academic world of disciplinary knowledge and the practice world of teaching ...by identifying amalgam knowledge that combines the knowing

2014. In J. Anderson, M. Cavanagh & A. Prescott (Eds.). Curriculum in focus: Research guided practice *(Proceedings of the 37th annual conference of the Mathematics Education Research Group of Australasia)* pp. 295–302. Sydney: MERGA.

of content with the knowing of students and pedagogy". This notion is strengthened through observations by Superfine (2008) that in reality, teachers tend to use curriculum documents as starting points for their planning, a point echoed by Sullivan, Clarke and Clarke (2012) who also described difficulties that teachers experience in trying to match tasks to curriculum descriptors. It also links well to the claim made by Tatto et al. (2008) about the over-emphasis on 'general pedagogy'. Sullivan et al. (2012, p. 27) conclude that the matching of tasks to needs should be "an important focus for future professional learning of both prospective and practising teachers". Whilst teachers seem to use the curriculum as a starting point for task choice it is reassuring to note that the majority of teachers use their own assessments as the starting point for planning (Clarke, Clarke & Sullivan, 2012). However, it may be that teachers and PSTs need to be better able to identify the specific mathematics knowledge demonstrated by children in such assessments and to use that as the basis for their task choices, supported by curriculum documents.

It is argued in this paper that the motivation and impetus behind good task choices lies in using children's responses to diagnostic tasks, or indeed everyday tasks and activities, coupled with teachers having a strong and connected content knowledge. The key criterion for judging the effectiveness of teaching in this instance is the matching of the task to the perceived need. Curriculum is a guide to show the sorts of understandings and concepts children should have and develop. Teachers need to be able to interpret children's responses in the light of the curriculum content descriptors and make informed decisions about tasks with very specific features to match very specific mathematical needs.

Methodology

Data were generated from one of the assessment tasks for the second mathematics education unit in the Bachelor of (Primary) Education course at one Australian university. The participant sample consisted of 53 pre-service teachers (PSTs) who were in the second year of their four-year course. The PSTs were required to complete a Child Study, which consisted of a diagnostic assessment based on the Mathematics Assessment Interview (MAI) (Australian Catholic University, 2010) and which focused on aspects of place value understanding. Participants had to administer the diagnostic interview, analyse the results, and structure a tutoring program of at least seven one-on-one sessions of up to one hour with the child who was the subject of the interview. Their final written report of the diagnostic and tutoring process was the source of the data for this study which sought to answer the following research question: *To what extent does a diagnostic assessment of children's understanding of place value assist novice pre-service teachers to make appropriate choices of learning tasks and associated resources?*

A content analysis of the PSTs' written reports was performed to see if they could identify very specific mathematical ideas that children knew and didn't know and if they could choose tasks and resources to match the identified mathematical needs. A manual analysis was performed on key words and ideas and they were clustered to identify emergent themes.

Results and Discussion

Before considering details of the PSTs' analysis of the diagnostic work done with the children it is important to present an overview of the gains made by the PSTs regarding their ability to interpret such work in relation to curriculum documents. In terms of the unit

assessment task, which formed the research question, there were four levels of attainment by the 53 PSTs who participated.

- Identified on numerous occasions the very specific mathematics involved and were able to match the child's needs to very specific and appropriate tasks. Clear linking was made to AC: M (n = 35).
- Identified at least one specific aspect of the mathematics known by the child (or needing to be known) and linked that to a specific and appropriate task. Some linking to curriculum documents was evident (n = 9).
- Identified the mathematics involved but was too general (e.g., 'needed to know more about place value'). Task choice contained several appropriate selections but not necessarily explicitly linked to particular aspects of mathematics (n = 5).
- Identified some of the mathematics known and/or required by the child but tasks were procedural in nature and not appropriate for the identified needs (n = 4).

The specific mathematics identified by the PSTs fits into one of three following themes with specific questions from the MAI providing the assessment points. Each theme is now discussed with examples of assessment decisions and associated task choices made by PSTs being provided.

Theme 1: Difficulties with Reading and Writing Numbers

Several questions in the MAI informed this area with children having to read and write numbers between two and seven digits using different contexts. As well as noting that children experienced difficulty in writing and/or reading numbers, some PSTs stated specifically that children lacked knowledge of the cyclic pattern for reading and writing numbers and that several also had difficulty when zero was a place holder, particularly in an 'internal' place such as in a number like 23 067. A range of resources used in tutoring tasks conducted by the PSTs included Arrow Cards, Gattegno Charts, place value charts, mats or 'houses', Multipurpose Arithmetic Blocks (MABs), number expanders, bundling sticks, ten frames and 10 000 grids. These resources in themselves are useful and appropriate for developing an understanding of how to read and write numbers but it is in the descriptions and reasons given by the PSTs that there is strong evidence of their ability to make good task choices.

PST Janie asked her child subject to make numbers such as 1365 using Arrow Cards and MABs whilst also recording the numbers on a place value mat. She stated that "Arrow Cards complement the use of MABs as they allow students to see the numerical representation of a number which they can then relate to the physical representation". She then described how the child originally chose MAB longs instead of flats but when he made the number with Arrow Cards he corrected his original MAB representation. Another PST, Belle, also used Arrow Cards and MABs and incorporated the Gattegno Chart noting the following: "The Gattegno Chart was used and allowed the child to demonstrate counting by 1's, 10's, 100's and 1000's. It allows children explore larger numbers and make a connection to how a number looks like (sic) and how it can be said". The choice of multiple resources for tasks was common in the work of a number of the PSTs and this reflected sound understanding of how to best develop the targeted mathematical concept. PST Mel also made use of MABs with the place value mat and the 10 000 grid. She had identified her child subject's misconception that millions followed thousands when reading and writing numbers and asked the child to represent numbers into the thousands by cutting

up a 10 000 grid and recording the number on a place value mat. Figure 1 provides an example of this task.

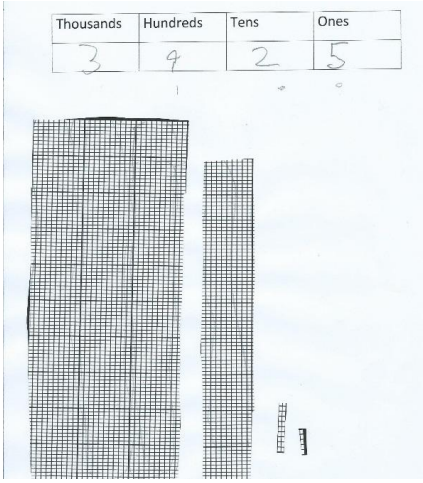

Figure 1: Child's work sample showing a model of a number using a 10 000 grid

PST Alex identified the same misconception when assessing her child subject. She used the place value chart to help him see the cyclical pattern noting that "Once the numbers were put into the place value board, he began to see that he was missing the tens of thousands and hundreds of thousands before he got to the millions". She then asked the child to use the 10 000 grid to model numbers and build the concept that there are numbers between the thousands and the millions as well as of 'how big a million actually is'. PST Sam provides another example of the use of multiple resources and models. She used MABs to consolidate her child subject's understanding of the 'ten times multiplicative relationship' between the places but had noticed that the child still had difficulty in writing numbers correctly when showing them on the place value mat. She realised that the particular place value mat being used may have contributed to this and used a different mat showing the cyclical '100-10-1' pattern being repeated. The child's response using the original mat is shown in Figure 2 as well as the replacement mat that PST Sam used.

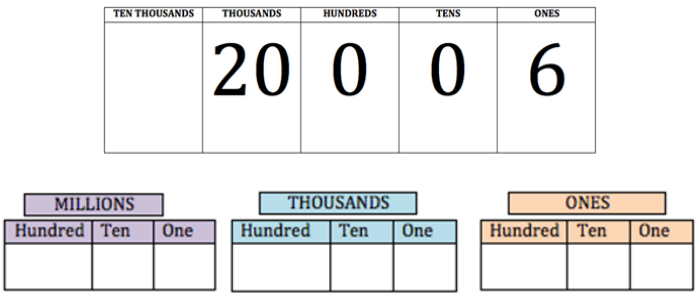

Figure 2: Different place value mats used by PST Sam reflecting improved choice of resources

Also, PST Sam decided to use Arrow Cards to reinforce the understanding that had been built by using the 'cyclical mat'. The child was then able to consistently read and write numbers, and represent them on the place value mat but also to partition them, the

latter, according to Sam, being attributable to the use of the Arrow Cards. Sam's ability to think flexibly about the use of resources and to make sound decisions on the basis of her observations is a good example of how PSTs were able to make appropriate choices of tasks and resources.

Theme 2: Interpreting Numbers and their Values

The MAI questions that informed this area required children to order sets of two, three and four digit numbers, interpret two digit numbers with bundling sticks, and allocate an approximate value to a given point on number lines with two, three and four digit end points. PSTs focused on several aspects of the specific mathematics involved such as relative values of different digits and relative magnitude of numbers, the notion of the ten group as an entity and the idea of 'numbers between numbers', the latter point being linked by several PSTs to the need to explore patterns in number charts. For example, PST Louise noted that her Year Three child subject "was unable to make informed estimates of unknown numbers on a number line [which] demonstrated that she had no concept that numbers are evenly spaced". Consequently, she introduced the child to a 1-200 chart where the child circled each ten after 100 to develop the idea that there are one hundred numbers from each hundred to the next.

PST Annie who noted that her child subject also struggled with the number line tasks also employed a similar strategy. To develop the idea of 'numbers between numbers', she used a range of tasks including Up To and Through 100 and later the use of resources such as a 401-600 chart. Prior to that, Annie had interpreted the child's problems with number lines as follows: "This accentuates the idea that she was not able to interpret numbers [and] it is highly important that the first thing to establish is her ability to see what each digit in a particular number represents". She decided to use the calculator task Wipe Out requiring the child to identify particular digit values. When the child subtracted four instead of 400, PST Annie introduced MABs. She made the following observation:

> I worked with her using the MAB blocks and asked her to show 479 using the MAB blocks, then asked the student to take out the four in the number using the MAB blocks. She then responded by taking out 4 of the 100 blocks. She soon realised that to wipe out the 4 in 479 she needed to subtract 400 rather than 4.

This was a strong choice of task and resources and showed how this PST interpreted the child's responses, recognising the need to consolidate the key understanding about individual digit values before progressing further.

The importance of consolidating essential underpinning ideas is highlighted by the work of PST Maria. She cited her Year Three child subject's response to the ordering task (Order four number cards, randomly spread – 156, 408, 97, 813), noting that she ordered them 97, 403, 813, 156 explaining that '97 has only two numbers so it goes first and 403 comes next because it has a zero in it'. Maria decided to explore the child's misconception and asked her to explain how she ordered a set of two digit numbers (19, 74 and 36) and the child responded that 'This is tricky because they all have high numbers in them'. Maria's assessment was that "It is clear that she understands the value of single digit numbers but does not understand that their value changes depending on their position in a multi-digit number". PST Maria decided to consolidate this key understanding with a set of tasks using ten frames and bundling sticks in combination with a place value mat. As the child created each number model with the resources, she recorded it on the place value

chart. This enabled the child to see the significance of the ten group as an entity as well as the role of zero as a place holder when bridging tens. PST Maria commented that "Using the manipulatives allowed her to see the relative size difference between a two in the tens column and a two in the ones column". She also described the success of the tasks as evident when the child was asked to order some three digit numbers (198 and 213): "During the diagnostic session, she would have chosen 198 as larger because it has larger digits. Now she answered correctly explaining that 'this has two hundreds and this one has one hundred'".

PST Maria's analysis provides another strong example of how PSTs are able to identify the need to consolidate some very important underpinning mathematical ideas but most importantly how they are able to make appropriate choices of tasks and resources to address children's needs. PST Camille also demonstrated this ability as her Year Two child subject appeared to be able to read and write three digit numbers but had trouble ordering them. This was particularly evident when the number had 99 in it or ended in a zero. Camille decided that it was necessary to consolidate grouping and trading rules and embarked on a series of tasks using bundling sticks and writing numbers for the models. This was found to be successful as the child was able to see a physical representation of what occurred when bridging tens and hundreds and could match it to the numerical representation.

The importance of the ten group as an entity has been highlighted in the work of both PST Maria and Camille. PST Jen also saw that as an essential element that was missing in the understanding of her child subject. He appeared to have learned how to read numbers to six digits but could not complete the bundling task requiring him to show the number 36 with pre-bundled and loose sticks. Jen stated that "This conveys that he may have not yet formed a solid understanding of the decade pattern or skip counting in groups of ten. This is further implied when he had extreme difficulties in finding ten more or ten less than a number in later questions". She suspected that the child had likely developed a procedure for reading numbers but lacked the underpinning conceptual understanding. Hence, she structured tasks using dot cards to reinforce his subitising skills and the concept of grouping as well using MABs to build numbers identified when colouring the counting patterns on hundred charts. Again, these were all very appropriate task choices.

Theme 3: Understanding and using Counting Patterns

Some mention has already been made of the identification of counting patterns but this theme specifically relates to the MAI tasks where children had to identify a number that was 10 more or less, and 100 more or less than a given number. Many PSTs recognised that a child's inability to complete these tasks was due to their lack of knowledge and/or application of the skip counting pattern, rather than a problem with adding or subtracting. This in itself demonstrated strong content knowledge on the part of the PSTs and situated them well to make sound task choices. PST Renee identified the issue when her Year Three child subject counted on his fingers to find a number 10 more than 592 as did PST Jo when she observed the following about her Year Six student:

> It is alarming that at a Year Six level Zeke [pseudonym] is using his fingers to arrive at answers for these questions [and] it is just as alarming that this has gone un-noticed until now. At a Year Six level he should be 'trusting the count' and using his knowledge of patterns in the numbers system, rather than using his fingers and other accessible manipulatives.

PST Trish observed her Year Four child subject 'drawing an imaginary sum in the air' and also concluded that the child needed a lot of exposure to the skip counting patterns in order that, as many other PSTs also noted, they would 'trust skip counting'.

Task choices made by PSTs to develop this important understanding varied but were all appropriate and based on identifying and using the skip counting patterns in number charts. Some used 1-100 and 1-200 charts while others used a range of charts including 101-200, 301-400, 501-600 and 1001-1100. Tasks included colouring the skip counting patterns and completing missing sections by using the patterns (e.g, Jumbled Charts). Another task used was 'From Here to There' where different pathways are given (e.g., 10 more, one less, 20 less, one less etc.) and a child has to follow the counting pattern, writing the number each time. Other PSTs made use of a calculator as described here by PST Renee: "I asked him to create a number sequence by skip counting more and less by 10's, using the calculator [to] predict and check the numbers from the calculator display – this was to help him see the patterns that exist in numbers when skip counting more or less by 10's with numbers more than 100".

PST Jessica also noted the dependence of her Year Six child subject on using a written algorithm to complete the '10 more or less' tasks and decided that this was restricting him in developing his conceptual understanding. She developed a series of tasks based around the number charts initially to 100 and then beyond and noted success when the child realised that 'the ones place stays the same number and the tens place goes up by one' when he counted by tens. The work of another PST, Lorrie, highlighted the importance of focused questioning to reinforce the selection of appropriate tasks. When using the constant function of a calculator to generate numbers 10 and 100 more/less, she noted:

> During each transition, the child was asked 'What are you noticing about the digits when you add/subtract another group of 10/100?', 'How does this help you predict the next number?' and 'How do you know that this is the next number?' It was important to ask such questions, as it began ensuring that the procedural knowledge was matched with the correct conceptual understanding."

As her Year Four child subject became more confident with exploring and applying patterns, PST Lorrie posed tasks like 'What is 100 more than 905?" and asked questions such as 'Why is there now a zero where the nine used to be?' This enabled the child to bridge to thousands and see that the pattern continued. Further exploration of patterns continued when the child successfully completed tasks like 1005 – 199. She observed: "It was encouraging to see how the child's understanding was intrinsically linked to what she knew about the pattern, rather than attempting to mentally count back/on or by needing to manually solve using paper and a pen".

Summary

There is strong evidence of the PSTs' ability to make appropriate and informed choices of tasks and associated resources in response to their initial diagnosis of the children's work. It was also encouraging that a number of the PSTs persevered with a range of tasks to consolidate a particular aspect of the child's learning that was identified. They realised that it was essential to 'take a step back' rather than trying to push forward where there may have been some underpinning conceptual understanding that had not been properly developed. This is quite a mature thing to do and demonstrates an approach not really expected of PSTs in only their second year. Also a number of PSTs tried to plan for the whole sequence of required tutoring sessions but quickly realised that they would not

possibly cover them. They showed a preparedness to follow the direction indicated by the child's learning rather than adhere to their original plan.

Conclusions

Sullivan et al. (2012) described the difficulty teachers have in matching tasks to specific situations. However, the results described here suggest that PSTs are clearly capable of doing so when those choices are situated in the context of diagnostic assessment of children's learning. Tatto et al. (2008) identified such diagnosis and the analysis and evaluation of those responses as key elements of a framework for developing the MPCK of PSTs. It is suggested that such diagnostic work is an effective starting point for PSTs to choose appropriate tasks and resources. Content knowledge of PSTs has often been viewed from a deficit perspective. However, as Anakin and Linsell (2014, p. 3) state, perhaps it is time that a 'growth-oriented disposition' is adopted and that we should "deliberately position pre-service teachers as active, responsible, and collaborative agents who are capable of engaging with their learning of mathematics in metacognitive ways". The Child Study described in this paper did just that in giving the PSTs an opportunity to demonstrate their MCK and MPCK in the context of 'doing what effective teachers do' with a focus on how they can be guided to use what they know. This is akin to the 'bridge' suggested by Ball et al. (2008) and provides evidence that practices described here should be widely incorporated in PST courses.

References

Anakin, M., & Linsell, C. (2014). Foundation content knowledge: Pre-service teachers as half-empty or becoming fluent? [under review].

Australian Catholic University. (2011). *Mathematics Assessment Interview.* Australian Catholic University, Melbourne

Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it so special? *Journal of Teacher Education, 59*(5), 389-407.

Beswick, K., Callingham, R., & Watson, J. (2012). The nature and development of middle school mathematics teachers' knowledge. *Journal of Mathematics Teacher Education, 15*(2), 131-157.

Callingham, R., Beswick, K., Chick, H., Clark, J., Goos, M., Kissane, B., Serow, P., Thornton, S., & Tobias, S. (2011). Beginning teachers' mathematical knowledge: What is needed? In J. Clark, B. Kissane, J. Mousley, T. Spencer & S. Thornton (Eds.), *Mathematics: Traditions and [New] Practices* (Vol. 2, pp. 900-907). Alice Springs: Australian Association of Mathematics Teachers and Mathematics Education Research group of Australasia.

Clarke, D.J., Clarke, D.M., & Sullivan, P. (2012). How do mathematics teachers decide what to teach? *Australian Primary Mathematics Classroom, 17*(3), 9-12.

Schulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4-14.

Sullivan, P., Clarke, D.J., & Clarke, D.M. (2012). Choosing tasks to match the content you are wanting to teach. *Australian Primary Mathematics Classroom, 17*(3), 24-27.

Superfine, A. C. (2008). Planning for mathematics instruction: A model of experienced teachers' planning processes in the context of a reform mathematics curriculum. *The Mathematics Educator, 18*(2), 11-22.

Tatto, M. T., Schwille, J., Senk, S., Ingvarson, L., Peck, R., & Rowley, G. (2008). *Teacher Education and Development Study in Mathematics (TEDS-M): Policy, Practice, and Readiness to Teach Primary and Secondary Mathematics. Conceptual Framework.* East Lansing, MI: Teacher Education and Development International Study Center, College of Education, Michigan State University.