Comparing the Development of Australian and German 7-Year-Old and 8-Year-Old's Counting and Whole Number Learning

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This paper compares the counting and whole number knowledge and skills of primary school children in Australia and Germany at the end of Grade 1 and Grade 2. Children's learning was assessed using the Early Numeracy Interview and associated Growth Point Framework. The findings highlight substantial differences between the two groups that vary for the four whole number content domains that have been investigated. These variations are likely due to different curriculum emphases in the two countries.

Introduction

Understanding the mathematical knowledge and capabilities of young children is essential for designing high quality curriculum and teaching methods that enable all children to thrive mathematically at school. Many studies demonstrate that young children learn and use informal mathematical ideas as part of their everyday lives, but countries differ in how they approach more formal whole number learning with young children. A key question is whether these differences matter. Selter, Walther, Wessel, and Wendt (2012) found that at the end of Grade 4 in Australia there were more children in the lowest (9.7% vs. 5.8 %) and highest mathematical competence levels (9.8% vs. 5.2%) than in Germany, while the mean scores in arithmetic (and overall) did not significantly differ between the two countries (p. 103). This suggests that while overall the outcomes of mathematics education in Australia and Germany may be quite similar, the experience and outcomes for children at the higher and lower ends of the competency spectrum may be quite different. In order to explore further the impact of different curriculum and teaching approaches on children's whole number learning, the authors compared the development of 7 to 8-year-old Australian and German children. The children's whole number learning was measured using the task-based *Early Numeracy Interview* and associated Growth Point Framework (Clarke et al., 2002) that was first developed in Australia and then translated into German (Peter-Koop, Wollring, Spindeler & Grüßing, 2007).

Gaining Insights About Young Children's Mathematics Knowledge Using the Early Numeracy Interview

It is well established that teachers need access to high quality information about their students' mathematical knowledge in order to plan effective instruction and to monitor their progress. It is also known that formal written tests are limiting in providing this information about young children as they do not provide information about the strategies that children choose and apply when solving computation problems. For these reasons, the *Early Numeracy Interview* (Clarke et al., 2002; Peter-Koop et al., 2007) was designed especially for young children, is task-based and interactive, derived from extensive research, and enables young children's mathematical learning to be measured in multiple domains. This assessment instrument was originally developed as part of the Early Numeracy Research Project (ENRP) (Clarke et al., 2002; Department of Education,

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Employment and Training, 2001). The principles underpinning the construction of tasks and the associated mathematics Growth Point Framework were to:

- describe the development of mathematical knowledge and understanding in the first three years of school in a form and language that was useful for teachers;
- reflect the findings of relevant international and local research in mathematics (e.g., Fuson, 1992; Gould, 2000; Mulligan, 1998; Steffe, von Glasersfeld, Richards, & Cobb, 1983; Wright, Martland, & Stafford, 2000);
- reflect, where possible, the structure of mathematics;
- allow the mathematical knowledge of individuals and groups to be described; and
- enable a consideration of children who may be mathematically vulnerable (Gervasoni & Lindenskov, 2011; Peter-Koop & Grüßing, 2014).

The development of the interview and Growth Point Framework has been widely reported and is explained in detail in Clarke et al. (2002). The assessment includes four whole number domains (Counting, Place Value, Addition and Subtraction Strategies, and Multiplication and Division Strategies), three measurement domains (Time, Length, and Mass); and two geometry domains (Properties of Shape and Visualisation). Children's growth point data for the four whole number domains are explored in this paper.

The Australian and German Primary School Systems

Children begin school in Australia as a whole cohort in February, after the summer holidays (typical ages are from 4 years 6 months – 5 years 6 months). Australian children are encouraged to complete 15 hours of pre-school in the year before they begin school. This is subsidised by the government. Formal mathematics education begins when children begin school.

In Germany children begin school at 6 years as a whole cohort at the start of the school year in August and after the summer holidays. Most children (over 90%) attend kindergarten prior to school enrolment for at least one year, but more typically for 3 years (between the ages of 3 and 6). Kindergarten education does not follow a mathematics curriculum and is not compulsory. However, kindergarten curricula increasingly acknowledge the importance of early numeracy learning for later success in school mathematics and most kindergarten children would experience activities that involve counting, cardinal, and ordinal numbers as they evolve in every-day situations and in their play. In some cases there is even early support with respect to their mathematics learning prior to school.

Whole Number Learning in Australian and German Mathematics Curricula

The primary school mathematics curriculum in Australia is set by each State and Territory, but follows the framework provided by the Federal Government in consultation with the States. The Australian Curriculum: Mathematics (ACARA, 2013) focuses on the domains of number and algebra, geometry and measurement, and probability and statistics. The curriculum also incorporates four proficiencies: understanding, fluency, problem solving, and reasoning. There is a variety of textbooks used in primary schools, but it is also common for teachers not to use a textbook at all, but rather devise their own tasks or draw on a variety of resources, including textbooks.

Like Australia, the German mathematics curriculum is set by each State following the "National Standards" (KMK, 2005); that is, the curriculum guidelines agreed to by all States. While there is a clear focus on arithmetic in Grades 1 and 2, other content areas include space and shape, measurement, pattern and structure as well as chance and data. Like Australia, the National Standards and respectively the state-based curricula incorporate cross-content proficiencies: communicating and reasoning, problem solving and modelling as well as using representations. In Germany the vast majority of primary mathematics teachers would use one of the major textbooks available for each grade level.

Curricula and Approaches for Teaching Whole Number Concepts and Arithmetic

Teachers in Australia use a variety of teaching approaches for whole number concepts and arithmetic. One common approach is using problems connected to everyday experiences. It is also common for teachers to encourage the use of manipulatives and pictures for modelling a problem to assist children to find a solution. The use of tokens, blocks, and counting frames are customary. Children are encouraged to work in pairs or small groups to discuss their strategies and solutions. Many teachers use a framework, such as the ENRP Growth Point Framework, to evaluate the development of children's whole number learning and arithmetic strategies, and plan experiences that enable children to replace counting-based arithmetic strategies with basic and derived strategies such as building to ten, doubles and commutativity. Initially children work with whole numbers in the range of 1-20 and then expand to increasingly greater number ranges. At this point Multi-base Arithmetic Blocks (MAB) are often used to model the problems and support children's calculation strategies. The Grade 1 Australian Curriculum Mathematics emphasises counting to 100 by 1s, 2s, 5s and 10s, building concepts for numbers to 100, using partitioning to count collections to 100, and representing and solving addition and subtraction problems using a range of strategies including counting-on and partitioning. In Grade 2 the curriculum emphasises investigating number sequences from any starting point, building concepts for numbers to 1000, arranging collections up to 1000 in hundreds, tens and ones to facilitate efficient counting, solving simple addition and subtraction problems using a range of efficient mental and written strategies, recognising and representing multiplication as repeated addition, groups and arrays, and division as grouping into equal sets, and solving simple problems using these representations.

The vast majority of German primary mathematics teachers use a mathematics textbook. In Grade 1 the focus is on whole number arithmetic with numbers up to 20. Counting activities, comparing sets, getting to know and learning to write the numerals from 0 to 9 as well as matching numerals to sets is the focus of the first 4 to 5 months of school. After that, firstly addition and then subtraction is introduced with the aim to help children understand the underlying concepts and to increasingly develop and use heuristic strategies based on derived-facts to replace initial counting-based arithmetic strategies. In most classrooms manipulatives such as the arithmetic rack would be used to model addition and subtraction strategies based on derived facts. In Grade 2 the focus is on addition and subtraction strategies with 2-digit numbers as well as the introduction of multiplicative concepts. This includes understanding and automatising the multiplication facts, as well as understanding division as the counterpart of multiplication and associated with distribution and sharing. Children are invited to share their computation strategies with a partner or small group and discuss multiple strategies for how to solve problems such as 57 – 29. These strategies are also discussed in class and applied to similar problems, emphasising advantages and disadvantages of these different strategies. With respect to the multiplication facts, while teachers would adopt/use different strategies to introduce these (either by tables or by taking a rather holistic discovery based approach), they would spend extensive time and effort on the automisation process.

Methodology

In order to compare the whole number learning of the Australian and German children, *Early Numeracy Interview* data were compared for children who had completed Grade 1 and Grade 2 and who were present for both interviews. This was after the second and third year at school for the Australians and at the end of the first and second years at school for the German children. The 637 Australian children attended school in the States of Victoria and Western Australia, attended schools in low SES communities, were present for both interviews, and were assessed in 2010 and 2011 after the summer holidays (at the beginning of the school year), as was the standard practice. It is noted that, after the summer holidays, it is typical for some children, who may have learnt procedurally, to have some lower growth points than at the end of the school year, but also for some children to reach higher growth points. The 334 German students attended schools in a region in the northwest of Germany and included children from low SES communities to suburbs with predominantly middle class families. The children were assessed before the summer holidays (at the end of Grade 1) in 2013. The Australian students were part of the *Bridging the Numeracy Gap in Low SES and Aboriginal Communities* longitudinal study (Gervasoni, Parish & Upton et al., 2010), and the German students were part of a longitudinal study on children's mathematical development from one year prior to school until the end of Grade 2 (first results of this study are reported in Peter-Koop & Kollhoff, 2015). We do not claim that the two cohorts are closely matched due to the different countries, cultural backgrounds, school starting ages, and curricula. Rather, the selection of participants is pragmatic for enabling the research questions to be investigated. Generalisability of results is not claimed. Importantly, children in both cohorts were about the same age, were assessed using the same instrument, and the results were analysed using the same Growth Point Framework. The research questions guiding the data analysis and comparison are:

1. Does the data show relevant differences between the performances of the Australian and German children in the four whole number domains with respect to each year level and over time?
2. Do any observed differences reflect the different curricula or approaches to teaching whole number concepts and arithmetic in Australia and Germany?

Data Collection and Analysis

The whole number tasks in the Early Numeracy Interview (ENI) take between 20-30 minutes per child and for the studies described in this paper were administered by classroom teachers in Australia and by pre-service teachers in Germany, who all followed a detailed script. The classroom teachers and pre-service teachers were competent with using the interview and had participated in associated professional learning. Throughout the assessment interview process the interviewer continued with the next tasks in a domain for as long as a child was successful, according to the script. The processes for validating the growth points, the interview items and the comparative achievement of students are described in full in Clarke et al. (2002).

A critical role for the interviewer during the assessment was to listen and observe the children, noting their solutions, strategies and explanations for completing each task. These responses were noted in detail on a record sheet and next independently coded to:

- determine whether or not a response was correct;
- identify the strategy used to find a solution; and

- identify the growth point reached by a child overall in each domain.

The assessment data for both groups were analysed to determine children's whole number growth points, according to the ENRP framework (Clarke et al., 2002). The growth point data was then entered into an SPSS database for analysis. Of particular interest for this paper is comparing the distribution of growth points for the Australian and German children to identify any differences.

Results

Figures 1 and 2 show the children's growth point distributions for the four whole number domains across two years of schooling. The data was collected after the first and second years of school for the Germans and after the second and third year of school for the Australians.The children from both countries were approximately 7 years old for the first assessment and 8 years old for the second assessment. For the purpose of discussing the comparative data, and although the Australians were assessed at the start of Grade 2 and Grade 3, we refer to these assessment periods below as 'end' of Grade 1 and Grade 2.

Counting Knowledge and Skills

Figure 1 shows that both groups clearly develop their counting knowledge from Grade 1 to Grade 2. Although the spread of growth points is similar, the median growth point (GP) for the Germans at the end of Grade 1 is GP2 (count 20 items) but for the Australians is GP4 (skip count by 2s, 5s and 10s from zero). One year later the median growth point is GP4 for both groups, and both distributions are similar, except that 19% of the German children reached GP6. We wonder whether the German children's increase in counting knowledge was influenced by the curriculum focus on automatising the multiplication facts that are quite frequently presented in tables that emphasise the counting sequence.

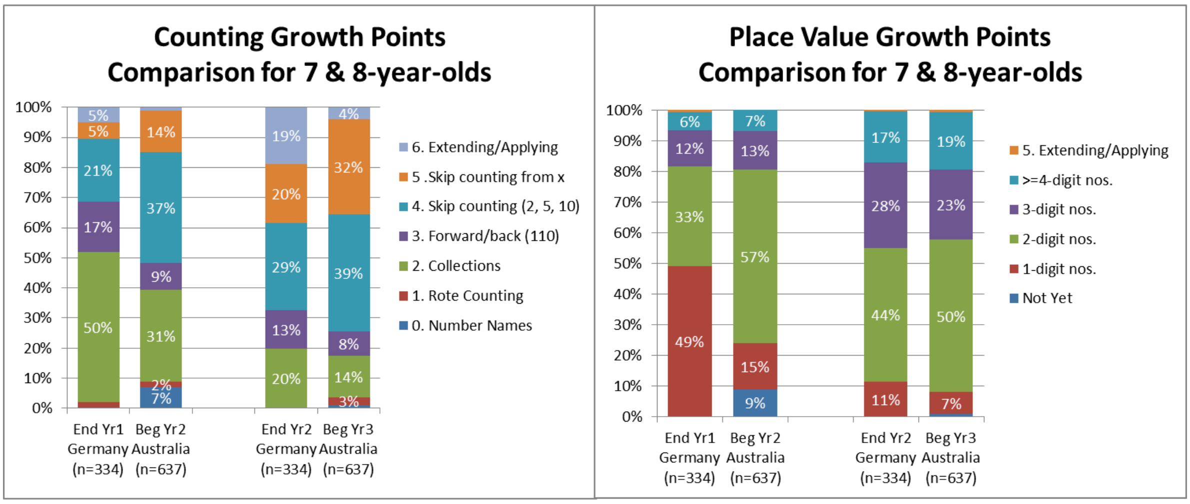

Figure 1. Counting and place value growth point distributions for German and Australian 7 year-old and 8 year-old children.

In order to gain insight about the longitudinal development of children's whole number knowledge, we traced children's knowledge in the Counting Domain for the two preceding years (i.e., immediately before starting Grade 1 (6-year-olds) and one year prior to that (5-year-olds; for details see Peter-Koop, Kollhoff, Gervasoni, & Parish, 2015). The growth point distributions for the 5 year-old children are fairly similar, with the major difference being the number of children able to count 20 teddies (GP2) or count forwards and backwards past 109 (GP3). One year later, as six year-olds, most German children increased one growth point, but a large group of Australian students increased two growth points (typically from GP0 to GP2 or GP2 to GP4). Nearly half of the German 6 year-old children who were attending kindergarten were not yet able to count 20 objects. This type of counting activity is a significant focus of the Australian primary school curriculum, but was not a focus in German kindergartens. Figure 1 highlights that the ability of German children to count 20 items changed dramatically after they began school.

Place Value Knowledge and Skills

While there is a large difference between the two groups concerning the percentage of students who understand 2-digit numbers (GP2) at the end of Grade 1, the spread of knowledge in both groups is almost the same after Grade 2. It is interesting to note that the curriculum in Germany in Grade 2 is limited to numbers up to 100, however 45% of the German children can deal with 3- and 4-digit numbers without that being taught explicitly in school mathematics. In contrast, the curriculum in Australia focuses on numbers to 1000 (GP3) and only 42% of children understand 3- and 4-digit numbers. This suggests that the German curriculum in this domain may be more suitably focused for Grade 1 and Grade 2.

Addition and Subtraction Strategies

The development of Australian and German children's addition and subtraction strategies appears to follow a different trajectory from Grade 1. By the end of Grade 1 nearly half of the Germans have replaced counting-based strategies with basic and derived strategies, compared with only one-quarter of the Australians. The predominant strategy (72 %) for the Australian children at the 'end' of Grade 2 is GP2 (count on) while 80% of German children use more advanced basic and derived strategies (GP5). The Grade 2 German curriculum focuses directly on the development of heuristic strategies and this appears to be reflected in the data.

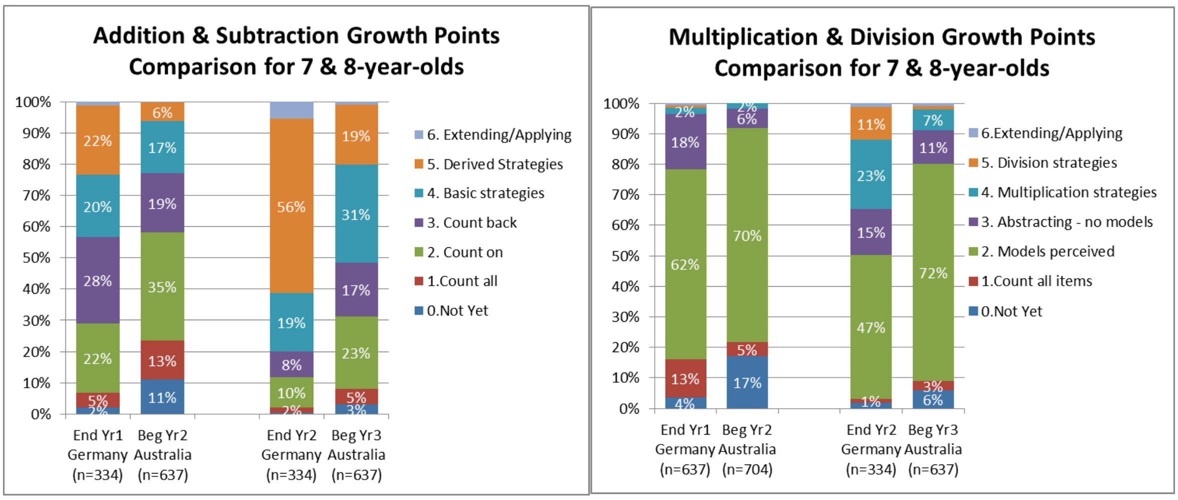

Figure 2. Addition and subtraction strategies and multiplication and division strategies growth point distributions for German and Australian 7-year-old and 8-year-old children.

The Grade 2 Australian curriculum also focuses on children solving simple problems using efficient mental and written strategies, but the heuristic strategies are not as clearly described. Of note is that the Grade 1 Australian curriculum focuses on representing and solving simple addition and subtraction problems using a range of strategies including counting on, partitioning and rearranging parts. This focus on representation and counting-on is in stark contrast to the German situation that emphasises heuristic strategies based on derived-facts to replace children's initial counting-based arithmetic strategies.

Multiplication and Division Strategies

At the end of Grade 1, the German and Australian growth point distributions for Multiplication and Division Strategies are quite similar except for the larger group of German children able to use the abstract strategy (GP3) in multiplicative situations. This confirms the trend that German children appear to develop more advanced arithmetic strategies in addition, subtraction, multiplication, and division strategies, while the Australian students reach higher growth points in Counting and Place Value at the end of Grade 1.

After Grade 2, 50% of the German children can solve multiplication and division problems without using any manipulatives (GP 3 and higher) compared with 20% of the Australian children. It is also interesting to note that there are hardly any German children on GP0 and GP1, compared with nearly 10% of the Australian children.

Discussion and Conclusion

The comparisons between the counting and whole number knowledge and skills of German and Australian children highlight some interesting differences. While it is important to note that the Australian children by the end of Grade 2 have spent an additional year at school and certainly show more elaborate competencies in the domains Counting and Place Value at the end of Grade 1, the German children catch up by the end of Grade 2.

While the German children in Grade 1 were more advanced in the Addition and Subtraction Strategies domain, with few differences between groups in Multiplication and Division, their competencies in these two domains significantly increase by the end of Grade 2. Of some concern is that almost 50% of Australian students are still using counting-based strategies (GP1-GP3) at the end of Grade 2 in Addition and Subtraction, compared with about 10% of German children. Typically, this persistent use of counting-based strategies is one criterion for identifying children who are mathematically vulnerable and who may benefit from an intervention program (Gervasoni, 2004).

We hypothesise that the noted differences between the German and Australian children's learning can partially be explained by different emphases in the two curricula at these levels, characterised by a strong focus on the learning and teaching of heuristic computation strategies in Germany, but of counting and place value concepts for numbers up to 1000 in Australia. Further, in Australia, children are more likely to be given manipulatives to help them model calculation strategies. Perhaps this reduces children's opportunities to replace counting strategies with more abstract heuristic strategies. It is likely that the Australian and German children's differing opportunities to formally explore whole number arithmetic at school matters, in at least the short term. It is also possible that the greater Australian curriculum focus on counting and place value for numbers to 1000 by the end of Grade 2 is misplaced. Indeed, the German children perform just as well in these domains, without this higher curriculum expectation.

The findings raise some interesting questions about the influence of curriculum documents and their emphases. Although comparisons between these Australian and

German children are mitigated by differences in cultural backgrounds, country, SES status, and school starting ages, some important differences have emerged. These are most likely explained by differences in curriculum emphases and possibly teaching strategies. Of interest is whether the noted trends and differences persist or diminish over time. This is a profitable area for further research.

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