Exploring relationship between scientific reasoning skills and mathematics problem solving

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Reasoning is considered to be an important proficiency in national mathematics curricula both in Australia (ACARA, 2014) and Malaysia (MOE, 2013). However, the nature of reasoning that supports learning and problem solving in mathematics is an area that requires further study (Schoenfeld, 2013). In this study we explored the link between *Scientific Reasoning Skills* (SRS) and mathematics problem solving performance among a cohort of Malaysian students. As expected, there was a positive relationship but the level of correlation between these two variables was moderate. Although the High-Achievement group performed significantly better than their peers in the Low-Achievement group in their solution outcomes, overall, all students exhibited low-levels of SRS. These findings suggest that while SRS could play a role in problem solving, components of SRS need further analysis in order to better explain how reasoning in science could facilitate problem solving processes.

Introduction

With increasing attention to logical arguments and justifications in mathematics, the study of reasoning that underpins these processes constitutes an important area of research (Schoenfeld, 2013; Santos-Trigo & Moreno-Armella, 2013). In this study, we focussed on reasoning that is associated with the development of arguments and justifications in the context of problem solving. Data on reasoning and how that reasoning is used by students during the course of their solution search was expected to deepen current understandings about the construction of arguments and generation of justifications by learners. In this study, we generate data relevant to this issue by analysing scientific reasoning processes that students could activate during their mathematical problem-solving attempts.

Literature review

Reasoning in Mathematics

Current research emphasises the importance of students engaging in reasoning in all strands of school mathematics (National Council of Teachers of Mathematics, 2009; Bieda *et al*., 2013; Santos-Trigo & Moreno-Armella, 2013; Stylianides *et al*., 2008). Ball and Bass (2003, p. 42) argued that 'mathematical reasoning is inseparable from knowing mathematics with understanding.' Several scholars have elaborated on the connection between learning mathematics with understanding and reasoning. Lakatos (1976) noted that complete mathematical understanding includes the engaging processes of thinking, in essence doing what makers and users of mathematics do: framing and solving problems, patterns recognition, making conjectures, examining constraints, making inferences from data, abstracting, inventing, explaining, justifying, challenging, and so on. This observation about understanding and reasoning was recently reaffirmed by Schoenfeld (2013) when he commented that one variable seemed to have strongest impact on students learning - the amount of time students spent in explaining and justifying their ideas. While definitions used in the context reasoning differ, they deal in one way or another with a broad range of

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thinking skills involving arguments that drive the evolution of solutions to mathematical problems. In this sense, there are aspects of mathematical problem solving that could draw on scientific reasoning (Hand, Prain & Yore, 2001).

Scientific Reasoning Skills (SRS)

The range of SRS that students bring to learning and problem solving can be expected to assist them in making progress in multiple ways. SRS 'encompasses the reasoning and problem-solving skills involved in generating, testing and revising hypotheses or theories, and in the case of fully developed skills, reflecting on the process of knowledge acquisition and knowledge change that result from such inquiry activities' (Morris, Croker, Masnick & Zimmerman, 2012, p. 65). Scientific reasoning differs from other skills in that it requires additional cognitive resources as well as an integration of cultural tools. Further, scientific reasoning emerges from the interaction between internal factors (e.g., cognitive and metacognitive development) and cultural and contextual factors. According to Lawson (2004), scientific reasoning pattern is defined as a mental strategy, plan, or rule used to process information and derive conclusions that go beyond direct experience. In a similar vein, Hand, Prain and Yore (2001) argued that scientific reasoning involves the ability to construct powerful arguments for learners' actions. Thus, SRS is related to cognitive abilities such as critical thinking and reasoning that assist students in producing knowledge during problem solving through evidence–based reasoning. Given the connectedness between knowledge generation via arguments and reasoning that gird these arguments, students with higher levels SRS could be expected to be superior problem solvers.

Conceptual framework

Lawson's (2000) Classroom Test of Scientific Reasoning provided the framework to guide the analysis and interpretation of data in the present study.

Lawson's Classroom Test of Scientific Reasoning

In this study, we focus on a set of domain-general reasoning skills that are commonly needed for students to make progress with mathematical problems which includes exploring a problem, formulating arguments, manipulating and isolating variables, and observing and evaluating the production of new information. Lawson's Classroom Test of Scientific Reasoning (LCTSR) provides a theoretical lens for assessing a range of SRSs. The test was designed to examine a set of general reasoning ability dimensions which are crucial for the solution of problems in STEM including conservation of matter and volume, proportional reasoning, control of variables, probability reasoning, correlation reasoning, and hypothetical-deductive reasoning. The validity of the LCTSR had been established by several studies (e.g. Lawson, Bank & Lovgin, 2007).

LCTSR allows for the observation of three levels of reasoning: *concrete, transitional* and *formal* operational reasoning. The *concrete* operational reasoning refers to thinking pattern that enable one to understand concepts and statements that make a direct reference to familiar actions and observable objects, and can be explained in terms of simple associations (for example, all squares are rectangles but not all rectangles are squares). Students, at this level of reasoning, are also able to follow step-by-step instructions provided each step is completely specified (for example, solving two linear equations). Students are also able to relate his/her viewpoint to that of another in familiar situations (for example, students respond to difficult mathematical problems by applying a related correct rule). At this stage, students are unconscious of his/her own reasoning patterns, inconsistencies among various statements he/she makes, or contradictions with other known facts.

In contrast to *concrete* reasoning, *formal* reasoning patterns enable students to construct possible explanations as a starting point for reasoning about a causal situation. They can reason in a deductive manner to test their hypotheses. In other words, they can postulate causal factors, deduce the consequences of these possibilities and then empirically verify which of those consequences, in fact, occurs. Lawson (1978) categorised students at this stage as 'reflective thinkers'. For example, in solving mathematical problem, students' reasoning can be initiated with development of representations, use of symbols and planning a course of action.

The *transitional* operational stage is where students remain confined to *concrete* thinking or are only capable of partial *formal* reasoning. For example, proportional reasoning is the ability to compare ratios or develop arguments about the equality between two ratios. At *concrete* operational stage, students are not aware of ratio dependence and seek solutions by guessing. At the *transitional* stage, students are aware of objective dependence and seek solutions by estimation and later calculation, but assume that the change in one quantity produces the same change in the other quantity. In the *formal* stage, proportionality is discovered and applied to obtain correct solutions. Clearly, in all three levels of reasoning, students generate qualitatively different types of information that are driven by arguments and justifications.

Purpose of the Study

The review of literature indicates that reasoning skills are transferable across science and mathematics (Hand, Prain & Yore, 2001), and that we could learn a great deal about the role of reasoning in mathematical problem solving by examining potential links between the two (Lehrer & Schauble, 2000). The purpose of the study was to identify the levels of SRS attained by a cohort of upper Malaysian secondary school students (16-17-year-olds) and examine the impact of SRS on their mathematical problem-solving performance. We sought data relevant to the following three research questions:

1. What are the levels of SRS among upper secondary school students?
2. Is there a relationship between SRS and mathematics problem-solving performance?
3. Does achievement level of students affect their SRS and mathematics problem-solving performance?

Methodology

Design

This study employed a blend of descriptive and correlational research design as our interest was to generate information about the relationship between one independent (Achievement level) and two dependant variables (SRS and Mathematics Problem Solving).

Participants

A total of 351 students from 14 Malaysian secondary schools participated in the present study. Participants in this study were upper secondary school students or Year 11 students (16-17-year-olds). Participants were assigned to High or Low achievement groups on the basis of their performance in a centralised Malaysian examination, Lower Secondary Evaluation Examination (LSEE). The High-achievement group (Grades A or B in LSEE) comprised Science stream students (n = 98) and the Low–achievement group (Grades C or D in LSEE) were Non-Science stream students (n = 253). This is based on the Malaysian Education Evaluation System in placing students into Science and Non-Science streams. Under this system, students needed to score high marks in mathematics in order to go into the Science Stream in comparison to their peers in the Non-Science stream.

Tasks

This section provides details of two tasks that were used in this study. As discussed earlier, there were two tests used in order to generate scores for two dependent variables: Test 1 - Scientific Reasoning Test (SRT); Test 2 - Mathematics Problem-solving Test (MPST).

Task 1 - Scientific Reasoning Test (SRT)

The SRT was used to measure the students' level of SRS. It has been adapted from LCTSR (Lawson, 2000). We wanted to determine the internal consistency of the items which involved the generation of inter-item correlation matrix and computing of Kuder-Richardson 20 internal-consistency reliability coefficient. The final test had a KR-20 coefficient of 0.856. The test consisted of 12 paired items and was designed in a 'two-stage' multiple-choice format to illustrate problem scenarios. With each scenario, the first question focuses on the scenario content, while the second question asks for reason as to why the first answer is correct. Each answer for the first question has a corresponding reason in the second question.

For example, in one of the tasks, students' reasoning about conservation of volume was evaluated. Firstly, students have to think based on their experience or previous knowledge about where the water will rise to when the glass marble is put into cylinder. Then, students have to justify as to why the water rose at that level. This involves students applying the conservation reasoning to perceptible objects and properties. Making prediction and giving explicit explanation are important to successful completion of this item. Prediction, explanation and the generation of relevant new information are important processes of mathematical problem solving. Thus, we argue that these reasoning skills would contribute to the solution outcome of mathematical problems.

Scoring Rubric for SRT

The range of scores of SRS level is 0-12 which decomposes into three levels as suggested by Lawson: 0-4 (Concrete); 5-8 (Transitional); 9-12 (Formal).

Task 2 - Mathematics Problem Solving Test (MPST)

The MPST was designed to measure students' mathematics problem-solving performance by drawing on SRSs. The test was prepared by a panel of experienced mathematics educators, experienced teachers and mathematics curriculum experts. We were concerned to ensure that the solution of the problems necessitated the activation of one or more levels reasoning that was postulated in the framework of SRS. The items for the test were selected from a pool of resources such as textbooks, reference books and examination papers. The test consisted of 40 items that covered all core mathematical strands in the Malaysian Mathematics Syllabus (Year 8 – Year 11). The reliability of the test was established by following a process that was similar to SRT. The reliability index for MPST was 0.895.

For example, Item 18 required students to work out the perimeter of an irregular shape that was located within a rectangle. The solution required students to generate arguments about different ways to determine the perimeter and test their hypothesis. Students were categorised as having concrete reasoning level if they needed reference to familiar actions, objects, and descriptive properties. At this level, their reasoning was initiated with observations and step-by-step moves. For Item 18, students may only show a superficial understanding of concepts of perimeter, area of a rectangle and a circle without any way linking these to solving the problem. Students were categorised as having formal reasoning level if they can be initiated with possibilities, used symbols to express ideas, planned a lengthy procedure given the overall goal while being critical of his/her own reasoning patterns. In this case, students may systematically plan to find perimeter of the irregular shaded region. This will involve finding the curve length of a semicircle and a quadrant using the formulae for the area and using the given information of the radius length. Students were categorised as having a *transitional* reasoning level if they remained confined to concrete thinking or are only capable of partial formal reasoning such as they only understood and applied concepts of perimeter and area of a rectangle and area of a circle in a new context. Students responses for MPST were scored as 1- correct response; 0- incorrect response.

Procedures

There were three phases in the study. The first phase was concerned with the development and fine-tuning of MPST. The details are explained in the MPST task section. During the second phase, we pilot tested both the tests to allow for familiarisation of the data collection processes, to validate the instruments and to establish their reliability. The third phase involved the administration of the two tests. Both tests were administered during regular mathematics classes. Researchers and classroom teachers assisted with the administration of the tests. Students were invited to complete the SRT in the first week of their regular mathematics lesson. They were allowed a maximum of 40 minutes for SRT. The MPST, a one-hour paper and pencil test, was administered in the second week, again, during their regular mathematics lesson.

Results and Analysis

Three research questions were of interest to the present study. Data relevant to these research questions are presented below.

Research Question 1: What are the levels of SRS among Upper Secondary School Students?

Table 1 shows the overall level of SRS exhibited by the participating Year 11 students. The findings showed that 330 (94%) of the students achieved Level 1 (*concrete*) of SRS, 20 (5.7%) Level 2 (*transitional*) and 1 (0.3%) Level 3 (*formal*). The overall mean level of the SRS was 1.76. This indicates that majority of the participating students were functioning at the *concrete* level of SRS.

Table 1
Level of SRS

SRS Level	N	Percentage(%)	SRS Mean Score	Standard Deviation
Concrete	330	94.0	1.50	1.18
Transitional	20	5.70	5.65	0.81
Formal	1	0.30	9.00	-
Total	351	100.00	1.76	1.55

Research Question 2: Is there a Relationship between the SRS and Mathematics Problem Solving Performance?

Overall, the correlation between the SRS and mathematics problem solving performance was significant indicating a positive relationship between the two variables [(r = 0.593), p<0.05]. The coefficient of correlation (r = 0.593) indicating that there was a moderate positive relationship between the SRS and mathematics problem solving performance .This suggests that if a student had a high score in SRS, he/she are expected to achieve a high score in MPST.

Research Question 3: Does Achievement Level of Students Effect their SRS and Mathematics Problem-Solving Performance?

A t-test analysis was conducted to compare the mean scores of the overall level of SRS for the two Achievement levels (High/Low). Analysis as presented in Table 2 showed there were differences in mean overall SRS between High and Low-Achievement groups [t (349) = 9.260, p < 0.05]. The mean SRS level for High-Achievement group (mean = 2.99) was better than the corresponding score for peers in the Low-Achievement group (mean = 1.28). However, SRS score for both groups of students was 1.76 (Table 1) suggesting the students had acquired concrete reasoning level.

The total scores for MPST were converted to percentages. Mean percentages for the High-Achievement group and the Low Achievement group were 81.02 and 46.86 respectively (Table 2). The results also showed there were differences in mean MPST percentages between the High and Low-Achievement groups [t(349) = 16.789, p < 0.05]. The mean MPST percentage score for the High-Achievement group was higher than the Low-Achievement group. Taken together, students in the High-Achievement group produced higher scores for SRS and MPST than their peers in the Low-Achievement group.

Table 2
SRS score and MT score Vs Achievement Group

Dependent Variable		Achievement Group		t-value	p-value
		High (n= 98)	Low (n=253)		
SRT score	Mean	2.99	1.28	9.260	**
	SD	1.66	1.21		
MPST score	Mean	81.02	46.86	16.789	**
	SD	13.42	18.32		

**p<0.01

Discussion

The study was designed to generate data relevant to issues about relation between the level of SRSs and mathematics problem solving among a cohort of Malaysian students. The first research question addressed the level of SRS among upper secondary school students. We found participants to have acquired moderate levels of SRS. Almost all the students (94.0%) were in the concrete reasoning level and others were in the transitional (5.7%) and formal (0.3%) reasoning levels. The second research question addressed the relationship between SRS and mathematics problem solving. The results indicate that there was a moderate positive correlation between the SRS and mathematics problem solving ability as measured by the MPST. Data analysis relevant to Research Question 3 showed that students in the High-Achievement group performed significantly better than their Low-Achievement peers in both the MPST and SRS. Given the positive correlation between SRS and MPST, it was expected that the higher MPST scores of particularly the High-Achievement group can be attributed to their superior SRS. However, the SRS scores for all the students including the High-Achievement group was relatively low suggesting they were operating at concrete level.

Interestingly, the higher SRS scores for the High-Achievement group (in comparison to the Low-Achieving Group) is still low in terms of the overall SRS level that they had achieved. The mean SRS score for this group was 2.99 which falls well into the concrete reasoning level. However, as shown in Table 2, despite relatively low SRT scores for the High-Achievement group, the score on MPST for this group was significantly higher (mean = 81.02) in comparison to the Low-Achievement group.

We offer two possible explanations for the above pattern of results. Firstly, it might be that students in High-Achievement group were using reasoning and information generating strategies that do not involve the use of SRSs, a claim that needs further investigation. A second possibility is that concrete level SRS may be sufficient for the solution of the type problems that were provided in our MPST. If the latter is indeed the case, the suggestion is that we may have to develop more complex and sensitive problems in order to constrain students to activate transitional and formal levels of scientific reasoning during their solution attempts. In our next phase of this study, we are planning to pursue this hypothesis.

The scores for SRS and problem-solving for students in the Low-Achievement group were low in comparison to their high-achieving peers. In the absence of further data about how SRSs could foster problem solving in mathematics, it is too early to argue that low-achieving students could benefit from instructions to improve their SRSs. We also suggest that the scoring of SRS and MPST needs fine-tuning in order to make the comparison more sensitive to the skills underpinning the two variables.

In our analysis of level of SRS and mathematical problem solving, we did not consider the cultural context of the participating students. The students in this study had three types of linguistic backgrounds - Malay, Mandarin and Tamil. It would be interesting to explore the link between students' linguistic background, scientific reasoning skills and mathematical problem solving outcomes. In the present study, we drew on Lawson's work concerning the three levels of scientific reasoning skills on the assumption that these levels would be sufficient in order to capture the multitude of reasoning that could be activated during novel mathematical problem solving. As mentioned above, although all students were operating at Level 1 of SRS, their performance in MPST, particularly for the High-Achievement group, was high. It would seem that students were indeed engaging in substantial reasoning when they completed the MPST tasks.

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