Teachers' Use of a Pedagogical Framework for Improvement in Mathematics Teaching: Case Studies from YuMi Deadly Maths

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This paper describes the pedagogical framework used by YuMi Deadly Maths, a school change process used to improve mathematics teaching and thus enhance employment and life chances for socially disadvantaged students. The framework, called the RAMR cycle, is capable of being used by mathematics teachers for planning and delivering lessons and units of work with minimal training and external support, as demonstrated by three case studies. These, and other cases, suggest that the YuMi Deadly Maths approach is an effective model for scaling up professional development programs where school participation is voluntary and costs have to be minimised.

The mission of the YuMi Deadly Centre (YDC) of the Queensland University of Technology (QUT) is to work with Indigenous and low socioeconomic status (SES) schools to enhance mathematics learning outcomes, close the gap between Indigenous and low SES and other schools, and improve employment and life chances of Indigenous and low SES students. Researchers at YDC have developed a pedagogical approach to the teaching of mathematics in both the primary and secondary years, called YuMi Deadly Maths (YDM) that embraces the big ideas of mathematics to achieve deep learning of powerful mathematics (YuMi Deadly Centre, 2014). It is used by YDC in professional development (PD) programs that seek to improve the capacity of teachers to equip every student with the mathematical knowledge needed to pursue post-school options in science, technology, engineering, mathematics (STEM), and business (e.g., Cooper, Nutchey, & Grant, 2013). Since its genesis in 2010, YDM has been introduced into over 200 schools.

This paper describes the pedagogical framework used by YDM and presents three instrumental case studies (Stake, 1995) about the use of that framework. It builds on a paper that describes the development of the YDM approach to mathematics and mathematics pedagogy and the processes followed by YDM to bring about school change (Cooper & Carter, 2016).

The YDM Approach

Mathematics and Mathematics Pedagogy

YDM is based on three ontological propositions about the nature of mathematics: (a) mathematics is a connected structure of ideas formed into a schema; (b) mathematics is a language that concisely describes real-life situations; and (c) mathematics is a tool for problem solving. It uses a social constructivist epistemology in which mathematical knowledge is seen as the social invention of humans, where the importance of culture and context in developing meaning is emphasised (Vygotsky, 1978). In the context of school mathematics, learning is the development by the student of a set of connected mathematical schemas (Piaget, 1977; Skemp, 1976), influenced by personal experiences and collaboration with more knowledgeable others (teachers), who guide the student in the process of acquiring and adapting schemas (Davydov, 1995; Jardine, 2006).

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As the initial work of YDM was with Indigenous and low SES schools, the social constructivist perspective led to an exploration of the connections between culture and mathematics. A pedagogical approach was sought that valued the cultural capital that students bring to the classroom and challenged the Eurocentric nature of Australian school mathematics. The result is a view of mathematics that: (a) starts with a person identifying a real-life problem to which they seek a solution; (b) creates an abstract representation of the problem using a range of mathematical symbols; (c) uses mathematics to explore and communicate particular attributes and behaviours relating to the situation; and (d) reflects and validates the mathematics back to reality to determine if it is worthwhile in the context of the original problem and applies, extends, and transfers the mathematics to other situations.

The YDM pedagogy sees the abstraction and reflection processes as creative acts, framed by the cultural perspective of the actor (Matthews, 2009). It is also based on the philosophy that deep learning of mathematics occurs when students have a structural, or relational, understanding (Skemp, 1976) of mathematics. It was also influenced by the seminal works of many others (e.g., Alexander & Murphy, 1998; Ashlock, Johnson, Wilson, & Jones, 1983; Baturo, Cooper, Doyle, & Grant, 2007; Bruner, 1966; Payne & Rathmell, 1975). This background, together with the view of mathematics and culture discussed earlier, led to the development of a pedagogical framework, or teaching cycle, that followed the sequence: reality, abstraction, mathematics, reflection, called the *RAMR cycle* (Cooper & Carter, 2016; YuMi Deadly Centre, 2014). It is summarised in Figure 1.

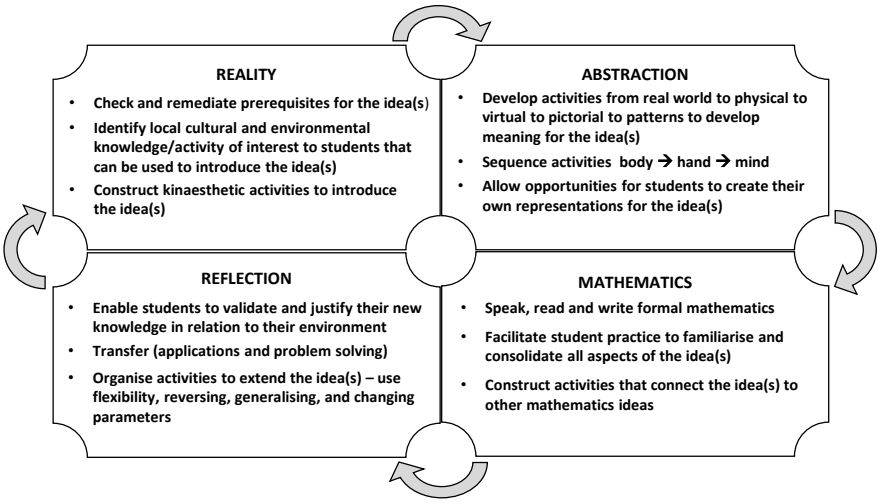

Figure 1. RAMR cycle.

The RAMR cycle begins and ends with the reality of the students' lives. It commences with the selection of a stimulus of interest or relevance to the students. Students explore the situation with kinaesthetic activities using the whole body and then manipulatives to build mental representations of the mathematics idea(s). It then moves to consolidation, which is seen as making connections, and finally reflects back to the students' reality. The two core processes are abstraction and reflection, with reflection ensuring that the new mathematical idea(s) are extended as far as they can be through four generic actions of flexibility, reversing, generalising and changing parameters. The framework is not fixed, either in theory, or in most schools' practices. Teachers can use it as much for a unit plan as a lesson plan. They move back and forth between the parts of the cycle, and the extension (reflection) activities can be used throughout the cycle.

Professional Development (PD) and Change

A pedagogical approach that is easy to implement and results in positive student responses is crucial to persuading teachers of the value of the YDM approach. However, given the social-constructivist philosophy of YDM, we do not seek to instruct teachers in a textbook or formula for implementing YDM. We believe that teachers are in the best position to determine what works for their students and to select appropriate activities to complement their lesson plan. Consequently, teachers are encouraged to use the RAMR pedagogical framework to scaffold the development of their own lesson and unit plans. As the case studies in the second half of this paper show, the RAMR cycle provides teachers with the confidence to undertake their own planning using the YDM approach, with a minimum of external support.

The implementation of YDM is a combination of centrally organised PD, school visits by YDC researchers, teaching resources (such as: books describing the YDM approach; sample lesson plans; and suggested approaches to, and activities for, teaching particular concepts), a discussion board and blog for teachers to share ideas with each other, and informal ad hoc online and telephone contact with YDC researchers (see Cooper & Carter, 2016). Practical considerations (including funding limitations) prevent the delivery of the YDM PD sessions to every teacher of mathematics in a school. Consequently, a train-the-trainer approach is used, working in geographical clusters of 4 to 12 schools. Each school selects four staff to be trained (one curriculum leader and three mathematics teachers is recommended). YDC provides four, three-day blocks of training (PD1, PD2, PD3, and PD4) across two years for these teachers, and the principal or an administrator is encouraged to attend the first days of PD1 and PD3. Schools are asked to prepare a plan to enable the trainers to trial the YDM pedagogy themselves using an action-research approach and then to provide training in YDM to the other teachers of mathematics in their school. They are invited to report to YDC on their progress each semester and to consider sharing their experience with other teachers at an annual YDC Sharing Summit. To summarise, schools are asked to select four staff to be trainers, change agents and researchers, and to provide time and space for all other mathematics teachers to become involved.

The process of scaling up PD in this way relies on the transmission of information along a chain from the YDC researcher to the teacher attending the PD, to other teachers in their school, and finally to their students. Yet YDC controls only the first link in this transmission chain, that is, what is central, formal and planned. The subsequent links take place in the school, often in an informal and ad hoc way. Furthermore, it is the outcome of the third link (student improvement) that determines whether schools and teachers persevere with YDM. As the case studies in this paper show, our experience is that the YDM PD process works and teachers are able to use RAMR and successfully teach with limited in-school support.

Case Studies

Given that YDC relies on the scaling up approach, the question arises as to how teachers use the RAMR cycle for their lesson planning independently of YDC researchers. In other words, what occurs in the second and third links of the chain referred to in the previous paragraph that YDC does not control. The remainder of this paper presents three instrumental case studies (Stake, 1975) drawn from a YDC project. They are based on the reflections shared with YDC by three teachers involved in the project: Becky, Megan, and Jenny (pseudonyms). The teachers were from primary schools in a large provincial city in Queensland, although the YDM program applies to Years F to 9. All three teachers were in their first year of YDM

training, Becky having completed PD1, and Megan and Jenny having completed PD1 and PD2.

Becky's Experience

Becky is a teacher of Year 1. This example starts with her first attempt to plan a lesson using the RAMR cycle. The lesson content was telling the time to the hour, using digital and analogue clocks. She chose this content because it suited her teaching plan, not because it had been demonstrated in the YDM training. In fact, at the time of delivering this lesson, Becky had not yet attended the YDM PD on the topic of measurement.

Becky wrote in her reflective journal that "I initially found it difficult to use the RAMR planning sheet because I was unsure of how much detail to put in, what activities fitted where and time durations." However, she consulted a YDM-trained teacher from a nearby school who shared a planning tool with her. This gave Becky the confidence to incorporate the RAMR framework into her school's planning template, as shown in Figure 2, and to amend her plan.

<table>
<tr><th colspan="4">Year One Maths Plan</th></tr>
<tr><td>Maths Strand:
Measurement</td><td>Term: 2</td><td>Week: 8</td><td>Lessons: 6</td></tr>
<tr><td colspan="2">Year Level Content Descriptors:
Using units of measurement
Describe duration using months, weeks, days and hours.
They tell time to the half hour.</td><td colspan="2">Goal:
Students will read and label o'clock on analogue and digital clocks.</td></tr>
<tr><td colspan="2">Reality
(How it relates to real life)</td><td colspan="2">Abstraction
(Whole Body and Hands on Activities)</td></tr>
<tr><td colspan="2">- → times for activities at school at school
- → home time
- → favourite shows on tv – tv guide on pay tv, newspaper
- → catching the bus to get to school and home
- → movie times
- → list and find pictures of clocks in the school, home and community</td><td colspan="2">- → human clock
- → paper plate clock
- → use arms to show times
- → draw clocks (analogue and digital)
- → interactive clocks on whiteboard
- → clocks with different numbers</td></tr>
<tr><td colspan="2">Reflection
(How the students are going)</td><td colspan="2">Mathematics
(Mathematics needed to do the task)</td></tr>
<tr><td colspan="2">- → pre-test/post-test</td><td colspan="2">- → daily practice of o'clock (analogue, words and digital)
- → clocks with missing parts – children complete</td></tr>
</table>

Figure 2. Becky's lesson plan.

Becky noted an immediate improvement in the engagement of her students:

> I have 17 boys and 7 girls in my class and have found the engagement level during the body and hand lessons to be greatly improved. My boys like to "do" not listen so this way of teaching new concepts is well suited.

Becky's first attempt showed a misunderstanding of the reflection stage of the cycle. Pre-post (diagnostic) testing is intended to inform the teacher's planning and evaluation, rather than being a reflection activity for students. As she noted: "I am still learning how to do this properly but can definitely see a clearer way to plan and implement YuMi Deadly Maths in my class." With more experience of the RAMR cycle, Becky might have planned reflection activities such as interpreting analogue clocks with Roman numerals or where some numbers do not appear on the clock face and considering in general terms what it means if the minute hand is not at 12.

However, Becky's first attempt at using YDM resulted in a positive response from her students, encouraging her to continue. This is an important outcome as a lack of success in the

early stages of using YDM may result in teachers giving up. After trying the RAMR approach with some more lessons, Becky wrote:

> I am really impressed with the level of discussion I hear when we are working in groups now. They are using the terms we discuss and telling me when they have seen things in their world. They love doing the activities.

This case study demonstrated that it is possible for teachers to make an immediate difference using the RAMR cycle as a planning tool, even with minimal training.

Megan's Experience

Megan taught a Year 6/7 composite class (at the time, Year 7 was part of the primary years in Queensland). She described how she used the RAMR cycle to plan a unit on integers, summarised in Figure 3. Megan used data from a pre-test to determine what her students already knew about negative integers. This informed the reality stage of her plan. She also used this information in the reflection stage where students investigated unfamiliar uses of negative integers. Megan planned many body, hand, and mind activities in the abstraction stage (including some demonstrated in the PD sessions), before formalising ideas in the mathematics stage of the cycle. However, she noted:

> I suspect I moved the students into this stage too quickly. They were obviously still needing the body to help them understand. This dilemma is created by a crowded curriculum and the need to meet curriculum and assessment requirements. While I can see the enormous benefits of moving through the RAMR cycle as the students' needs dictate, this often does not fit with the assessment/teaching schedule.

Megan's comments about the crowded curriculum reflect those made by other teachers in the early stages of implementing YDM.

Megan called the RAMR cycle planning process "yummification". She concluded that YDM:

> is about respecting and using the students' realities and prior understandings and building on them so the ideas are understood and connected with other ideas and experiences within mathematics and the real world.

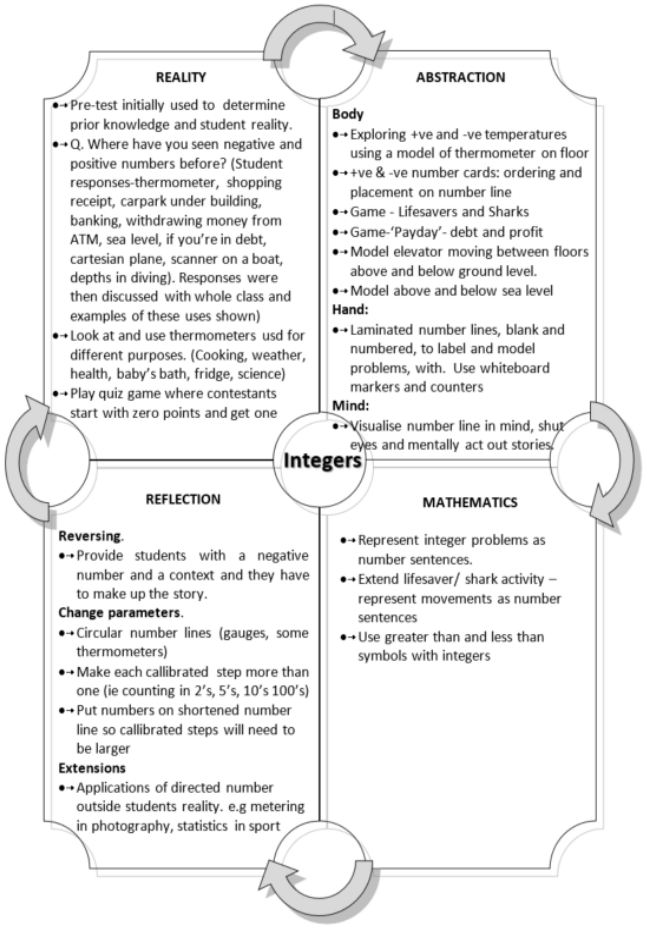

Figure 3. Megan's unit plan (summary).

Whilst only partially through her YDM training, Megan had had more practice than Becky in planning using the RAMR cycle. Her willingness to use the framework to plan an entire unit of work suggests that she was adequately prepared to assume a role as a YDM facilitator, trainer and leader in her school (the final link in the YDM transmission chain).

Jenny's Experience

Jenny was a teacher of Year 4 who shared her experience of using the RAMR cycle to plan and teach a unit on fractions. Her plan outline is shown in Figure 4. In her reflection journal Jenny provided more detail about most aspects of the plan in Figure 4. A pre-test helped Jenny to establish that "At the beginning of this unit the children really had a good concept of fractions as far as finding half, quarter etc of whole objects." She used a large floor mat divided into 36 squares (a resource with many applications recommended by YDM) for "body activities" that explored a variety of different fractions, followed by "hand activities" such as drawing pictures:

> We spend a lot of time drawing the pictures … and the children's confidence grew quickly. They were very keen to "showcase" their knowledge. This concept continued … as we drew pictures first, then looked for patterns in the numerator and denominator …. Simultaneously we were also making the connections between fractional numbers on a number line and discussing the infinite possibilities of numbers that can fit on a number line between two given numbers, such as 5 and 6, depending on how many parts were separating them.

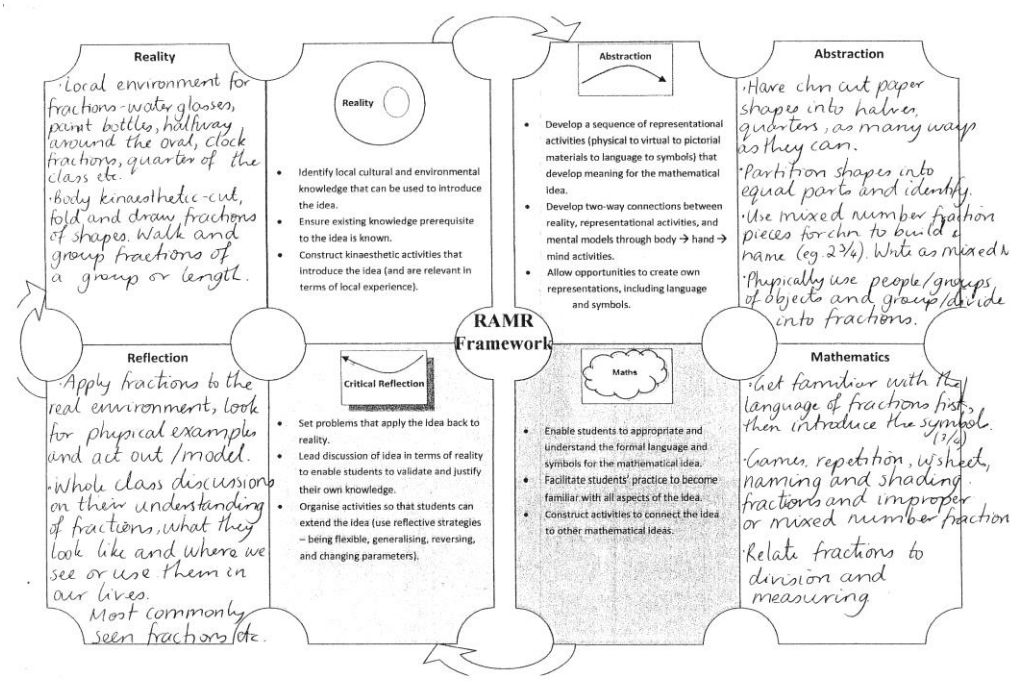

Figure 4. Jenny's outline.

In addition to teaching fractions, Jenny introduced her Year 4 students to another of the "big ideas" of mathematics – infiniteness. Making connections such as these is an important part of the YDM approach.

Like Megan, Jenny found that more time was needed: "We were going to spend 10 lessons on the fraction unit, but in the end required more like 15 before I felt the children were well equipped". However, Jenny also observed the benefits of the YDM approach:

> One of the most powerful influences that YuMi has given my class and I, is that now when I say we are about to do maths, the children are excited. They know it's not just pen and paper, that they will be manipulating objects, exploring and involved in some hands on activity. My attitude to teaching maths has also changed and I relish the new ways I can explore it with the children. This by far, has made the whole experience particularly valuable for my classroom practices and pedagogy.

Discussion and Conclusion

Becky, Megan and Jenny have all demonstrated that the RAMR cycle can be used for planning lessons and units of work with minimal training. Becky's experience, confirmed by other similar cases, showed that the RAMR cycle enables teachers to become autonomously active in the pedagogy after one three-day PD, even when the relevant topics have not yet been covered in the PD sessions. The strength of the YDM approach, incorporating the RAMR pedagogy, is that it works without having to provide external in-school resources. This is because the PD focuses on *how* to implement YDM as well as on *what* YDM is, and it provides enough theory and practice for teachers to succeed in planning and trialling the ideas without expert help.

Megan and Jenny both commented that YDM and RAMR required more time than they had initially planned. YDM learning is based on building knowledge schemas that enable later knowledge to be more easily learnt (Skemp, 1976). As most students do not have these schemas when YDM is introduced, their formation can result in topics taking more time. However, this time is made up in later years when the need to reteach prior learning is minimised. For example, by informally introducing the "big idea" of infiniteness in her Year 4 unit, Jenny has started to prepare the ground for the more formal introduction of this concept in the secondary years. Pre-empting activities of this kind save time later. It is for this reason that YDM is presented as a whole school program so that time invested in the early years can be caught up in later years.

Unlike other cases of scaling up PD reported in the literature (for example, Cobb, Jackson, Smith, Sorum, & Henrick, 2013), YDM is not mandated (or funded) by school or district systems. In most cases, the program relies on schools choosing to invest in their teachers of mathematics and financing the program from their own resources. This is problematic for many schools, especially those schools serving lower SES and geographically remote communities where staff turnover is high. Yet these are the schools with greatest need where we have observed that there are more inexperienced teachers and more than 80% of secondary teachers allocated to mathematics classes have had no formal training in teaching the subject. Even if external sources of funding cover the costs of delivering the YDM program (such as the recent PRIME Futures program that targets more than 60 schools with high levels of Indigenous enrolments throughout Australia, part of a larger Indigenous STEM Education project managed by CSIRO in partnership with the BHP Billiton Foundation), schools must finance the significant costs of releasing teachers to attend the PD sessions and for in-school training and planning. In these circumstances, it is important for YDC to minimise costs and to achieve success early in the PD program.

It is apparent from the cases presented in this paper that the YDM processes offer a promising model for scaling up PD programs where school participation is voluntary and costs have to be minimised. The provision of PD to geographic clusters of schools, the adoption of the RAMR pedagogical framework, and the use of an action research approach to in-school training activities appears to overcome many of the problems associated with limited school and teacher contact. Our findings are that schools become active in YDM after 12 days of PD spread over two years provided that there is: (a) a school plan for implementing YDM, supported by the principal; (b) reasonable staff continuity; and (c) at least one curriculum administrator and two or three teachers with enthusiasm for the program.

Another significant finding is that YDM methods improve mathematics teaching and learning for all students. Although the development of YDM sought to value the cultural capital that Indigenous and low SES students bring to the classroom and to challenge the Eurocentric nature of Australian school mathematics, in all of the examples in this paper

students were treated equally and the benefits of the YDM approach were shared by all. It appears that pedagogies developed to overcome social disadvantage can improve outcomes for *all* students.

These conclusions are based on more than the three cases presented in this paper. With more time and space there were many other similar cases that could have been included. At present, YDM training is occurring in over 30 schools across Queensland and over 200 schools have experienced the two-year program in the past six years. In most of these situations, lack of funding has limited YDM to the provision of training and resources with no in-school follow up. Yet, approximately 70% of these schools are still using YDM pedagogy. Many of them have made dramatic and positive changes in their mathematics teaching.

Acknowledgement

Since the early 2000s, many researchers have been involved in or related to the work that has led to YDM, including Annette Baturo, Bronwyn Ewing, Edlyn Grant, Gillian Kidman, Chris Matthews, David Nutchey, Grace Sarra, and Elizabeth Warren, as have administrators and practitioners including Kim Alexander, Robyn Anderson, Jan Cavanagh, Charlotte Cottier, Gillian Farrington, and Jim Lowe. This paper could not have been written without their contributions.

References

Alexander, P. A., & Murphy, P. K. (1998). The research base for APA's learner-centered psychological principles. In N. M. Lambert & B. L. McCombs (Eds.), *How students learn: Reforming schools through learner-centered education* (pp. 25–60). Washington, DC: American Psychological Association.

Ashlock, R. B., Johnson, M. L., Wilson, J. W., & Jones, W. L. (1983). *Guiding each child's learning of mathematics: A diagnostic approach to instruction*. Columbus, OH: Charles E. Merrill Publishing Co.

Baturo, A. R., Cooper, T. J., Doyle, K., & Grant, E. (2007). Using three levels in design of effective teacher-education tasks: The case of promoting conflicts with intuitive understandings in probability. *Journal of Mathematics Teacher Education, 10*(4–6), 251–259.

Bruner, J. S. (1966). *Toward a theory of instruction.* Cambridge, MA: Harvard University Press.

Cooper, T., & Carter, M. (2016). *Large-scale professional development towards emancipatory mathematics: The genesis of YuMi Deadly Maths*. Paper submitted to a conference.

Cooper, T., Nutchey, D., & Grant, E. (2013). Accelerating the mathematics learning of low socio-economic status junior secondary students: An early report. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Mathematics education: Yesterday, today and tomorrow* (Proceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia, Melbourne, pp. 202–209). Melbourne: MERGA.

Cobb, P., Jackson, K., Smith, T., Sorum, M., & Henrick, E. (2013). Design research with educational systems: Investigating and supporting improvements in the quality of mathematics teaching and learning at scale. *Yearbook of the National Society for the Study of Education, 112*(2), 320–349.

Davydov, V. (1995). The influence of L. S. Vygotsky on education theory, research, and practice. *Educational Researcher, 24*(3), 12–21.

Jardine, D. W. (2006). *Piaget and education primer.* New York: Peter Lang Publishing, Inc.

Matthews, C. (2009). Stories and symbols: Maths as storytelling. In *Professional Voice 6*(3), 45–50. Retrieved from https://issuu.com/aeu-vic/docs/pv_6_3/1?e=1350839/5379655

Payne, J., & Rathmell, E. C. (1975). Mathematics learning in early childhood: Number and numeration National Council of Teachers of Mathematics Yearbook (Vol. 37, pp. 125–160).

Piaget, J. W. F. (1977). *The essential Piaget*. New York: Basic Books Inc.

Skemp, R. (1976). Relational understanding and instrumental understanding. *Mathematics teaching, 77*, 20–26.

Stake, R. E. (1995). *The art of case study research*. Thousand Oaks, CA: Sage Publications Inc.

Vygotsky, L. S. (1978). Interaction between learning and development. (M. Lopez-Morillas, Trans.). In M. Cole, V. John-Steiner, S. Scribner, & E. Souberman (Eds.), *The mind in society: Development of higher psychological processes* (pp. 79–91). Cambridge, MA: Harvard University Press.

YuMi Deadly Centre. (2014). *YuMi Deadly Maths: Overview – Philosophy, pedagogy, change and culture*. (Unpublished booklet supporting a PD program). Brisbane: QUT.