Custodians of Quality: Mathematics Education in Australasia Where from? Where at? Where to?

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As a contribution to honour the foresight of Ken Clements and John Foyster in founding MERGA so many years ago this paper is not a research paper in the usual sense. Rather it sets out to sample the context of Mathematics Education in Australasia and beyond (then and now) and to highlight some challenges as seen by this author. In this personal view I do not intend to expand in detail upon particular strands of research in which I have been involved, although for purposes of illustration examples will be drawn from time to time from this and other work. MERGA is about both people and scholarly activity, and so this paper will make reference to both – for history, culture, and challenge are essential components of the development of any organisation.

The MERGA Community

In considering how it has evolved within and from the traditions established by its history it is useful to view MERGA as a community of practice (Lave & Wenger, 1991), whose defining characteristics are summarised as follows:

> Communities of practice are groups of people who share a concern or a passion for something they do, and learn how to do it better as they interact regularly. (Wenger, 2006, p. 1).

In his terms three characteristics are central to a community's structure and purpose.
The Domain: Identity is defined by a shared domain of interest, and therefore commitment to a shared competence that is a distinguishing feature of the group.
The Community: To pursue interests in their domain, members engage in joint activities and discussions, help each other, share information, and build relationships that enable them to learn from each other.
The Practice: Members of a community of practice are practitioners – they develop a shared repertoire of resources, experiences, stories, insights, and ways of addressing characteristic problems that arise in their domain.
The *community* is constituted by these components in combination, and is cultivated by developing them in parallel. This paper draws from all these elements in developing its theme.

Beginnings

Mathematics Education had been through a decade and more of at times turbulent activity. The relatively calm seas of mathematics teaching in Australia were ruffled from overseas in the early 1960s and beyond by influences emanating from the UK, the USA, and continental Europe. The return of Bob McMullen (NSW) from a period working on a US based project in Chicago foreshadowed the introduction of elements of *New Mathematics* thinking into school curricula through the *On Course Mathematics* series, and the following years saw a proliferation of activity in all states.

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At this time mathematics teaching in most states was historically supported by academic mathematicians in university positions, who gave valued support to teachers, for example, through activities of state mathematics teaching associations. It also meant that their influence on syllabus committees kept curricula across all levels of schooling effectively constant, and courses were generally geared towards those who would proceed with further studies in mathematics. The inaugural conference of the Australian Association of Mathematics Teachers was held at Monash University in 1966, while the first of the annual Mathematical Association of Victoria (MAV) December conferences took place at the University of Melbourne in 1965 - where all delegates were housed in a single lecture theatre in the Old Arts building.

A change in emphasis from *Mathematics Teaching* to *Mathematics Education* (in Victoria) occurred in 1965 with the inaugural meeting of the School Mathematics Research Foundation (SMRF), with an interdisciplinary working group including Monash University mathematicians, selected practising teachers, and a psychologist from the State Department of Education. The initial purpose was to produce exemplary text material for the new courses being introduced at senior level – initially published in two volumes under the title *Mathematics for Today and Tomorrow* (School Mathematics Research Foundation, 1967). This was arguably the first Australian text produced under an umbrella that could legitimately be called *Mathematics Education*.

It is possible with the benefit of hindsight to identify characteristics of belief systems driving thinking at this time. In an environment still dominated by direct mathematical influences a belief prevailed that if mathematics was presented accurately then this would facilitate understanding – effectively being necessary and sufficient for this purpose. Motivation was assumed to be absorbed within the former.

For example the following definition of continuity is found on page 122 of *Mathematics for Today and Tomorrow Book 1.*

> We say f is *continuous* at *a* if *a* belongs to the domain of f, and if for each neighbourhood of $f(a)$, there is a neighbourhood of *a* such that for all x in the neighbourhood of *a*, $f(x)$ belongs to the neighbourhood of $f(a)$, i.e., lim $f(x) = f(a)$ as $x \rightarrow a$.

We can recognise here a product of its era, with students assumed interested and able to absorb, hold, and manipulate such content without drama. The significance of cognitive load and an understanding of information processing characteristics had yet to emerge! The psychologist was a behaviourist, albeit a delightfully humane one.

In the early years there were only two senior university appointments in mathematics education in Australia – on the east coast Theo MacDonald (Professor at Monash University) - and the west coast Roly Mortlock (Associate Professor at the University of Western Australia). It was from within the above background a few years later that Ken and John crystallized the need for a mathematics education group whose research interests reflected the wider perspectives and needs that were by now engaging attention. And there was another need as well – it is fair to say that at MERGA 1 delegates did not even know people from their own state, let alone those from further afield.

Advice from New Zealand colleagues indicates that history there followed a broadly similar path, with the early emphasis fairly described as focusing on mathematics teaching, rather than mathematics education in its wider sense. The first postgraduate qualification in mathematics education, M. Ed. Studies (Mathematics), was offered at Massey University from the mid 1980s.

MERGA conferences became truly Australasian when colleagues from across the Tasman began to attend regularly and in numbers, following a watershed year in 1992 at the University of Western Sydney. MERGA has been immeasurably enriched by the widened membership, with awards such as Early Career, Practical Implication, and Career Research Medals now appearing on both sides of the Tasman, along with increased representation on the Executive. In more recent times increasing connection with colleagues from Singapore, culminating in the hosting of the 2012 conference, has continued to enrich the MERGA community.

In reflecting on positives in the development of MERGA we are drawn immediately to its focus on quality. The impetus provided for the dissemination of the fruits of scholarly activity, through for example output in quality journals, in turn generated invitations to join editorial boards and review panels, and to nominate for positions on executives of international research organisations. (By 2000 the number of Australasians invited to take leadership roles within topic and working groups at ICME 9 in Tokyo was equal to the number from North America and greater than that from any European country). The insistence for more than twenty years that material published under the MERGA banner be strictly refereed, whether journal articles, four yearly research reviews, or conference proceedings has been instrumental both in maintaining and promoting the quality of Australasian research, and in providing a strong culture for the nurturing of new scholars in the field. Other more idiosyncratic factors may have also helped – for example the ability to turn the tyranny of distance to advantage. Being geographically remote from both Europe and North America in particular, has enabled the MERGA community to maintain an identity independent of both. The absence of pressure to support or cite particular research directions or sources, or to accept moral imperatives or ideological positions favoured within certain traditions and locations, has enabled independence with respect to the international community, whereby arms length judgments on matters related to scholarly activity can be pursued. (Not that we have avoided making our share of home grown mistakes - which need to be noted acknowledged and left behind).

Finally we should be thankful for an Executive that continues to carry the community forward, by exercising vision in finding means, incentives and avenues to enhance the quality of what can be collectively achieved. This culture is important in forestalling any sense of complacency, in identifying new challenges, and in providing a supportive environment for new members. Indeed we are in debt to Ken and John for their foresight and action.

Between two Stools

Having visited the past what of the present? The academic context within which mathematics education is practised has changed markedly in this part of the world. At secondary level, where once the major influence came from involvement with mathematicians, now those identified as mathematics educators (with a few exceptions) are located within schools of Education. At primary level a similarly situated current situation has replaced one in which teacher education and certification was once provided by major employing authorities. In neither case did research or theoretical positioning as we have come to understand them, formerly play a meaningful role in theory or practice. Where once discussions were most usually with mathematicians for whom “if it is good mathematics it will be good pedagogy” the present community of mathematics educators lives in an academic environment with colleagues driven by a variety of frameworks, including various socio theoretical perspectives, and post modern views of the world. Given that the resulting environment creates pressures and tensions for scholars subject to a cross fire of paradigms, what are the implications for individuals for whom both MATHEMATICS and EDUCATION are spelled in capitals? What can mathematical knowledge contribute within the wider field of educational and social thought above and beyond contributions to do specifically with teaching and learning?

Perhaps by demanding continued accountability of educational theory and practice. This author is not against socio-theoretical perspectives as such, nor constructed realities when appropriate, having worked both individually and with colleagues in developing and using constructs from such frameworks. However we (mathematicians and mathematics educators) find ourselves at times consigned to a supposedly moribund mentality that is not deemed capable of thinking outside some assumed rigid squares. Consider the following:

> Few mathematics education writers have addressed the very useful notions of reality, simulation and knowledge in our contemporary technocultural environment elaborated by theorists like Baudrillard, Lyotard, Lacan, Derrida, Zizek, Bakhtin, McLuhan, and others. These theorists of post modernity trouble the sense of a transparent, self-evident reality and its representations in a number of ways. (Gerofsky, 2010, p. 63).

This paper has some very useful things to say, particularly with respect to the shortcomings of word problems as commonly presented within curriculum implementations. There is objection however when supposed views of the world are ascribed to mathematicians (and mathematics educators), which are just as inflexible as any they are accused of holding. In 1996 a paper was published in the highly respected cultural studies journal Social Text. It argued that the scientific entity quantum gravity was a social construct, and included profuse annotations to writings of social theorists, including a majority of those referred to in the passage above – who used references to mathematical theories to support their contentions. A short time later the author Alan Sokal (a mathematics and physics professor) released the purpose behind the paper which he described as:

> A mélange of truths, half-truths, quarter-truths, falsehoods, non-sequiturs and syntactically correct sentences that have no meaning whatsoever. (Sokal & Bricmont, 1998, p. 248)

The paper was a spoof: its purpose was to identify and expose abuses of scientific and mathematical concepts that had been used to support various post-modern perspectives, particularly those that argued that all knowledge is socially constructed with no external referents. It also sought to provide tools for deconstructing the deconstructions of social theorists. The original exercise has been expanded into two major books (Sokal & Bricmont, 1998: Sokal, 2010), sparked in no small measure by the attacks of those held responsible and featured in the original article. What message does this have for a group of mathematically capable educators, if and when we see mathematics caricatured in ways that distort not only mathematics itself, but views of the world that it is alleged to support? Especially when such views stand to exert theoretical and practical pressures within the teaching and research environment in which we operate.

Fast forward to the report *Mathematics, Engineering and Science in the National Interest* (Office of the Chief Scientist, 2012) which identified five key areas deemed necessary for the purpose of strengthening mathematics and science teaching. Along with *inspirational teaching*; *teaching techniques*; *gender issues*; and *scientific literacy* which obviously fall within the province of MERGA interests, the fifth area nominated was *inspired school leadership*. What can mathematics education contribute here?

Well there have been some strange suggestions in the leadership literature that claim support from the mathematical theory of chaos (e.g., Reilly, 1999; Sullivan, 1999). Together with some startlingly erroneous mathematical attributions regarding the behaviour of non-linear systems, heroic claims were advanced such as a particular introduced policy in a school (an input) acting as a 'chaotic attractor' (an outcome), and that "the science of chaos tells us that signs of disorder might well be signs that the system of education is healthy and on its way to a much improved new order" (Sullivan, 1999: 421). An alternative interpretation might be to suggest inept management! In the vein of the objections addressed by Sokal, these works claimed mathematical support for a range of inferences that would give comfort to any leader or administrator, with megalomaniacal tendencies, who wanted to rail road their own versions of leadership preferences. A response (Galbraith, 2004) was published in the Journal of Educational Administration with the title *Educational Administration and Chaos Theory: Let's be careful.*

While we may mostly find ourselves physically separated from mathematics as it is pursued in universities, times can arise when we need to stand up and be counted, in terms of requiring accountability from those who invoke mathematics dubiously in support of understandings of the world and pursuits of knowledge in the social domains that we inhabit.

What approaches then can we usefully take when sitting on either stool? This author's experience suggests that on curriculum matters mathematical issues need to be addressed directly and substantively with mathematicians – not avoided or diverted. It isn't (naturally) a question of the amount of mathematical knowledge that we possess, but it is a matter of mathematical integrity, in that arguments presented need to demonstrate that the ideas and content being put forward are demonstrably consistent with quality mathematical theory and practice – as well as educationally desirable, whatever level of education is under discussion. That is – address mathematical issues in mathematical terms.

Things are probably less clear from the other stool, partly because of the difficulty of communicating the substance and purpose of mathematical critique to some who want to claim mathematical support for cavalier social theorising. But there is one level at which we can all demand accountability – in pointing out the difference between metaphor and model, and the ways that confusion between these different levels of authority continue to muddy debates. At the least we can demand that any claiming insights for social theories on the basis of outcomes of specific mathematical theories (e.g., chaos or catastrophe theory) are obliged to share the substance of the mathematics that legitimates the inferences at the levels of detail claimed. And identifying the Emperor's tailor in advance can often save the need to examine individual clothes.

Epistemic fallacy

Following from the above we note that one of the ongoing sagas within our field derives from beliefs as to the nature of *reality*, with consequential implications for views as to nature of mathematics, its learning and teaching. For example Sarantakos (1998) predicates a description of constructivism with the statement:

> There is no objective reality; the physical world exists but is not accessible to human endeavour. (Sarantakos, 1998, p. 37)

There follows an elaboration of constructivism containing some non-controversial suppositions and some that invite substantial argument. One does not need to be a positivist to observe that successfully landing astronauts on the moon represents a rather big challenge to the claim that *reality* is entirely constructed and not accessible to human endeavour. Indeed Sokal has angered critics by inviting them to test the belief that the laws of physics are social constructions alone, per medium of the window of his 21st floor apartment! What such claims do is remove external criteria from notions of testability and accountability, by making everything subject to internal constructions – effectively privileging epistemology at the expense of ontology. Concern about the implications of this view led to the development (from the seventies) of Critical Realism (e.g., Bhaskar, 1975).

Accepting that ontology is concerned with definitions of fundamental categories of reality a distinction is made between formal and domain ontology. The former implies something general about reality; the latter is concerned with different areas of reality.

Epistemology defines how we know and reason about a reality in question, so that each domain ontology will have a specific epistemology associated with it. For example maps used by a biologist studying colonies of bees will have a different meaning from maps used by a demographer in studying human settlement, and different again from maps used by a sociologist in studying inter personal behaviour. The epistemic fallacy (Bhaskar, 1975; Bryant, 2011) concerns the conflating of ontology and epistemology – in confusing the nature of an underlying reality with knowledge of it. (The fallacy occurs when statements about being are analysed in terms of knowledge of being - so that ontological questions are avoided through being transposed to epistemological ones.)

In mathematics education this involves the replacement of the consideration of what the nature of mathematical and educational entities are - by how the terms are interpreted and described both for themselves and with respect to their places in educational actions and settings – and in particular how we obtain knowledge about them. While epistemological issues remain important in their own right, they must not be used to obscure or direct attention away from essential ontological underpinnings. Allowing the epistemic fallacy permits appeals against Caesar to be decided by Caesar.

An example - Mathematical Modelling

Anyone reading within the field of mathematical modelling in education can be forgiven for wondering whether different authors have the same thing in mind in their use of the term. They often don't. In fact the modelling field is muddied by the presence of a multitude of epistemological positions – often not explained – that impact with confusing effect on parties not well versed in the field as such. This should reflect upon the individuals concerned, not the field of mathematical modelling which often unfortunately receives the referred pain. Confusion is illustrated by the attempt to address a worthy motive – to classify various curricular implementations of modelling (e.g., Kaiser & Sriraman, 2006) that arose from activity within a working group at a CERME conference. Because the participants started from what they did, and then sought to relate it to modelling, the result might be described as a well-intentioned epistemological bog. Some Australasian examples are included in Stillman, Brown, and Galbraith (2008).

If we examine purposes behind mathematical modelling in education and what it sets out to achieve, then in essence there are just two – distinguished by Julie and Mudaly (2007) as *modelling as content* (empowering students to become independent users of their mathematics) and *modelling as vehicle* (modelling used to serve other curricular needs/more properly called mathematising in many cases).

> Central to the debate is whether mathematical modelling should be used as a *vehicle* for the development of mathematics or treated as *content in and of itself* … The purpose for embedding mathematics in context is not the construction of mathematical models per se but rather the use of contexts and mathematical models as a mechanism for the learning of mathematical concepts, procedures … Mathematical modelling as content entails the construction of mathematical models of natural and social phenomena without the prescription that certain mathematical concepts, procedures or the like should be the outcome of the model-building process. (Julie & Mudaly, 2007, p. 504).

Implications of the vehicle perspective are portrayed unequivocally below:

> Engagement in classroom modeling activities is essential in mathematics instruction only if modeling provides our students with significant opportunities to develop deeper and stronger understanding of curricular mathematics. (Zbiek & Conner, 2006, p. 89-90).

Curricular statements promoting the goal of students able to employ mathematics to address problems in their personal and work lives and as active citizens (ACARA, 2013; CSSSI, 2012) are endorsing the treatment of mathematical modelling as content – that is real world problem solving where realities are not always within-school constructed ones. Here the goal is not only to solve particular problems; it is to teach students how to successfully apply their mathematical knowledge to new situations, including problems of their own choosing – it involves a cumulative process over time. However when, as with our National Statement, the epistemological and methodological consequences for this purpose are not recognized let alone mandated, the result is a selection of applications related features that appear among a plethora of objectives buried in curriculum detail. This ensures that they become just another competing priority in the classroom struggle for survival – a vehicle to be used, compromised, or discarded according to circumstances.

An example of the cross-purposes that have emerged is illustrated by Sfard (2008), who argued that the minute an out-of-school problem is treated in school it is no longer an out-of-school problem and so the search for authentic problems to be modelled is necessarily in vain - as they lose their authenticity. This assertion provides a particularly useful example for it lies at the heart of what mathematical modelling in education is about, and can be addressed at two levels. Firstly counterexamples exist to demonstrate it is a misplaced generalisation. The most potent evidence for authenticity is when students, having been taught modelling (as content) in school, independently apply the skills learned to problems of choice in their personal world. Burkhardt (1981; 2006) gives examples involving junior students, and the present author has previously shared examples in which a senior student employed the modelling process to redesign his hydroponic cultures, and a primary school class successfully presented a case to their local council for a new crossing on the basis of statistical data collected, interpreted, and synthesised. More profoundly, the passage demonstrates how the acceptance of modelling as vehicle assumptions (concerning the nature of school classrooms) distorts perceptions of what can be achieved through modelling as content approaches which will involve actions outside the classroom when required by the needs of a problem. The privileging of a constructed and conservative classroom reality violates the ontological foundations of mathematical modelling as real world problem solving, for which the fullness of reality cannot be constrained to the former.

Another example - Cognitive architecture

The previous example considered realities as they impact on mathematical problem solving in the classroom and beyond. As a second illustration consider reality as it relates to biological attributes associated with teaching and learning – specifically *cognitive architecture* through its impact on information processing.

As a component of human biology the brain and its functions exist independently of what we are able to say about them – such as motor control, automatic processing, long-term memory (LTM) and working memory (WM) capacities. The development of automaticity, for example, involves a shift in brain usage and a reduction in brain activity in contrast to the initial processing of tasks that is procedure based, and reliant on the completion of sequential steps. Schneider (2003) indicates that the development of automaticity generally reduces the load on the working memory by 90%. Figures 1a and 1b (after Schneider, 2003) show how an MRI scan of the brain activity appears before and after the automaticity of a skill is acquired - note the dramatic reduction of activity in the brain as automaticity is developed. In ontological terms this is a fair demonstration of the existence of a *real* phenomenon that is not constructed.

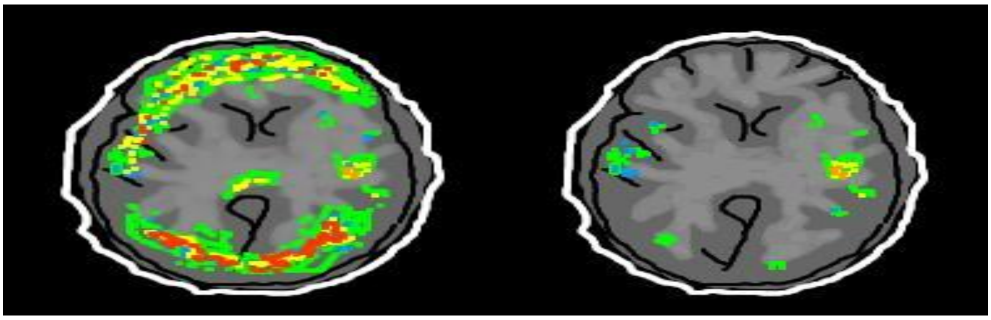


Figure1(a) and (b). MRI brain scans (after Schneider, 2003)

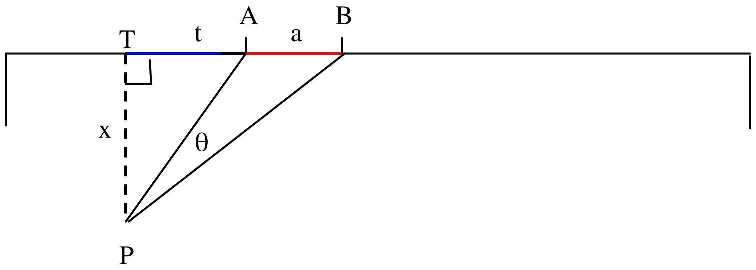
What we are able to learn about information processing abilities, and how to use them effectively in education, is then a product of what we set out to find and how we go about it – the epistemological and methodological dimensions and success or failure to learn more is governed by how well we operate within these domains.

The following examples illustrate ways in which information processing realities are useful in understanding implications associated with mathematical learning tasks.

> Example 1a: If tan $60^o = \sqrt{3}$ and tan $45^o = 1$ find in exact form the value of tan 15^o. (Accompanied by a formula sheet that included **tan (x – y) =** $\frac{\tan x - \tan y}{1 + \tan x \tan y}$)

This question came from a senior level mathematics paper. The answer of tan $15^o = 2 - \sqrt{3}$ is obtained by writing $15^o = (60^o - 45^o)$ and substituting in the given formula. The format enables a solution without requiring the formula to be memorised – but what is the point? Formulae such as these are useful only if they can be invoked for problem solving purposes as in Example 1(b) below, and this requires storage in LTM. The other is a purposeless manipulation, seemingly motivated by knowledge of working memory limitations.

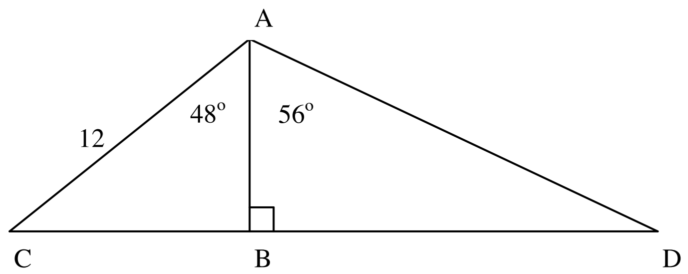
Example 1b: Find the optimum position from which to attempt to convert a Rugby try scored at T.



Noting that tan <BPT = (a + t)/x and tan <APT =t/x then tan θ can be computed and hence maximised - if the formula for tan (x – y) is known (that is it is in LTM and can be retrieved). If the form is not recognised the strategy is not available. Such a formula when part of a network of knowledge stored in LTM is available for recall for problem solving purposes. A calculator can do the processing, but cannot identify what makes the processing relevant.

The particular example is not important – but misplaced information processing implications and activities affecting the learning and retrieval of mathematical knowledge certainly are. Powerful observations on how mathematical expertise is developed and creatively employed have been with us for many years (e.g., Hadamard, 1945). These can now be viewed afresh and understood more completely in the light of increased understanding about the role of memory in problem solving and creative mathematical work. The next example looks at how knowledge of information processing was used to research a quite specific aspect of a mathematical topic.

Example 2: After (Owen & Sweller, 1985).



Students in two groups were required to find the length of BD.

Group A: Were to solve by first (realising the need to find and then) finding AB from the left hand triangle and then using it in the right hand triangle to find BD.

Group B: Were simply directed to find all unknown sides in the diagram (among them of course BD).

Outcome: Those given instructions of the Group B format were consistently more successful on this and a range of examples of similar type. Group A students need to hold strategies in their heads, while also deciding which calculations to perform, and then performing them. Group B students have a lessened cognitive load, needing only to perform separate calculations. A strategy for finding BD (if required) is then supported by the existence of specific data from which to generalise – a chunking strategy. The questions asked and methods used reflect a research methodology grounded in the existence of hardware structures (cognitive architecture) external to the specifics of the questions. The outcome is a theoretically grounded teaching methodology.

The next example is derived from a personal experience.

Example 3: Some years ago I was involved in helping a student with a language processing problem that extended from the primary years through secondary school. It meant she could not handle multiple choice items as the comparisons imposed a cognitive load she could not begin to cope with.

A strategy adopted was to convert multiple choice situations into constructed responses – to work out an answer and then scan visually for its presence among the alternatives – don't even look at distractors as such. As an illustration consider the following item from Year 5 NAPLAN 2008.

Which number is four thousand and seventy six? 4067, 4760, 4706, 4076.

The strategy (which would here select 4076 simply on the basis of its visual presence among the alternatives) enabled the student to cope reasonably well with multiple choice requirements that otherwise presented an impossibility for her. The difference was palpable – with the reduction of information processing demands on working memory a quantifiable factor. Reflecting on the outcome a consequential question suggests itself: What implications are there for the analysis of distractors, when an approach effectively eliminates them by converting multiple choice questions to constructed responses? (That question only suggested itself when writing this section – perhaps it has been done.)

In summary, cognitive architecture continues to impose itself as a key influence in mathematics learning – as a structure with an existence independent of various epistemological and methodological frameworks that aim to enhance learning. It remains astounding that supposed new *panaceas*, for example outcomes based education, or aspects of technology based learning, can be presented as portents of a brave new world without serious engagement with the information processing issues that inevitably attend them. As succinctly put by a student in a research study involving computer learning:

> I have a mental block against performing like a trained 'circus animal' and just pressing the right buttons. I need to know why? What for? What am I trying to find out? (Povey & Ransom, 2000, p. 52).

Serendipity

One feature of a community of practice is that the nurturing of new members at one end runs side by side with pushing the frontiers of knowledge at the other, and with everything in between. Opportunities for new initiatives arise in a multitude of ways – if only we can recognise them. The best mathematics teaching and learning environment I was ever involved in was at the then Melbourne College of Education in the context of teaching within the BSc/BA and BSc(Ed) mathematics programs at Melbourne University. Discussions around teaching had the kind of depth I later came to associate with research programs, although this could not have been recognised at the time. An example was the work involved in identifying the need for, and the subsequent design of teaching examples geared to clarify subtle conceptual differences that regularly caused confusion within particular topics. Feedback was informal – I remember information coming from the Mathematics Department that BSc(Ed) students whose university entry scores were below those of the straight Science degree students, were forming a disproportionate number of better performing students in second year courses.

With the benefit of hindsight, in a different era, this evidence would suggest the development of a systematic research program with the methodology theorised and implemented in terms of some descriptor – perhaps *scholarly teaching.* What it does in retrospect, is sensitise us to possibilities which continue to surround us through anecdotal evidence and practical experience, and that are amenable to systematisation into productive research initiatives. We recall the famous statement by Kurt Lewin that "nothing is as practical as a good theory". The obverse of this, (the source now eludes me) is that "nothing is as theoretical as good practice", and opportunities generated by increasing awareness of the latter is something we can perhaps enhance within our community. It is useful to recall that some telling theoretical advances have first appeared in teaching journals – such as Richard Skemp's development of Instrumental and Relational Understanding (Skemp, 1976), and in a minor classic by Sawyer (1963). I recall an article by David Kent stimulated by reflecting on students' creativity in misusing simple procedural instructions in mathematical processing – The Dynamic of 'Put' (Kent, 1978). All three articles first appeared in the UK journal *Mathematics Teaching.* What potential avenues await research oriented readers who scour present day teaching journals, and reflect deeply on successful practice?

Culture and the Curriculum

During the seventies the present author was commissioned by the MAV to write a position paper arguing the case for mathematics to be retained as a subject in the Victorian curriculum – the alternative push was for it to be absorbed into a broad based structure called General Studies. If that now sounds strange it is interesting to recall that after the Russian revolution of 1917 moves took place to embed discipline areas within the cultural and politically correct ethos of the era.

> In 1923 it was decided to abolish mathematics as a subject. The whole school programme was reorganised around such themes as Man and Nature, Work and Society. Mathematics was to arise naturally in the study of these themes. It did not work out too well; Pythagoras Theorem was embedded in a section dea1ing with the Constitution of the Soviet Union, while fractional and negative indices were under Imperialism and the Struggle of the Working Class. Children brought up under this scheme did not do well … (Sawyer, 1978, p. 260).

While the current situation is less extreme, responses to plans to rework the Australian national curriculum to incorporate selected themes across key subject areas including mathematics, have been generating public debate, and can be informed by such historical precedents. Enhancing the notion of global citizenship (a worthy ideal) includes intentions for children to become "Asia literate", and emphasising Australia's indigenous cultures "as a key part of the nation's history, present and future". However how such intentions stand to be realised have direct implications for the content, approach, and quality of mathematics curricula and pedagogy.

Examples of content descriptions provided by the Asia Education Federation (2014) include the following for mathematics.

- Calculating population growth rates in Australia and Asia and explaining their difference.
- Considering the history and significance of pyramids from a range of cultural perspectives including those structures found in China, Korea and Indonesia.

Regarding the first of these, a news item appeared on April 23 2013 indicating that with a birth every one minute and 44 seconds, a new migrant arriving every two minutes and 19 seconds, and a death every three minutes and 32 seconds, the 23 million mark for the Australian population would be reached just after 10:00 pm that day. Various stated implications followed including the prediction of the nation's population in mid-century.

Several genuine population problems suitable for both junior and senior levels can be generated from these data, including ones showing that some of the data are problematic as a basis for predictions – the death rate data can be shown to imply an average lifespan of about 155 years! (Available current data for Singapore imply an average life span of about 292 years). Birth rate data imply fertility levels (children per female) that also have implications for population predictions, and flow on effects for migration issues. These data raise interesting questions about lifestyle and planning in both Australia and Singapore – and other countries as well. For example: What age profiles are implied by the data? What if fertility rates change? What are implications for education provision, health care? Aged care? What ramifications are there for policies on planned immigration? And so on. These all depend first on the capacity to undertake appropriate mathematical calculations as a pre-requisite – noting that calculating is the initial attribute mentioned. By contrast the second example does not imply that any significant treatment of the mathematics of pyramids will necessarily occur.

The indigenous dimension has different implications, and MERGA is well equipped with people, from both sides of the Tasman, to address these. I recall an exchange in Educational Studies in Mathematics a few years ago focused around ethnomathematical issues, regarding whether, and in what circumstances, culturally specific mathematics advantaged or disadvantaged learners. The exchange involved a contribution from New Zealand authors (Adams, Alanguis & Barton, 2003) in response to an initial argument from authors in the UK (Rowlands & Carson 2002). I recall finding some persuasive arguments on both sides.

What we need to argue is that increased mathematical power for students is the primary goal, and this only ensues if disciplinary mathematics is developed or invoked at some suitable level in some specific ways. This in no way argues against general statements that enhance the value of mathematics in the eyes of particular groups, and promotes a wider understanding of its role in culture. But in responding to cultural pressures from influences that lack authoritative curricular knowledge, organisations such as MERGA need to act as integrity watch dogs.

Given a hammer everything becomes a nail

In reflecting on submissions for publication, and indeed published work, a reader is struck by the increasing number of idiosyncratic schemes that are proposed and used in various forms – for example as theoretical frameworks within which to locate specific research programs. Table 1 summarises the result of a modest amount of time spent skimming through books and journals. A sample of twelve schemes suffices for present purposes – the actual number is many more than this.

Table 1
Schemes and their descriptors

Scheme	Description
AA	Academic Agency
ACODESA	Collaborative learning, Scientific debate, Self-reflection
APOS	Action, Process, Object, Schema
CHAT	Cultural History Activity Theory
DGE	Dynamic Geometry Environment
IMPROVE	Metacognitive Self-questioning method
HCK	Horizon Content Knowledge
MKT	Mathematical Knowledge for Teaching
PUFM	Profound Understanding of Fundamental Mathematics
SOLO*	Structure of Observed Learning Outcomes
SRL	Self Regulated Learning
TRTLE	Technology Rich Teaching and Learning Environment

The above heading is not totally fair as a generalisation, as individuals differ in the extensiveness of claims made on behalf of their pets – from balanced and targeted to outrageously optimistic. Some seem almost to promise a magic bullet, and a question which occurs regularly to this reader when reviewing such claims is the following: What does this approach/scheme/conceptualisation add that represents a compelling addition to existing knowledge? What actually is new?

By way of illustration, applying such criteria to argue on behalf of the SOLO Taxonomy (Biggs & Collis, 1982), might read something like the following. The Taxonomy (which applies to other areas of learning apart from mathematics) has something important to say both theoretically and practically about each of two massive influences on learning theory and practice – Piagetian and Vygotskian psychology. A major contribution within the former was to move the focus from the person to the response. The emphasis is moved from "s/he is a concrete operational thinker" to "that is a multistructural response – how can we help her/him to improve its quality?" A significant consequence being that an individual is not type cast – it is perfectly reasonable to operate at an extended abstract level in some contexts while employing more primitive response types in others – for example in learning within new areas. Don't we all?

Using a Vygotskian lens, having identified a current response type as (e.g. multistructural), we can then use this knowledge to structure a ZPD geared to enabling a higher level facility, such as a response demonstrating relational or extended abstract qualities. Vygotsky says to us that "Teaching really matters" and SOLO speaks to this belief strongly and practically. As further icing on the cake by its very nature the Taxonomy has theoretical contributions to make to the design of questions geared to assess quality of performance in any domain. We will have our own preferences, but for reasons such as these I believe that the SOLO Taxonomy could be advanced as among the most significant contributions to educational theory and practice to emerge from Australasia.

In summary the proliferation of schemes vying for the attention of researchers in all spheres of activity, points to the need to analyse and distil the plethora of claims made on their behalf. One goal in particular would be to identify and classify attributes that represent distinct and unique conceptual properties, with the potential to provide the most potent advances across the various facets of our field – an Occam's razor approach. In such an exercise the disentangling of ontological and epistemological issues looms as a key challenge. Is anyone interested in a theoretical dissertation (meta-analysis) along these lines? Before doing so the Blind Men and the Elephant (e.g., Saxe, 1882), and available from any number of other sources, might be regarded as compulsory reading!

Outrage

In one of his 2013 communications Jerry Becker posted the following from prolific US education author Alf Kohn under the title: *Encouraging Educator Courage* (Kohn, 2013).

> It pains me to say this, but professionals in our field often seem content to work within the constraints of traditional policies and accepted assumptions — even when they don't make sense. Conversely, too many educators seem to have lost their capacity to be outraged by outrageous things. Handed foolish and destructive mandates, they respond only by requesting guidance on how to implement them.

So I thought it would be useful to conclude this paper by reflecting on the theme of *outrage* as suggested by Kohn. Where and from whom can we find statements that indicate outrage at situations identified within Mathematics Education?
Here are some examples that might qualify.

1. From a societal perspective, the school mathematics curriculum is worse than regrettable; it is scandalous. (Burkhardt, 2008, p. 2091).
2. You won't believe what a room full of psychometricians dreams up, thinking that they're testing problem solving, reasoning, and modeling with mathematics. We have a math board that's trying to chip away at that and make sure there's some mathematical integrity to the tests. (Schoenfeld, 2013, p. 9).
3. It is as if mathematics education was a badly performed play. We immediately see that the actors are amateurs, that the script is bad and the directing poor. But we also identify, in this failed performance, a brilliant idea which would no less than change the world, were it only made real by a proper crew. (Lundin, 2012, p. 83).

Burkhardt is clearly outraged – on grounds that he continues to elaborate in his writings. We can sense outrage also in Alan Schoenfeld's restrained and temperate choice of words about the validity and effect of testing regimes. And we may indeed wonder whether some supposed productive moves in this direction really act as glass ceilings, to inhibit more profound changes that are needed. For example the inclusion of allegedly real world contexts among PISA items does not in itself stand to enhance success in the teaching of applications and modelling skills – solving messy problems involves much more than responses to contrived items wearing contextual clothes. Lundin's remarks represent outrage at the whole pantechnicon of Mathematics Education and the way it is structured and implemented. How do we dip our toes in that stream?

On a personal note this author is outraged that students continue to spend nine, twelve, even fifteen years studying mathematics and yet are unable to use their knowledge to address issues in their living environment that simple mathematics would clearly inform.

So in conclusion how do we decide individually and collectively on directions for the future – the where to? – in the title of this paper. Suppose we took our cue from Kohn and wrote down those things about the theory and practice of mathematics education that currently outrage us – and then shared the outrages with each other. What themes would emerge from such an exercise? What imperatives would these suggest? What could we do? Where would it take us?

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