Calculating for probability: "He koretake te rima" (Five is useless)

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In Māori medium schools, research that investigates children's mathematical computation with number and connections they might make to mathematical ideas in other strands is limited. This paper seeks to share ideas elicited in a task-based observation and interview with one child about the number ideas she utilises to solve a problem requiring probabilistic thinking. The explanations provided by the child demonstrate how early number and spatial patterns can impact on computation, ease of determining possible outcomes and assigning a numerical probability measure to an event.

Crites (1994) states that mathematical literacy is crucial for citizenship in today's society. An important component of mathematical literacy is number sense and the ability to apply it in a range of contexts is essential for coping confidently with the demands of an information-laden society.

Quantifying the probability of an event occurring is linked inextricably with number. Difficulties arise when measuring the chances of an event occurring for learners with limited number knowledge including that of fractional number (Langrell & Mooney, 2005). Reasoning in number and probability is vital for life beyond school (Gal, 2005; Jorgensen & Dole, 2011). For example, making decisions about whether or not a rain jacket is needed or where to invest for retirement are based on probability.

Since the introduction of the Numeracy Development Projects into New Zealand schools in 2000 there has been a major focus on the development of number strategies and knowledge for children learning mathematics. The national implementation of these projects meant that children beginning their formal mathematics education were expected to develop a strong base in numeracy as a foundation for learning a broad range of ideas in mathematics and statistics. Developing number sense and understanding connections between numbers, how they might be manipulated when calculating and noticing patterns with numbers is fundamental to effective mathematical thinking (Jorgensen & Dole, 2011).

Mulligan & Mitchelmore (2013) state that children's development in mathematics is heavily dependent on their awareness of pattern and structure "…mathematical pattern involves any predictable regularity involving number, space, or measure" (p.30). Mason, Stephens & Watson (2009) argue that children who have sound understanding about structure in mathematics not only recognise key ideas about properties in a relationship but they also have an awareness of how numbers might be manipulated.

Numeracy involves computation, interpretation, and making appropriate decisions with and about numbers to support mathematics learning. Learners need to be able to count, quantify, compute and manipulate numbers (Ministry of Education, 2007). By Year 8 Te Marautanga o Aotearoa (the national curriculum document for Māori medium schools in New Zealand) outlines the expectation that children should be working fluently with whole number and fractional number in a range of mathematical contexts including probability (Ministry of Education, 2008). Children are expected to have developed clear conceptual understandings of proportion and be able to utilise that knowledge to determine the likelihood of an event, for example, the probability of getting a 7 when throwing two die and adding the numbers together. Representing, interpreting and evaluating data to make

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informed decisions is dependent on having a sound understanding of whole number and fractional number, including percentages (Gal, 2004; 2005).

To develop probabilistic thinking learners will need to understand that the kind of thinking required with probability is different to that typically addressed in school mathematics (Langrell & Mooney, 2005). While exploring probability contexts requires the use of mathematics, this area of study is based on chance (Neill, 2010). Learners have to come to appreciate the idea that it is not possible to determine an individual outcome, but it is possible to predict the frequency of an outcome (NCTM, 2000). The reasoning that is required for such thinking develops over time and may be understood through a framework offered by Jones, Langrell, Thornton & Mogill (1997). The four levels of reasoning involve:

1. Subjective reasoning
2. Transition between subjective and naïve quantitative reasoning
3. The use of informal quantitative reasoning
4. The incorporation of numerical reasoning

The reasoning constructs may not be uniform and children may not follow an ordered progression of learning. Learners do however require frequent experience with actual experiments to develop their probabilistic thinking.

Determining the sample space is fundamental to aspects of probabilistic reasoning and requires the coordination of different cognitive skills. Children need to recognise that there may be different ways of obtaining an outcome. Research suggests that children should be presented with opportunities to actively participate and use physical material for exploring and investigating probability situations (Jorgensen & Dole, 2011; Neill, 2010). Being able to systematically and exhaustively generate possible outcomes is important to help consider the value of experimental probability and its relationship to theoretical probability (Barnes, 1998; Gal, 2004).

According to New Zealand curriculum documents assessment of children's learning in mathematics is to be based on multiple sources of evidence gathered over time (Ministry of Education, 2009). It is crucial that children are presented with opportunities to communicate their mathematical thinking, reasoning and solutions in a variety of ways (Hunter, 2009; Hunter, 2006). Making time for listening to children share their thinking can support assessment of learners' development in mathematics (Higgins & Weist, 2006; Reinhart, 2000). This practice includes listening to their thinking about probability (Barnes, 1998; Neill, 2010).

The ability to represent mathematical ideas using words, symbols or pictures supports children to communicate their thinking. Using different representations can encourage flexible thinking and provide teachers with artifacts constructed by learners, to support teacher judgments about children's learning (Suh, Johnston, Jamieson & Mills, 2008). Representations also serve as tools for justifying and making sense of mathematical ideas while supporting learners to construct knowledge (NCTM, 2000).

Children rate highly those opportunities for mathematics learning that incorporate physical movement and situations that include concrete materials (Attard, 2012). Ensuring that a task is accessible to everyone and that it facilitates reasoning and communicating, while incorporating multiple approaches, can also ensure that it is worthwhile (Breyfogle & Williams, 2008).

The purpose of this paper is to illustrate how one child's thinking in number supported her learning and understanding in a probability context.

Method

Data was collected as part of a larger study in a Year 7-8 class in a Māori medium setting (80-100% teaching and learning in the Māori language). The classroom teacher had base line information that indicated the class had gaps in knowledge with regard to the statistics component of assessment for Whanaketanga Pāngarau: He Aratohu mā te kaiako (Ministry of Education, 2010). She explained that due to a strong focus on number in recent years the children had limited exposure in their formal mathematics education programme to the development of probability ideas. She wanted to focus on helping children to work out the possible outcomes of an event and communicate their process and solutions appropriately and effectively.

This paper concerns one child's ideas for solving a probability task. The child had earlier completed a similar probability task involving the addition of numbers when two dice are thrown and finding the probability of different outcomes. The child was familiar with the researchers who had been into the classroom to observe and listen to them sharing ideas while completing the probability addition task.

This task (shared with the children in the Māori language) was about saving dolphins (six for each player) that had stranded themselves on some sandbanks (labelled 0-5 on same sheet for both players) and needed to be "saved". When players took turns to throw 2 dice and subtracted the numbers to find the difference, they could "save" one of their dolphins if it was on that numbered sandbank. Players have to decide before they start the game where to locate their six stranded dolphins (counters). The "winner" is the one who saves their six dolphins first.

The child for this case study was asked to play the game with one of the researchers in a space away from the other children. This situation was designed so that the researchers could record, listen and probe the child's thinking.

Results

Key ideas and recordings that emerged from this particular 12 year-old child Marino, about finding all possible outcomes and their significance for determining the likelihood of particular events are noted below.

1) Recognition of Spatial Patterns and Using that Knowledge

Upon throwing two dice and finding the difference between the numbers, Marino recorded the results in a tally chart. She noticed that some numbers appeared more frequently than others. When asked to explain why that might be occurring, Marino then recorded what she considered to be all possible ways of obtaining numbers zero to five as seen in *Figure 1.*

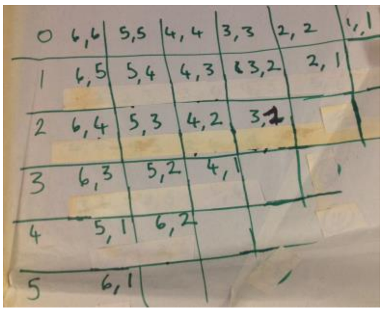

Figure 1. Marino's recording of different ways of getting outcomes 0-5

When asked how many possible outcomes there were in her diagram she said there were 21 because "5 and 6 =11 and 10 and 11 = make 21". She explained that the above recording resembled a triangle where the first 4 columns (beginning from the right hand side) had a pattern of 1, 2, 3 and 4 sets of numbers. She likened that pattern of numbers to the first 4 rows of another drawing (Figure 2) where the circles represented that same pattern of numbers. The spatial representation of the circles in each row i.e. 1, 2, 3, and 4 automatically indicated to her a total of 10. Continuation of the spatial pattern meant to her that the next two rows would equal 11. Therefore the total number of items had to be 21 because she stated that was a pattern that she had learned when she was much "younger".

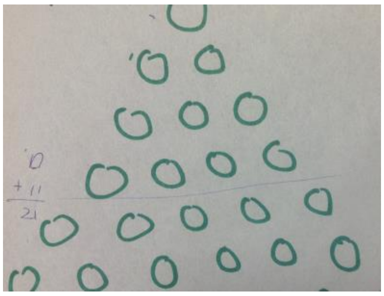

Figure 2. Drawing of circles to support addition of possible outcomes shown in Figure 1

2) Adding up all the Possible Outcomes

After being prompted to consider more possible combinations when throwing two dice, Marino then added to her recordings shown in Figure 1.

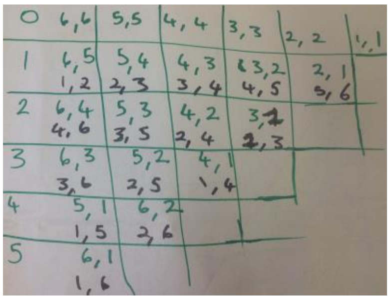

Figure 3. Total number of combinations noted by Marino when throwing two dice and subtracting the numbers

When asked how many outcomes there were when looking at her recording, Marino recognised and utilised another pattern. She started from the right hand side of Figure 3 and counted the number of combinations in each column and stated: "1, 3, 5, 7, 9, 11 is 36". Only when asked to explain her mental addition strategy to a very puzzled researcher, did she record "1, 3, 5, 7, 9, 11" on the sheet as shown in Figure 4. She then explained that "11 is 10 +1" (as shown to the right of 11), "…then you have 9+1 (left hand side column)) is 10 (right hand side column), then 7+3 is 10 and 5 is 36 altogether".

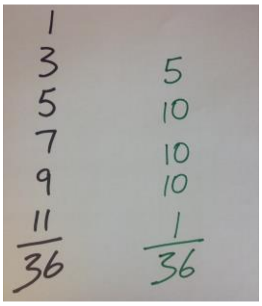

Figure 4. Numbers recorded by Marino when adding total number of combinations shown in Figure 3.

3) Alternative Presentation of Findings

When asked if she could present her findings in another way Marino was able to show quickly in an array the outcomes when throwing 2 dice and finding the difference as shown in the Figure 5.

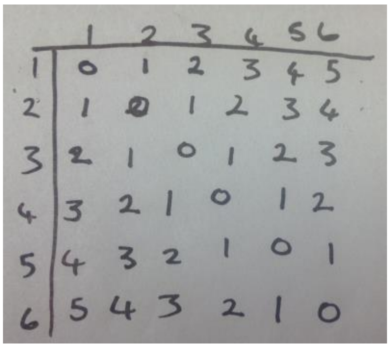

Figure 5. Array showing outcomes when throwing two dice and subtracting the numbers.

To find the "chances of getting a zero" Marino counted the number of zeros on the array and said six. She stated that there were 36 possible outcomes altogether on the array when throwing two dice and subtracting the numbers and the fraction for the chances of getting a zero was 6/36. When asked if 6/36 could be stated as another fraction, Marino said that it was the same a 1/6.

> "He ōrite ngā nama e rua nā te mea e ono ngā ono i roto i te 36"
> (The two numbers are the same because there are six sixes in 36).
>
> Marino stated "He koretake te rima…Ko te tahi te nama pai. Ko te tekau o te toru tekau mā ono ka puta mai te tahi".
> (…Five is useless…One is a good number. One will appear ten out of thirty-six times).

4) Making Connections between Fractions and Percentages

When asked what one sixth would be as a percentage, Marino showed in Figure 6 that it was about 16.5%. Marino explained that 1/3 of 100 is 33.5 If dividing that by 2 you get about 16.5. Therefore 1/6 is the same as 16.5 because a half of one-third is a sixth.

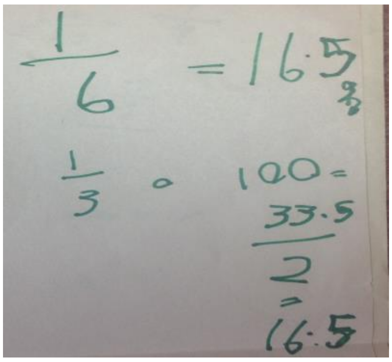

Figure 6. Marino's recording of the link between fractions and percentages.

Discussion

This student was able to recognise patterns and use her knowledge and strategy development in number to support her to calculate all possible outcomes and the chances of an event occurring in probability. For example, she was able to calculate how many combinations there were in the recording (Figure 1), based on connections she made with a spatial pattern. Her immediate recognition of the recorded sets of numbers in a triangle (Figure 1) and knowledge that the first four rows of the pattern (Figure 2) would equal 10 with the next two rows equalling eleven, made it easy for her to add the two totals together to quickly determine that there were 21 possible outcomes. This thinking indicated that the spatial pattern with perceived links to number was a mathematical idea that she had met in the past and had become an example of "predictable regularity" (Mulligan & Mitchelmore, 2013). She was familiar with the spatial pattern and its associated number sequence and knew from past experience that it would be 'true'.

The ease of calculation to 21 enabled Marino to then respond to prompting about the possibility of determining other combinations when throwing two dice. She was able to move forward and reason that there were other possible outcomes for this task and therefore reconsider her original total of outcomes. Accurate working out of the sample space is a crucial aspect of probabilistic thinking (Jorgensen & Dole, 2011).

Failing to understand all the possibilities that a particular context offers reinforces misconceptions about all possible outcomes. There was a need to support Marino to think more deeply about other combinations when throwing two dice so that she could appreciate that probability idea. A fundamental premise of supporting learners to develop probabilistic thinking is helping them to recognise and address misconceptions that might be held (Barnes, 1998; Neill, 2010).

Students with sound number sense can see numbers in a range of combinations and groupings (Jorgensen & Dole, 2011). When it came to calculating the total number of outcomes (36) Marino demonstrated use of a "Make 10" number strategy that she had learned earlier in her mathematics education. This part whole strategy is one that is promoted early for children to learn in their formal mathematics education in New Zealand. Her obvious awareness of the predictable regularity of the tens pattern and how numbers can be restructured to create it (Mason, Stephens & Watson, 2009; Mulligan & Mitchelmore, 2013), assisted her to apply that knowledge in a probability context and not be hampered by the mechanics of calculation.

The systematic recorded representation that Marino presented in Figure 5 showed the arithmetical difference that results when throwing two dice. The picture meant that she could easily count or quantify the likelihood of any number in the zero to five range appearing in this context. She was able to link these numbers to the 36 possible outcomes and make a fraction. Children need to draw on fractional number knowledge when the context demands, so that the probability ideas can be realised.

"Six out of 36" is a number idea that children have to take further when developing ideas about probability. They need to make meaning of such fractions according to the probability context that has been presented. While Marino abandoned the 'dolphin scenario', the context of throwing two dice and subtracting the numbers remained. The 'best' theoretical outcome still had to be ascertained by comparing the various numerical probabilities of each. The uncertainty of particular outcomes needed to be quantified if the focus of the learning was to be about developing probabilistic numerical reasoning (Jones, Langrell, Thornton & Mogill, 1997).

Assigning a numerical probability measure to an event can be demonstrated in a number of ways. Marino showed that she was able to make connections between fractions and percentages and understood key ideas of equivalence. She understood the relative size of fractions; that a third is equivalent to two sixths and is therefore approximately 33.5%. She understood that a sixth as a percentage could be found by dividing 33.5% by two. The ability to make rapid connections between common fractions (1/3 = 2/6) and then between fractions and percentages allowed Marino to express the probability of an event occurring in a variety of ways as expected of Year 7-8 children in New Zealand (Ministry of Education, 2010).

Conclusion

Research suggests the importance of children having sound number ideas if they are to explore and develop appropriate quantitative probabilistic thinking. The task-based interview has provided some specific examples of instances where number sense proved critical for determining the chance of an event occurring. The ease of accessing and understanding the probability ideas was enhanced by a facility with number. Working with a task that was easily accessible, that encouraged the use of concrete materials and inherently provided opportunities for reasoning and communicating, supported engagement with the probability concepts. Being able to record and express significant ideas in a variety of ways indicated a security with two related but distinctly different ways of thinking. Despite limited formal development in probabilistic thinking, early development with numerical and spatial patterns provided a platform to support investigation of probability ideas. A limitation is that this paper examines just one rich example of how a child's robust in number can support learning and understanding in probability. There would be merit in examining further task-based observation and interview data with a wider sample of learners to make a stronger argument for the significance of number understanding in a probability context.

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