Towards a Fresh Understanding of the Relationship Between Teacher Beliefs about Mathematics and their Classroom Practices

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This study proposes a new theoretical framework that incorporates thematic descriptors to form broad conclusions about teachers' mathematical beliefs and their classroom practices. Narrative accounts provided by beginning primary teachers were used to analyse the relationship between their beliefs and practices through the lens of this newly developed framework. The outcome of this study was a revision to the proposed framework and a challenge to other well-established frameworks that directly correlate teachers' expressed beliefs about the nature of mathematics and their teaching and learning practices.

This study is located in a theoretical context that considers the relationship between teachers' beliefs about the nature of mathematics and their mathematics classroom teaching practices. Teachers' beliefs about the nature of mathematics, per se, can influence their practices in mathematics teaching and over the past twenty years a number of researchers have proposed theoretical models that characterise this relationship (Askew, Brown, Rhodes, Johnson, & William, 1997; Ernest, 1989; Raymond, 1997). The focus of this study was to record and describe the expressed mathematical beliefs of a group of beginning primary teachers and their mathematics classroom teaching practices. The resultant purpose of the study was to determine the relationship between their beliefs and practices, and to analyse this relationship through the lens of a newly developed theoretical framework. Indeed, the significance of this study is that this framework appears to support a fresh understanding of the relationship between teachers' beliefs about mathematics and their classroom practices that is not evident in what might be considered more well-established models.

Literature Review

The study of teacher beliefs has only had a presence on the educational research agenda for the past twenty or so years (Forgasz & Leder, 2008). However, Thompson (1992), and more recently Forgasz and Leder, suggest that in many studies there has been no theoretical discussion of the nature of beliefs. One proposed reason for this scarcity of discussion is that there seems to be no agreed definition of teacher beliefs (Beswick, 2003). Some studies (Beswick, 2007; Cross, 2009) focussing on mathematics teachers' beliefs have used aspects of Green's (1971) belief system framework to understand how beliefs are organised. Green identifies three dimensions that interact with each other within a belief system. The first is that a belief system is comprised of *primary* beliefs and associated *derivative* beliefs. The second dimension is the strength of conviction with which beliefs are held. *Central* beliefs are the most strongly held, and *peripheral* beliefs are those most susceptible to change. The third of Green's dimensions describes how beliefs are held in clusters, in isolation from other sets of beliefs, and hence possibly in contradiction to them. This dimension of Green's framework has significant implications in this study as the clustering of beliefs as described by Green provides for the possibility that teacher beliefs

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about the nature of mathematics may not necessarily correspond to their beliefs about how mathematics ought to be taught and learned.

Many researchers have investigated the relationship between teacher beliefs and classroom practices (Beswick, 2003; Cross, 2009; Nisbet & Warren, 2000). It could be assumed that teacher beliefs would bear a direct impact on classroom practice; however, the relationship is often not a clear-cut one (Warfield, Wood, & Lehman, 2005). Rather, as Nibset and Warren (2000) argue, the relationship between teachers' beliefs and classroom practices is dynamic with each influencing the other. Thompson (1992, p. 140) concurs, asserting that "the research strongly suggests that the relationship between beliefs and practices is dialectic, not a simple cause-and-effect relationship". Raymond's (1997) research establishes that there is a robust, reciprocal relationship between teachers' beliefs and their classroom practices, but this relationship is not always consistent. She argues that teacher beliefs strongly influence classroom practices and more so than how their practices can influence their beliefs. This is confirmed by Fennema et al. (1996) who conclude that teachers can and do make procedural changes in their practices without changing their beliefs. Therefore, whilst teachers' beliefs can influence their classroom practices, a cause-and-effect relationship has not been clearly established.

Frameworks for classifying teachers' views regarding teaching and learning mathematics are generally represented on a continuum from the transmissionist approach where the teacher imparts the mathematical knowledge and students are expected to absorb and reproduce it, to the constructivist approach in which teachers are facilitators of learning and students construct their own mathematical knowledge. A simple model proposed by Perry, Howard, and Tracey (1999) reflects this dichotomy. In expanding this duality, Askew et al. (1997), characterise teachers' beliefs about teaching and learning mathematics as falling into one of three categories. They define these as orientations toward transmission, discovery or connectionist approaches. In their meta-analysis Kuhs and Ball (1986) identify four dominant and distinctive views of how mathematics should be taught: learner-focused; content-focused with an emphasis on conceptual understanding; content-focussed with an emphasis on performance; and classroom-focussed. Whilst these four views of mathematics teaching usefully describe the dominant beliefs held by teachers regarding mathematics teaching and learning, Kuhs and Ball's model does not explicitly link the relationship between teachers' beliefs about the nature of mathematics and their approaches to teaching and learning.

On the other hand, Ernest's (1989) often cited framework does emphasise the significance of this relationship. Ernest has distinguished three distinct beliefs that a teacher may hold with regard to the nature of mathematics that he orders to form a hierarchy: the problem-solving view; the Platonist view; and the instrumentalist view. Ernest (1989) proposes that teachers' beliefs about the nature of mathematics provides a basis for their beliefs about mathematics teaching. According to Ernest the instrumentalist view of mathematics is likely to be associated with a transmission model of teaching and the compliant mastery of skills by students. He makes similar links between the other beliefs about mathematics and models of teaching and learning; for example, "mathematics as problem-solving corresponds to a view of the teacher as facilitator, and learning as autonomous problem posing and problem solving, perhaps also as the active construction of understanding" (Ernest, 1989, p. 26). Finally, Raymond (1997) proposes a framework incorporating five categories of teacher beliefs as being traditional, primarily traditional, an even mix of traditional and non-traditional, primarily non-traditional, and non-traditional. In the Raymond framework, traditional beliefs are aligned with Ernest's instrumentalist beliefs about the nature of mathematics. On the other hand, the non-traditional beliefs are associated with Ernest's problem-solving view of the nature of mathematics.

Proposed Theoretical Framework

The proposed theoretical framework adopted in this study draws, to varying degrees, on other existing theoretical models. Similarly to Ernest (1989) and Raymond (1997), this framework links the relationship between views about the nature of mathematics to views of teaching and learning. Also in line with Ernest, and drawing on Askew (1997), this framework tenders three distinct categories for the nature of mathematics: the instrumentalist view, the connectionist view, and the problem-solving view. However, unlike Ernest this framework is not presented as a hierarchy, but rather a description of the varying nature of teacher beliefs. This framework is underpinned by the notion that teacher beliefs are too diverse to be able to construct just three categories for the views of teaching and learning that link directly with the three broad views of the nature of mathematics. As Thompson (1992, p. 137) explains: "a given teacher's conceptions of mathematics teaching is more likely to include various aspects of several models than it is to fit perfectly into the description of a single model". Thus, this proposed framework (Table 1) includes five categories of teaching and learning practices; three of these categories are explicitly associated with the views about the nature of mathematics and the other two are aligned with possible intersections that may occur with regard to these views.

Table 1
Proposed Theoretical Framework

Mathematics	Teaching	Learning
Instrumentalist	Mastery of skills and facts	Mastery of skills and facts by passive and compliant learners
	Skills and facts are augmented with conceptual understanding	Mastery of skills and facts by learners who are developing some conceptual understanding
Connectionist	Conceptual understanding	The learner constructs their understanding
	Conceptual understanding enriched with investigations	The learner may use investigations to constructs their understanding
Problem-Solving	Understanding built through investigation	The learner autonomously pursues mathematical investigations

Research Method and Procedures

Narrative inquiry was selected as the method best suiting the purpose of this study: to record and describe the beliefs about mathematics expressed by a group of beginning primary teachers and their mathematics classroom teaching practices. One advantage of adopting a narrative approach is that individual voices that are often underrepresented can be heard, respected and legitimised (Smulyan, 2004). A number of previous studies have successfully applied narrative methods to exploring the development of beginning mathematics teachers (Brown, 2005; Kaasila, 2007) and an appreciation of the value of narratives has grown as educational researchers have come to understand how they assist in more meaningfully rendering teachers' experiences. The participants in this study were six beginning primary mathematics teachers (Verity, Hannah, Angela, Donna, Jenny and Sarah) who were mid-way through their second year of teaching and the data were gathered during a single, approximately one hour, narrative interview with each participant.

Findings and Discussion

Beliefs about the Nature of Mathematics

In this study, four of the participants, Verity, Hannah, Angela and Donna, explicitly embraced the utilitarian nature of mathematics, articulating the view that mathematics is a practical phenomenon that permeates many aspects of everyday life. As Verity explained, "It is something that is practical [and] something we need to use everyday". Hannah had a similar view: "I like to see maths as something that [we] do use in everyday life". Sarah and Jenny, on the other hand, espoused the view that, at primary school, mathematics is actually numeracy, though neither indicated how they perceived any differences between numeracy and mathematics. There are, however, well-cited definitions of numeracy which may be useful in pinpointing Sarah and Jenny's view of the nature of mathematics. These definitions include that numeracy involves using some mathematics to achieve some purpose in a particular context (AAMT, 1997) and that it involves the ability to apply number skills to everyday purposes, together with the ability to interpret quantitative information in everyday life (Westwood, 2008). Given these definitions of numeracy, it would therefore be reasonable to surmise that both Jenny and Sarah also espoused the practical, everyday-use nature of mathematics. Thus, the participants in this study all appeared to embrace the utilitarian nature of mathematics.

In espousing the belief that mathematics is a pragmatic tool to be used for everyday purposes, all of the participants ought to be aligned most closely to the instrumentalist view of mathematics in the proposed theoretical framework. This is the view that mathematics is the accumulation of a set of facts, rules and skills to be used for some external purpose. Theorists such as Ernest (1989), would argue that if a teacher holds such a view, then their classroom practices would most likely be associated with the mastery of skills and facts by passive and compliant learners which aligns coherently with the instrumentalist view in the proposed framework. However, when the lens of this framework was applied to each of the participants' accounts this suggested alignment was not evident.

Mathematics Teaching and Learning

Angela, Jenny, Verity. For the most part, Angela saw that her role was to facilitate and guide her students' learning: "It is about knowing where they're at and starting where they're at and then building on that …" She was inclined to value process over product in mathematical learning, observing that students "don't always have the process that you would use [but] they do come up with different answers that are still right". In addition to this, Angela also encouraged mainly student-centred classroom interactions and provided an environment where her students were primarily active learners working with other students. She stressed that, "I do believe ... that kids work better when they're doing social stuff together when they're learning". Additionally, Angela encouraged the learners in her class to take a greater degree of responsibility for their learning. She believed that learners "build knowledge much more when they have the opportunity to do it at their own pace and actually explore what it means".

Whilst Jenny also saw her role primarily to facilitate, she did find that explicit teaching was still occasionally required: "it's nice to get the kids to get it themselves but then [for] some things ... they need that explicit instruction". Jenny, too, encouraged mainly student-centred interactions in her mathematics classroom, explaining that: "I try to get the kids doing working together, sharing. We always try to do mapping of what we know before we start ... what to talk about in the discussion and then going away and finding things out". Additionally, she emphasised understanding over repetition because learners "just don't get anything if they don't get to the meaning themselves". Jenny was committed to providing learning activities that involve "hands-on, manipulatives [and] meaning-making". Therefore, students in Jenny's class were actively learning mainly through investigation activities. As a result, she observed: "My classroom's always a mess and I know that ... but I think they should be out there moving around, doing different things".

Verity also embraced a role that principally involves facilitating and guiding her students' learning: "I guess I'm a believer in making sure children start from where they're at and move them on from there and just try and provide learning experiences where they are able to do that". Like Angela, Verity was also inclined to value process over product; hence, she observed, "if you looked in my children's maths books … a lot would look like gobbledy-gook". Verity's students were also mainly engaged in student-centred interactions, primarily learning mathematics by working with other students through "peer tutoring and partner work and group work". Investigations were an integral part of mathematics lessons in Verity's classroom where she provided "some really good guiding questions that they can explore". It was clearly evident the learners in Verity's class engaged with mathematics primarily as active learners when she observed, "I cannot think of a maths lesson I would [teach] without manipulatives".

Thus, using the proposed theoretical framework, it is evident that Angela, Jenny and Verity's expressed classroom practices all seem to indicate a view of teaching that is aligned with building a *conceptual understanding enriched with investigations*. Additionally, it would appear that they embrace the approach that *learners may use investigations to construct their understanding*.

Hannah, Sarah, Donna. Hannah encouraged teacher-centred and student-centred interaction, firstly assuming the teacher-as-explainer role: "[I] talk through it with them of what to do and put step by step", and then using collaborative discussions to clarify procedural and conceptual understandings: "then it's up to the class to do it, working together". She also provided her students with a variety of activities and tasks, and valued both product and process in mathematical learning. The learners in Hannah's class appreciated that there was more than one way to learn mathematics and they took responsibility, with the teacher, for their learning. She explained, "[I] let them assess their own work, [to find] what they think they need help with and what they think they do really well". Hannah's students were engaged in both individual and group work: "usually in small groups, like two or three, to work together on a group task ... and then some individual work". Additionally, it seemed that the learners in Hannah's class both understood and mastered skills and were able to demonstrate this when they "take turns coming up and putting [their work] on the board themselves to show that they know what they are doing".

Sarah, similarly, assumed the teacher-as-explainer role, stressing that: "direct instruction provides [students] with awareness and empowerment and that self-determination that maybe facilitation doesn't". She did, however, also use collaborative discussion with her students to clarify procedural or conceptual understandings. Sarah valued equally both product and process in "an environment where the answer ... isn't an answer that is either right or wrong, that acknowledges observations and recordings and stories". Additionally, she promoted mathematical learning through both repetition and understanding because "often it really gets down to the old boring and basic repetition". Sarah's belief about learners was that they ought to understand skills, for example, by "seeing the strategies they're using"; and master skills, by "know[ing] their number facts automatically". She believed there is more than one way to learn mathematics, and asserted that "a safe learning environment in maths is one that acknowledges different strategies".

In Donna's class, the learners were engaged in both individual and group work and took an equal responsibility with the teacher for their learning. As a primary mathematics teacher, Donna assumed the teacher-as-explainer role in conjunction with the use of collaborative discussion to clarify understandings. "We use a lot of explicit teaching when we start a topic and work a lot on the floor together to start with, then usually disperse into groups". Donna incorporated a variety of mathematical tasks, and seemingly valued both product and processes: "[I make] sure that their book work is structured really well ... when you're doing maths you need to just do it down one side of the page usually and then go to the other side ... [otherwise] you can get muddled up". Additionally, Donna created an environment where the students are more active learners because "you want them to learn in a different way rather than writing down things all the time and just doing question after question, so I try to find different things for them to do". Donna's class encountered mathematics through traditional texts: "I think kids enjoy mental maths", and through more investigative approaches such as the use of "manipulatives like clocks and measuring; we do lots of activities around those sorts of things".

Thus using the proposed theoretical framework, Hannah, Sarah and Donna's expressed classroom practices appear to indicate a view of teaching that aligns with building a *conceptual understanding*. In addition to this, they appear to embrace the view that the *learner constructs their understanding of a meaningful body of knowledge*.

Revised Theoretical Framework

Whilst all of the beginning teachers embraced the instrumentalist nature of mathematics, the accounts of their classroom practices were more closely aligned to less traditional models for mathematics teaching and learning. Thus, it appears that there is no direct correlation between their view of mathematics and their expressed teaching and learning practices as has been proposed in the theoretical framework. This finding is contrary to Ernest's (1989) suggestion that a teachers' views regarding the nature of mathematics will directly inform their mathematics teaching and learning practices. Other studies have also reported a high degree of correspondence between teachers' beliefs about the nature of mathematics and their classroom practices (Cross, 2009; Nisbet & Warren, 2000). Thompson (1992), on the other hand, notes that there can be varying degrees of consistency between teachers' expressed beliefs about the nature of mathematics and their teaching practices.

Thus, this study highlights an apparent lack of direct correlation between teachers' more traditional beliefs about the nature of mathematics and their less traditional classroom practices. If this is the case then a theoretical framework such as the one originally proposed for this study and informed by the work of others that directly links teachers' views regarding the nature of mathematics to their views of mathematics teaching and learning does not appear to realistically represent the range of teachers' mathematically related views. Hence, on the basis of the finding of this study, a revised theoretical framework (Table 2) is presented. In the revised theoretical framework the views of mathematics have been disengaged from the views of mathematics teaching and learning components as there appears to be insufficient evidence that a direct correspondence between these two aspects exist.

Table 2
Revised Theoretical Framework

Mathematics	Teaching	Learning
Instrumentalist	Mastery of skills and facts	Mastery of skills and facts by passive and compliant learners
	Skills and facts are augmented with conceptual understanding	Mastery of skills and facts by learners who are developing some conceptual understanding
Connectionist	Conceptual understanding	The learner constructs their understanding
	Conceptual understanding enriched with investigations	The learner may use investigations to constructs their understanding
Problem-Solving	Understanding built through investigation	The learner autonomously pursues mathematical investigations

The strength and significance of the revised theoretical framework is the manner in which it is reflective of Green's (1971) seminal belief systems framework. The third of Green's framework dimensions describes how beliefs are held in clusters, in isolation from other sets of beliefs, and hence possibly in contradiction to them. This dimension of Green's framework is clearly evident in this revised theoretical framework. Whilst others (Ernest, 1989; Raymond, 1997; Thompson, 1992) have described how teachers' beliefs about the nature of mathematics per se can directly inform their practices in mathematics teaching and learning, the clustering of beliefs as described by Green provides for the possibility that teacher beliefs about the nature of mathematics do not necessarily correspond to their beliefs about how mathematics ought to be taught and learned.

Conclusion

Belief theorists argue that if a teacher embraces an instrumentalist view of mathematics their classroom practices will most likely be associated with the mastery of skills and facts by passive and compliant learners as per the proposed theoretical framework originally adopted in this study. However, in examining the participants' accounts through the lens of this framework, it was evident that such a correlation did not exist, based on teachers' accounts of their beliefs and classroom practices. The findings appear to support the argument for varying degrees of correlation between teachers' expressed views about the nature of mathematics and their teaching practices. If this is the case then the proposed theoretical framework, as informed by the work of others that directly links teachers' views regarding mathematics to their classroom practices, is not realistically representative. Hence, on this basis a revised theoretical framework has been presented. In the revised theoretical framework the beliefs about mathematics have been disengaged from the teaching and learning components as, based on this study, there is insufficient evidence for a direct correspondence between these two aspects. However, there is a need for circumspection when offering the revised theoretical framework as the one limitation of this study was the small number of research participants involved. Nevertheless, this revised framework will provide a sound starting point for further investigations related to teachers' mathematical beliefs and classroom practices.

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